

# Exam Winter Semester 2022

## Student Group

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## Table of Contents

- Exam Winter Semester 2022** ..... 3
- Additional permitted Aids ..... 3
- Hits ..... 3
- Tasks ..... 3
- Exercise E4 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022) ..... 3
- Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022) ..... 3
- Exercise E1 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022) ..... 4
- Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022) ..... 4
- Exercise E6 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022) ..... 5
- Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022) ..... 7
- Exercise E1 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022) ..... 9
- Exercise E3 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022) ..... 13
- Exercise E5 Charging Capacitors (written test, approx. 16 % of a 60-minute written test, WS2022) ..... 17
- Exercise E4 Charging Capacitors (written test, approx. 16 % of a 60-minute written test, WS2022) ..... 18
- Exercise E2 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022) ..... 20
- Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022) ..... 20
- Exercise E3 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022) ..... 20

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written test, WS2022) .....	21
Exercise E6 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022) .....	21
Exercise E1 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022) .....	22
Exercise E7 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022) .....	25

# Exam Winter Semester 2022

## Additional permitted Aids

- non-programmable calculator,
- formulary (2 DIN A4 pages)

## Hits

- The duration of the exam is 60 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

## Tasks

### Exercise E4 Resistance of a Wire by Resistivity

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. Heating elements are used to heat the oven with a temperature of  $180^\circ\text{C}$ . The electric power dissipation (= heat flow) of  $P=40\text{ W}$  is necessary.

Calculate the current  $I$  needed to operate it.

The Nichrome wire has a resistivity of  $1.10 \cdot 10^{-6}\ \Omega\text{m}$ .

The heating element is  $3\text{ m}$  long and has a diameter of  $3.57\text{ mm}$ .

∴ Calculate the resistance  $R$  of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \\ \sqrt{\frac{P}{R}} &= \sqrt{\frac{40\text{ W}}{0.33\ \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \& \quad | \text{with } A = r^2 \cdot \pi = \\ \frac{1}{4} d^2 \cdot \pi \quad \& \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ 1.10 \cdot 10^{-6}\ \Omega\text{m} \cdot \frac{4 \cdot 3\text{ m}}{(3.57 \cdot 10^{-3}\text{ m})^2 \cdot \pi} \end{aligned}$$

### Exercise E1 Resistance of a Wire by Resistivity

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. Heating elements are used to heat the oven with a temperature of  $180^\circ\text{C}$ . The electric

**Result** power dissipation (= heat flow) of  $P=40 \text{ W}$  is necessary.  
 Calculate the current  $I$  needed to operate it for heating elements.  
 The Nichrome wire has a resistivity of  $1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$ .  
 The heating element is  $3 \text{ m}$  long and has a diameter of  $3.57 \text{ mm}$ .  
 Solution:  $R = 10^{-3} \text{ } \Omega$  (Wrong calculation)

**Solution**

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}}$$

$$R = \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \quad || \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad || \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi}$$

**Exercise E1 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator explains a temperature sensitive component. The thermistor has a resistance of  $10 \text{ k} \Omega$  at  $25^\circ \text{C}$ . Its temperature coefficients are:  $\alpha=0.01 \text{ } \frac{1}{\text{K}}$  and  $\beta=71 \cdot 10^{-6} \text{ } \frac{1}{\text{K}^2}$

**Result** The temperature inside the refrigeration system can reach down to  $-40^\circ \text{C}$ .  
 Calculate the resistance of the thermistor at  $-40^\circ \text{C}$ .  
 Resistance of the resistor  $R$  depends on the current and generated heat. Therefore, a solution is to use the heat flow up the refrigeration system.  
 Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.  

