Exam Summer Semester 2023

Student Group

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Table of Contents

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Additional permitted Aids

- non-programmable calculator,
- formulary (2 DIN A4 pages)

Hits

- The duration of the exam is 60 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

Tasks

Exercise E1 Resistivity and temperature dependent Resistance
(written test, approx. 7 % of a 60-minute written test, SS2023)

The resistance of the dielectric material of a film capacitor (polypropylene) is to be described by the
(Incidentally, the temperature dependence could also be approximated by a $B25$ value in an exponential formula. In this case, the best fit would be a $B25 = 15'000$ for T between $20 ~\rm °C$ and $100 ~\rm °C$)
The given film capacitor has a surface of $A=100 ~\rm mm^2$ and a distance between the plates of $d=0.8 ~\rm \mu m$.
The resistivity of the dielectric material is $\rho_{\rm PP}(20 ~\rm °C)=10^{17} ~\Omega m$.
For the given material the temperature coefficients in the range of $20 ~\rm °C$ and $55 ~\rm °C$ are given as $\alpha =-0.048 ~\rm 1/K$ and $\beta=+0.00057 ~\rm 1/K^2$.
Calculate the resistance R of the dielectric for $20 ~\rm °C$ and $55 ~\rm °C$.

Solution

Result

$$R(20 ~\rm °C) = 80 ~\rm G\Omega$$

$$\begin{align*} R(55 ~\rm °C) &= R(20 ~\rm °C) \cdot (1+\alpha\cdot\Delta T + \beta\cdot T^2 + ...)\\ &= 80 ~\rm G\Omega \cdot (1-0.048 ~\rm 1/K \cdot(35 ~{\rm K}) + 0.00057 ~\rm 1/K^2\cdot\Delta (35 ~{\rm K})^2) \end{align*}$$

Calculate the resistance for the dielectric material for \$20 ~\rm °C\$.

Solution

> \begin{align*} R(20 ~\rm °C) &= \rho\cdot {{d}\over{A}}\\ &= 10^{17} ~\Omega \rm m \cdot {{0.8\cdot 10^{-6} ~\rm m}\over{1 ~\rm m^2}} \end{align*}

Exercise E2 Analyzing a Scope Plot
(written test, approx. 12 % of a 60-minute written test, SS2023)

(in radiant and degree)?

Result

The measured current curve shall be visible as a dashed line.

Solution

The continuous line shows the voltage.

\begin{align*} \hat{U} &= 15 ~\rm V \\ \hat{I} &= 0.5 ~\rm A \end{align*} f = 1.66 ~\rm kHz \rightarrow f = 1.67 ~\rm kHz

\begin{align*} U_{\rm RMS} &= 10.6066 ~\rm V \rightarrow U_{\rm RMS} = 10.6 ~\rm V \\ I_{\rm RMS} &= 0.3535... ~\rm A \rightarrow I_{\rm RMS} = 0.35 ~\rm A \end{align*} \varphi = -30°

The RMS value for sinusoidal waves can be derived as: The phase shift is given by the voltage {{1} \over {2}} \sqrt{2} ... The phase shift is **negative**, the **voltage lags the current**.

A full period has an angle of $360°$ or 2π in $\rm radian$.

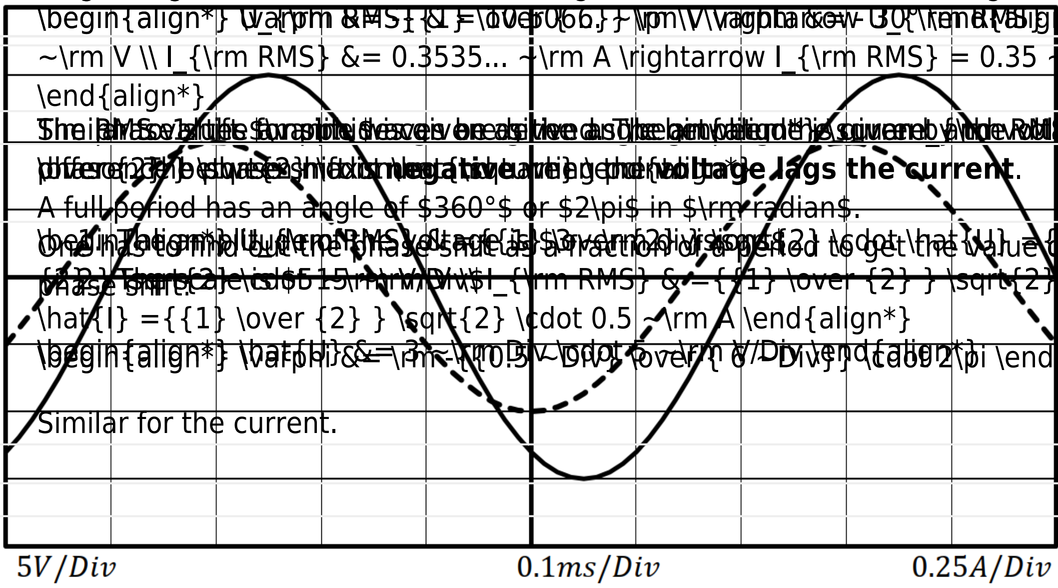
One has to find out the phase shift as a fraction of a period to get the value of the phase shift.