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad | \quad \Delta T = T_{\text{end}} - T_{\text{start}} \quad || \quad R = 10 \text{ k} \Omega \cdot \left(1 + 0.01 \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2\right)$$

**Exercise E2 Temperature-dependent Resistance**

**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A resistor exhibits a temperature coefficient of resistance of  $\alpha = 0.01 \text{ K}^{-1}$  and a nominal resistance of  $R_0 = 10 \text{ k}\Omega$  at  $T_0 = 25^\circ\text{C}$ . Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .  
 Result: The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

Resistance transfer resistor  $R$  depends on the circuit and generated heat. Therefore, a solution is to use a heat sink up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$R = 10 \text{ k}\Omega \cdot (1 + 0.01 \cdot (-40 - 25) + 71 \cdot 10^{-6} \cdot (-40 - 25)^2)$$

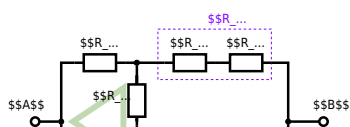
**Exercise E6 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall hold:  $R_1 = 200 \text{ }\Omega$ ,  $R_2 = R_3 = 100 \text{ }\Omega$ ,  $R_4 = 100 \text{ }\Omega$  and the voltage  $U = 10 \text{ V}$ .  
 Result:  $R_B = 132.8 \text{ }\Omega$ .

Solution

$$R_{\text{B}} = 132.8 \text{ }\Omega$$

Now a wye-delta transformation is necessary.

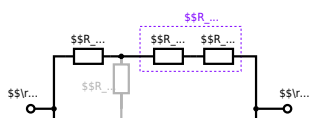


Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as: 
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series: 
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{eq}$  between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel (500 \Omega) \parallel (200 \Omega) \parallel (500 \Omega \cdot 200 \Omega) / (500 \Omega + 200 \Omega)$$

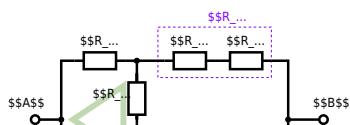
**Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved with  $R_1 = 200 \Omega$ ,  $R_2 = R_3 = 100 \Omega$  and the source  $B = 15 \text{ V}$ .  
 Result given:  $R_{\text{eq}} = 132.8 \Omega$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.

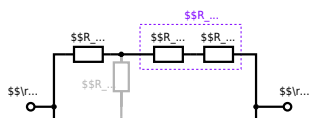


Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as: 
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series: 
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{eq}$  between A and B.

Solution



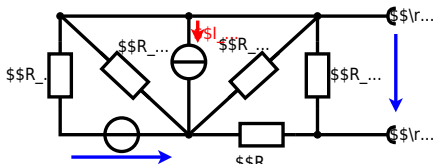
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E1 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

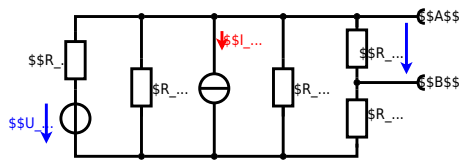
$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



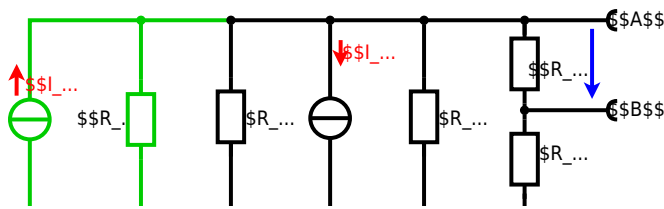
Calculated the internal resistance  $R_{\text{int}}$  and the source voltage  $U_{\text{oc}}$  of an equivalent linear voltage source on the connectors  $\text{A}$  and  $\text{B}$ .  $R_1=5.0 \text{ }\Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3= 10 \text{ }\Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \text{ }\Omega$ ,  $R_6=7.5 \text{ }\Omega$ ,  $R_7=15 \text{ }\Omega$  Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ : 
$$R_{135} = R_1 || R_3 || R_5$$
 
$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following: 
$$U_{24} = I_{24} \cdot R_5$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - I_4 \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - \left( \frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0\Omega$ , so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

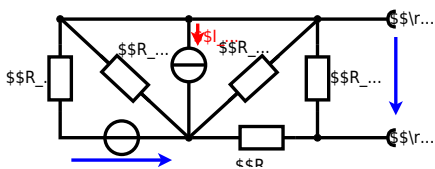
with  $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$ :

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \parallel R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

### Exercise E3 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.  
Result

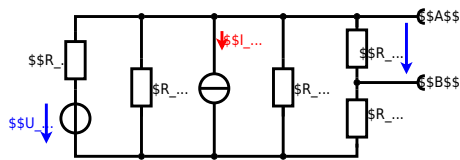
$$U_s = U_{AB} = 4.5\text{V} \parallel R_i = R_{AB} = 6\Omega$$



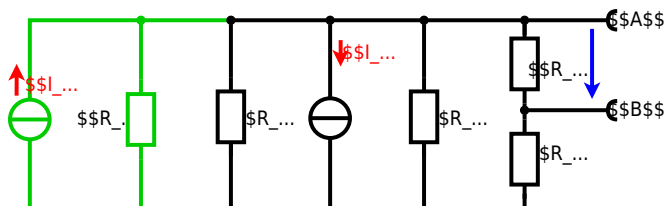
Calculate the internal resistance  $R_{in}$  and the source voltage  $U_{s}$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3=10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$ . Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :  

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - (U_2 \cdot \frac{R_1}{R_1 + R_3 + R_5} - I_4) \cdot R_1 || R_3 || R_5$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \cdot 15 \Omega \cdot 2.5 \Omega \cdot \frac{1}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

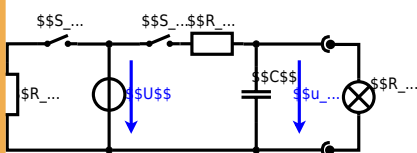
**Exercise E5 Charging Capacitors**  
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a RC circuit consisting of a DC voltage source  $U$ , a resistor  $R_1$ , a resistor  $R_2$ , and a capacitor  $C$ . The switch  $S_1$  is initially open. The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

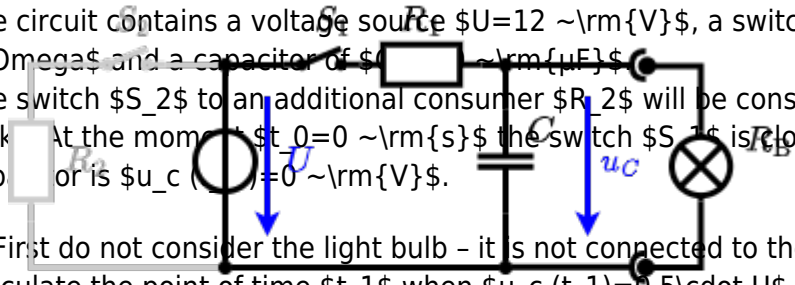
**Result:** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

**Solution:** The ideal voltage source  $U_{eq}$  is given by  $U_{eq} = U \cdot \frac{R_2}{R_1 + R_2}$  and the internal resistance  $R_{eq} = R_1 || R_2$ .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .

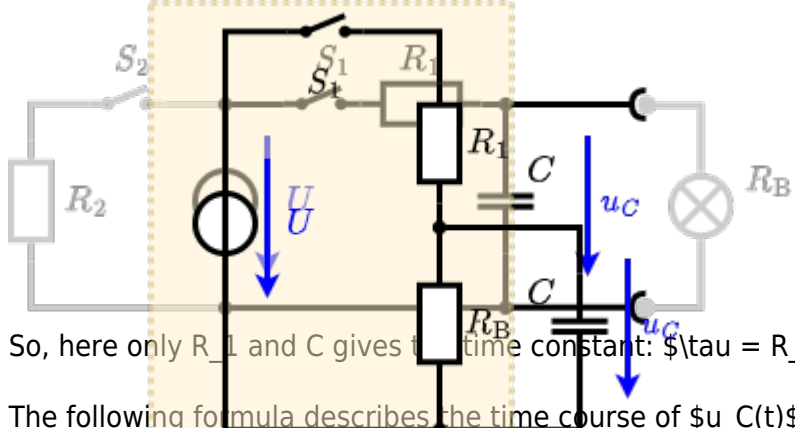


The circuit contains a voltage source  $U=12\text{ V}$ , a switch  $S_1$ , a resistor of  $R_1=20\text{ }\Omega$  and a capacitor of  $C=100\text{ }\mu\text{F}$ . The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first task. At the moment  $t_0=0\text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0)=0\text{ V}$ .



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1)=0.5 \cdot U$ .