\begin{align*} U_{\rm RMS} &= {{1} \over {2}} \sqrt{2} \cdot \hat{U} = {{1} \over {2}} \sqrt{2} \cdot 15 ~\rm V \\ I_{\rm RMS} &= {{1} \over {2}} \sqrt{2} \cdot \hat{I} = {{1} \over {2}} \sqrt{2} \cdot 0.5 ~\rm A \end{align*}

\begin{align*} \hat{U} &= 3 ~\rm Div \cdot 5 ~\rm V/Div \end{align*}

\begin{align*} \varphi &= -{{0.5 ~\rm Div} \over {6 ~\rm Div}} \cdot 2\pi \end{align*}

Similar for the current.



Use the correct symbols and units in your answers!

1. Calculate the frequency f of the periodic signals.

Solution

Frequency f is given by the period T. The period can be measured in the imagine of the scope.

1. The sine waves repeat after $6 ~\rm divisions$ (e.g. from falling turning point to falling turning point of one curve)
2. The scale is $0.1 ~\rm ms/Div$

\begin{align*} f &= {{1} \over {T}} \\ T &= 6 ~\rm Div \cdot 0.1 ~ms/Div \\ \rightarrow f &= {{1} \over {6 ~\rm Div \cdot 0.1 ~ms/Div}} \end{align*}

Exercise E3 Complex voltage dividers
(written test, approx. 16 % of a 60-minute written test, SS2023)

The following circuit is given, with the following values:

1. Calculate the complex impedance $\underline{Z}_L$.

2. **Draw the two impedances and the resulting phasor in the phasor diagram. Choose an appropriate scaling factor** and write it down.

- $R = 1.1 ~\rm k\Omega$
- $L = 3.5 ~\rm mH$
- $\underline{U}_\rm I = 5 ~\rm V$
- $f_0 = 150 ~\rm kHz$

Solution

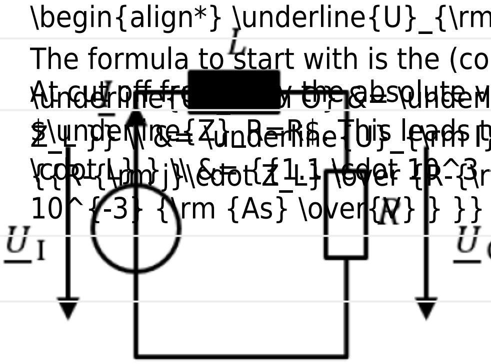
Result

\begin{align*} \underline{Z}_L &= {\rm j} \cdot 3.3 ~\rm k\Omega \end{align*}

\begin{align*} \underline{U}_{\rm O} &= 0.5 ~\rm V - j \cdot 1.5 ~V \end{align*}

The formula to start with is the (complex) voltage divider: \begin{align*} \underline{U}_{\rm O} &= \underline{U}_{\rm I} \cdot {{R} \over {R+{\rm j}\cdot Z_L }} \\ &= \underline{U}_{\rm I} \cdot {{R} \over {R+{\rm j}\cdot Z_L }} \cdot {{R-{\rm j}\cdot Z_L} \over {R-{\rm j}\cdot Z_L }} \\ \end{align*}

At cutoff frequency the absolute values of impedances $\underline{Z}_L$ is equal to $\underline{Z}_R=R$. This leads to: \begin{align*} f_{\rm cutoff} &= {{R} \over {2\pi \cdot L}} \\ &= {{1.1 \cdot 10^3 {\rm {V} \over {A}} } \over {2\pi \cdot 3.5 \cdot 10^{-3} {\rm {As} \over {V} } }} \end{align*}



1. Calculate the impedance $\underline{Z}_L$.

Solution

\begin{align*} \underline{Z}_L &= {\rm j} \cdot \omega \cdot L \\ &= {\rm j} \cdot 2\pi \cdot 150 ~\rm kHz \cdot 3.5 ~\rm mH \end{align*}

Exercise E1 Pure Resistor Network Simplification
(written test, approx. 12 % of a 60-minute written test, SS2023)

The circuit below shall be given.

The values in the circuit are

- $R_1 = 60 ~\Omega$
- $R_2 = 40 ~\Omega$
- $R_3 = 40 ~\Omega$
- $R_4 = 100 ~\Omega$
- $U_{\rm AB} = 10 ~\rm V$

2. Calculate the voltage at node $\rm K$, when switch $\rm S$ is closed.

Result

Solution

The equivalent resistance is similar to the circuit with opened switch. \begin{align*} R_{\rm eq} &= 150 ~\Omega \end{align*}

The voltage divider for node $\rm K$ has the same proportionality as the voltage divider for node $\rm K'$. Therefore, the potential of $\rm K$ is the same as for $\rm K'$. There will be no current flow through R_3. The resistance does not create a voltage drop and therefore does not interfere with the circuit.

1. Calculate the voltage at node $\rm K$, when switch $\rm S$ is open.
It might be beneficial to redraw the circuit first.

Solution

Rearranging the circuit one can get:

Once the switch $\rm S$ is opened, the upper part is a parallel circuit. Therefore, $R_{\rm eq}$ is given as: \begin{align*} R_{\rm eq} &= (R_1+R_2)||(R_1+R_2)+R_4 \\ &= {{1}\over{2}}\cdot(R_1+R_2)+R_4 \\ &= {{1}\over{2}}\cdot(60~\Omega + 40~\Omega) + 100~\Omega \\ \end{align*}

Exercise E2 Pure Resistor Network Simplification I
(written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit below shall be given.