Solution



So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1)=0.5 \cdot U$ :  

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5$ . An equivalent linear voltage source can be given with  $U_s = U \cdot \frac{R_B}{R_1 + R_B}$  and  $R_i = R_1 \parallel R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ . The internal resistance is given by substituting the ideal voltage source with its resistance ( $R_i = 0\text{ }\Omega$ , short-circuit).  

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1\text{ ms}/(10\text{ }\Omega \cdot 100\text{ }\mu\text{F})})$$

**Exercise E4 Charging Capacitors**  
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (as shown in the figure) consists of a DC voltage source  $U=6\text{ V}$ , a resistor  $R_1=20\text{ }\Omega$ , a capacitor  $C=20\text{ }\mu\text{F}$  and a light bulb  $R_B=20\text{ }\Omega$ . The switch  $S_1$  is open. The voltage across the capacitor is again  $0\text{ V}$  at the moment  $t_0=0\text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2=1\text{ ms}$  after closing the switch.

**Solution** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_B$ .  

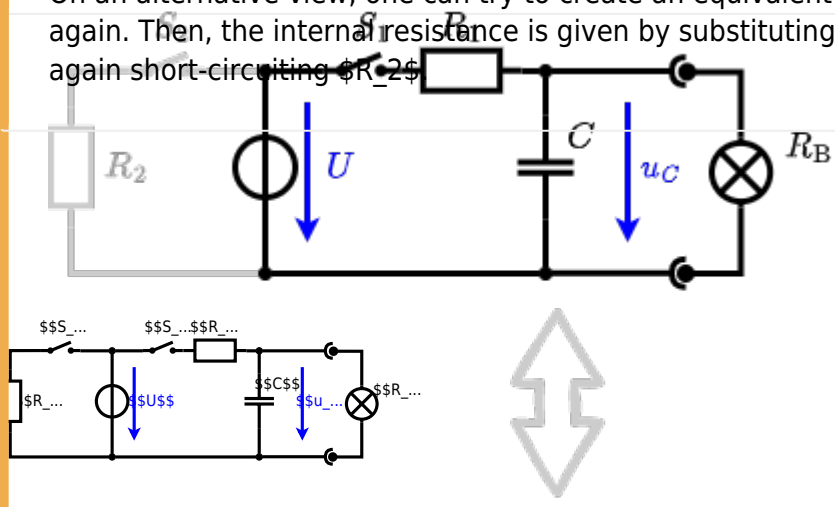
$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 3\text{ V}$$

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

Solution

The ideal voltage source is  $U = 12 \text{ V}$ . The internal resistance is  $R_1 = 20 \text{ }\Omega$ . The voltage across the capacitor is  $u_C$ . The voltage across the light bulb is  $u_B$ . The voltage across the resistor  $R_2$  is  $u_{R_2}$ . The voltage across the capacitor is  $u_C$ . The voltage across the light bulb is  $u_B$ . The voltage across the resistor  $R_2$  is  $u_{R_2}$ .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .

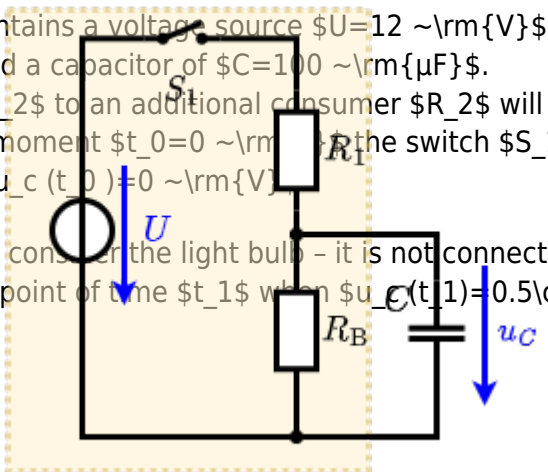


The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ }\Omega$  and a capacitor of  $C = 100 \text{ }\mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first asks. At the moment  $t_0 = 0$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_C(t_0) = 0$ .

First do not consider the light bulb - it is not connected to the RC circuit.

Calculate the point of time  $t_1$  when  $u_C(t_1) = 0.5 \cdot U$ .



Solution

An equivalent linear voltage source can be given with  $U$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ . The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \text{ }\Omega$ , short-circuit).  $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$ .

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2/(10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_C(t)$  which has to be  $u_C(t_1) = 0.5 \cdot U$ :  $u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$ . It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

### Exercise E2 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source  $\underline{U} = 50 \angle 0^\circ \text{ V}$  and the admittance  $\underline{Y} = 0.24 \text{ S}$  are both in the components. ( $\$R\$$  and  $\$X_L\$$ ) shall be given.

After analysis, the full width dimensioned complex impedance  $\underline{Z}$  shall be extracted and the magnitude  $|Z|$  in phase (in  $^\circ$ ) shall be given.  $\underline{Z} = (2 + j4) \Omega$

Solution  
.. Calculation of physical values of the two components.  
Solution  $\varphi_i = \arctan\left(\frac{-4}{2}\right) = -63.4^\circ$

Solution

$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{2 + j4} = 11.9 \angle -63.4^\circ \text{ A}$   
The current  $I$  has a magnitude of  $11.9 \text{ A}$  and a phase angle of  $-63.4^\circ$ .

resulting impedance  $\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle 0^\circ}{11.9 \angle -63.4^\circ} = 4.2 \angle 63.4^\circ \Omega$   
Therefore, the component  $R$  is  $4.2 \cos(63.4^\circ) = 1.6 \Omega$  and the component  $X_L$  is  $4.2 \sin(63.4^\circ) = 3.6 \Omega$ .

impedance  $\underline{Z} = R + jX_L = 1.6 + j3.6 \Omega$   
 $\underline{Z} = 4 \angle 60.9^\circ \Omega$

With the complex part  $\underline{Z} = 4 \angle 60.9^\circ \Omega$  and  $\underline{U} = 50 \angle 0^\circ \text{ V}$   
 $\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{4 \angle 60.9^\circ} = 12.5 \angle -60.9^\circ \text{ A}$

The phase  $\varphi_i$  shall be calculated as  $\varphi_i = \arctan\left(\frac{-3.6}{1.6}\right) = -66.1^\circ$

### Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source  $\underline{U} = 50 \angle 0^\circ \text{ V}$  and the admittance  $\underline{Y} = 0.24 \text{ S}$  are both in the components. ( $\$R\$$  and  $\$X_L\$$ ) shall be given.

After analysis, the full width dimensioned complex impedance  $\underline{Z}$  shall be extracted and the magnitude  $|Z|$  in phase (in  $^\circ$ ) shall be given.  $\underline{Z} = (2 + j4) \Omega$

Solution  
.. Calculation of physical values of the two components.  
Solution  $\varphi_i = \arctan\left(\frac{-4}{2}\right) = -63.4^\circ$

Solution

$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{2 + j4} = 11.9 \angle -63.4^\circ \text{ A}$   
The current  $I$  has a magnitude of  $11.9 \text{ A}$  and a phase angle of  $-63.4^\circ$ .

resulting impedance  $\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle 0^\circ}{11.9 \angle -63.4^\circ} = 4.2 \angle 63.4^\circ \Omega$   
Therefore, the component  $R$  is  $4.2 \cos(63.4^\circ) = 1.6 \Omega$  and the component  $X_L$  is  $4.2 \sin(63.4^\circ) = 3.6 \Omega$ .

The absolute value of the impedance is  $Z = \sqrt{R^2 + (X_L - X_C)^2}$  with  $R = 5 \Omega$ ,  $X_L = \omega L = 2\pi \cdot 50 \text{ Hz} \cdot 0.24 \text{ mH} = 0.754 \Omega$  and  $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \cdot 50 \text{ Hz} \cdot 4.68 \text{ nF}} = 680 \Omega$ .  
 The phase  $\phi$  is given by  $\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{0.754 - 680}{5}\right) \approx -89.9^\circ$ .

**Exercise E3 Impedances at different Frequencies**  
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with  $R_1 = 1 \text{ k}\Omega$ ,  $R_2 = 10 \text{ k}\Omega$ ,  $L = 4.7 \mu\text{H}$  and  $C = 40 \text{ nF}$  at  $f = 4 \text{ MHz}$ .  
 Result:  $Z = 10.0 \text{ k}\Omega$ ,  $\phi = 90^\circ$ .  
 A resistor  $R_1$  shall have the same absolute value of the impedance as a capacitor  $C_1 = 40 \text{ nF}$  at  $f_1 = 4 \text{ MHz}$ .

Solution  
 $Z = R_1 + jX_L - jX_C = 1000 + j(2\pi \cdot 4 \cdot 10^6 \cdot 4.7 \cdot 10^{-6}) - j\frac{1}{2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}}$   
 $Z = 1000 + j(117.6 - 994.7) = 1000 - j877.1 \Omega$   
 $|Z| = \sqrt{1000^2 + 877.1^2} = 1343.7 \Omega$   
 $\phi = \arctan\left(\frac{-877.1}{1000}\right) = -40.8^\circ$   
 A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $L$  combined is given by  $Z_{RL} = R + jX_L$   
 Parallel circuit means that the voltage is the same on  $R_2$  and  $C_1$ .  
 $Z_{RC} = \frac{R_2 \cdot (-jX_C)}{R_2 - jX_C} = \frac{10000 \cdot (-j994.7)}{10000 - j994.7} = \frac{-j9947000}{10000 - j994.7}$   
 $Z_{RC} = \frac{-j9947000 \cdot (10000 + j994.7)}{(10000 - j994.7)(10000 + j994.7)} = \frac{-j9947000 \cdot (10000 + j994.7)}{10000^2 + 994.7^2}$   
 $Z_{RC} = \frac{-j9947000 \cdot (10000 + j994.7)}{100000000 + 989410.09} = \frac{-j9947000 \cdot (10000 + j994.7)}{100989410.09}$   
 $Z_{RC} = \frac{-j9947000 \cdot 10000 - 9947000 \cdot 994.7}{100989410.09} = \frac{-99470000000 - 9894100900}{100989410.09} = \frac{-109364100900}{100989410.09} = -1082.8 \Omega$   
 $Z_{total} = Z_{RL} + Z_{RC} = 1000 - j877.1 - 1082.8 = -82.8 - j877.1 \Omega$   
 $|Z_{total}| = \sqrt{82.8^2 + 877.1^2} = 882.2 \Omega$   
 $\phi = \arctan\left(\frac{-877.1}{-82.8}\right) = 84.2^\circ$   
 Therefore, the resulting current of the parallel circuit is given as:  
 $I = \frac{U}{|Z_{total}|} = \frac{10 \text{ V}}{882.2 \Omega} = 11.33 \text{ mA}$   
 This current is shared between  $R_2$  and  $C_1$ .  
 $I_{R2} = I \cdot \frac{|Z_C|}{|Z_{RC}|} = 11.33 \text{ mA} \cdot \frac{994.7}{1082.8} = 10.4 \text{ mA}$   
 $I_{C1} = I \cdot \frac{|Z_R|}{|Z_{RC}|} = 11.33 \text{ mA} \cdot \frac{10000}{1082.8} = 104 \text{ mA}$   
 Back to the first formula:  $R_3 \cdot I_{R3} = X_{C3} \cdot I_{C3}$   
 $R_3 \cdot I_{R3} = \frac{1}{\omega C_3} \cdot I_{C3}$   
 $R_3 \cdot I_{R3} = \frac{1}{2\pi \cdot f \cdot C_3} \cdot I_{C3}$   
 $R_3 \cdot I_{R3} = \frac{1}{2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}} \cdot I_{C3}$   
 $R_3 \cdot I_{R3} = \frac{1}{100.53} \cdot I_{C3}$   
 $R_3 \cdot I_{R3} = 0.009947 \cdot I_{C3}$

**Exercise E6 Impedances at different Frequencies**  
 (written test, approx. 18 % of a 60-minute written test, WS2022)

**Resistor**  $R_1$  shall have the same absolute value of the impedance as a capacitor  $C_1 = 40 \text{ nF}$  at  $f_1 = 4 \text{ MHz}$ .

**Solution**

$$R_1 = 1.00 \text{ } \Omega$$

$$R_2 = 10.0 \text{ } \Omega$$

A series circuit means that the current is constant on every component.

The equivalent impedance for  $R$  and  $L$  combined is given by

$$Z_{RL} = R + j\omega L$$

Parallel circuit means that the voltage is the same on  $R_2$  and  $C_1$

$$Z_{RC} = \frac{R_2 \cdot (-j/\omega C_1)}{R_2 - j/\omega C_1}$$

Since  $Z_{RC}$  is perpendicular to  $Z_{RL}$ , the resulting current of the parallel circuit is given as:

$$I_{3R} = I_{2R} + I_{3C}$$

This can be simplified to:

$$I_{3R} = \frac{U}{\sqrt{R_2^2 + (X_{L2} - X_{C1})^2}}$$

Back to the first formula:

$$R_3 \cdot I_{3R} = X_{C3} \cdot I_{3R}$$

$$R_3 = \frac{X_{C3} \cdot I_{3R}}{I_{3R}}$$

$$R_3 = \frac{1}{2\pi \cdot f \cdot C_3} \cdot \frac{1}{\sqrt{R_2^2 + (X_{L2} - X_{C1})^2}}$$

**Exercise E1 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)**

**1. Calculate the current  $i(t)$  through the resistor  $R$  in the circuit shown in the figure. The voltage source is  $u(t) = 3.0 \text{ V} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$ .**

**Solution**

Result

$$Z = 48.2 \text{ } \Omega \quad Z_C = 19.8 \text{ } \Omega$$

Draw the circuit diagram of the given circuit with all components, voltages, and currents.

$$Z = \frac{U}{I} \quad I = \frac{U}{Z}$$

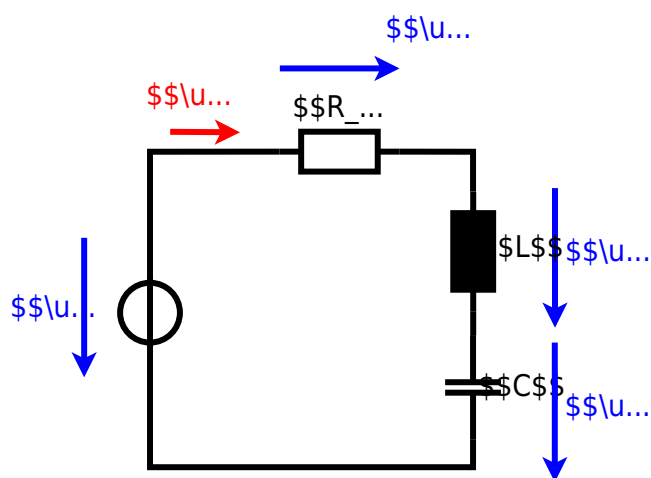
$$Z_C = \frac{1}{2\pi \cdot f \cdot C}$$

Result

$$I = \frac{3.0 \text{ V}}{\sqrt{48.2^2 + 19.8^2}} = 0.107 \text{ A} = 107 \text{ mA}$$

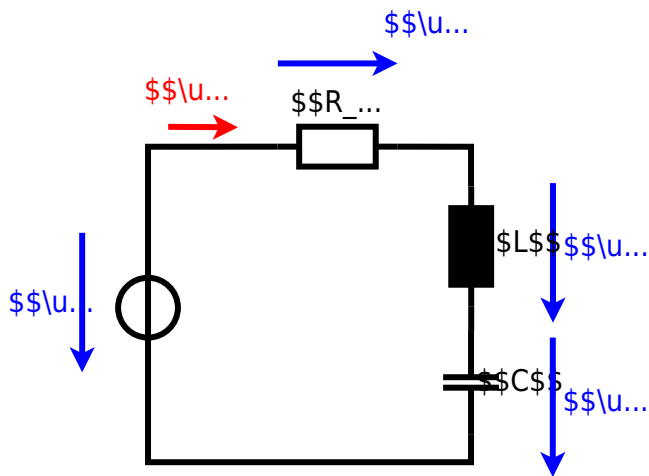
$$i(t) = 107 \text{ mA} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$$













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Last update: **2023/08/17 11:43**