2. Now the input voltage shall be given as $U_{\rm AB} = 60 ~\rm V$. What is the value for U in the circuit?

Result

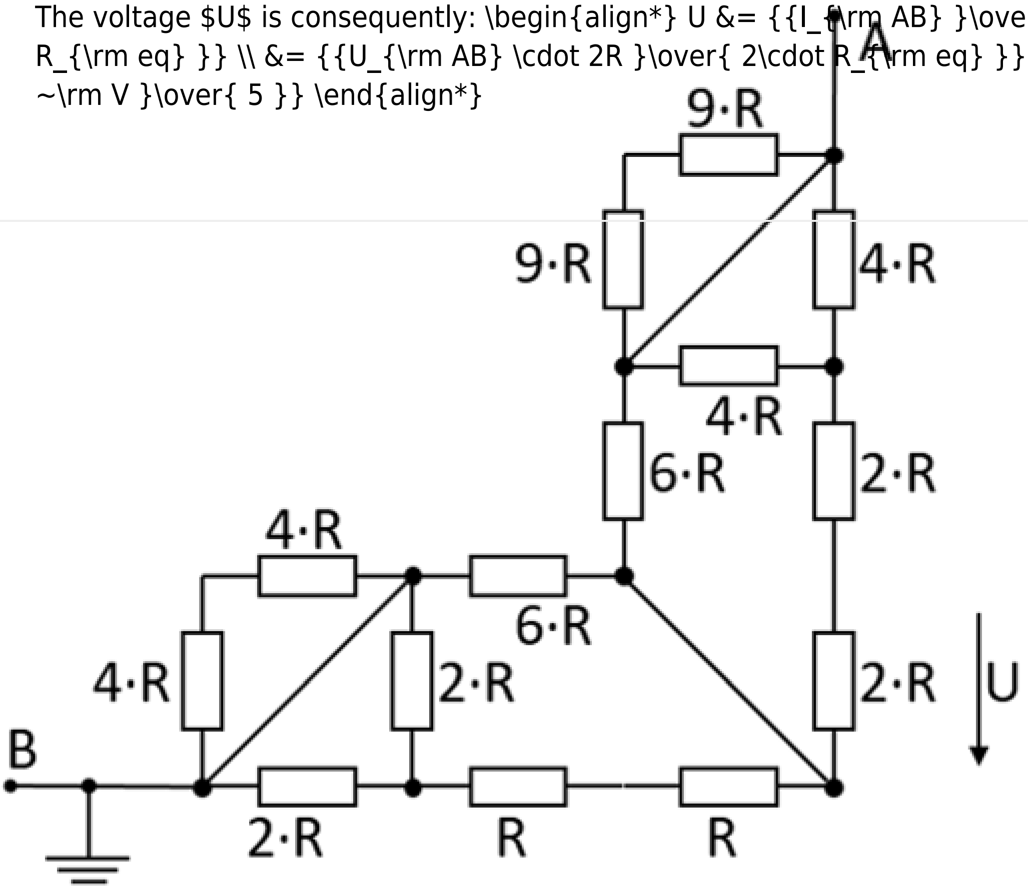
Solution

\begin{align*} R_{\rm eq} &= 5R \end{align*}

\begin{align*} U &= 12 ~\rm V \end{align*}

The current through the circuit is given as $I_{\rm AB} = U_{\rm AB} \cdot R_{\rm eq}$.
This current has to flow in summary through parallel branches. The voltage U in question in the upper right branch given by $(4R||4R)+2R+2R$. Its resistance is just the same as the upper left branch $6R$.
Therefore, half of the current flows to the left half to the right side.

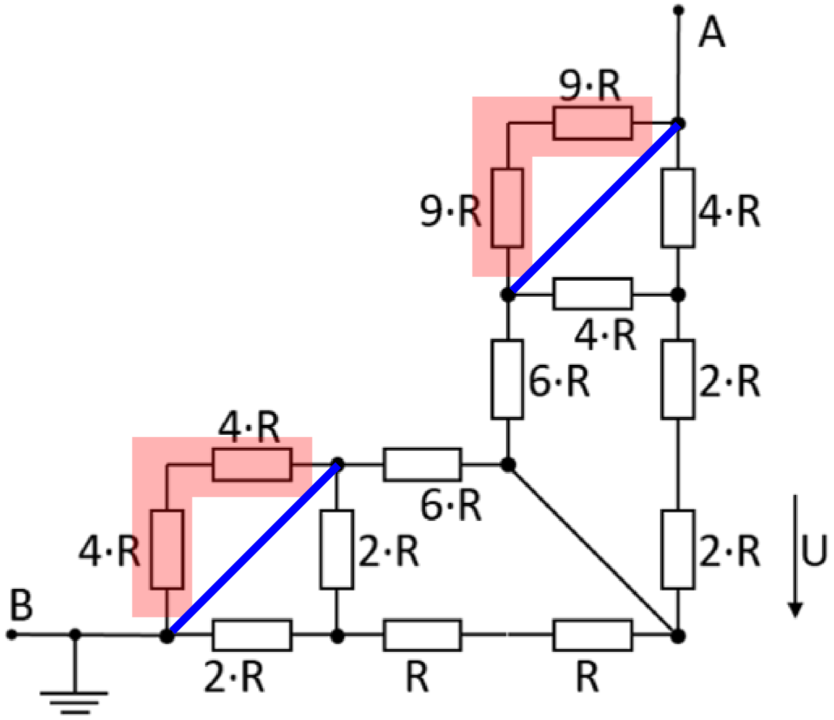
The voltage U is consequently: \begin{align*} U &= {{I_{\rm AB} }\over{ 2\cdot R_{\rm eq} }} \\ &= {{U_{\rm AB} \cdot 2R }\over{ 2\cdot R_{\rm eq} }} \\ &= {{60 ~\rm V }\over{ 5 }} \end{align*}



1. What is the equivalent resistance $R_{\rm eq}$?

Solution

Part of the circuit is shorted. Here the resistors (marked in red) are shorted by the connections marked in blue:



The circuit can then be rearranged for better interpretation:

Therefore, $R_{\rm eq}$ is given as: \begin{align*} R_{\rm eq} &= (2R||2R + R +

R)||6R &&+ 6R ||(2R + 2R + 4R||4R) \\ &= (R + R + R)||6R &&+ 6R ||(2R + 2R + 2R) \\ &= 3R||6R &&+ 6R ||6R \\ &= {{3R\cdot 6R}\over{3R+6R}} &&+3R \\ \end{align*}

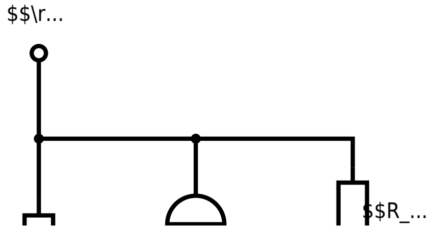
Exercise E1 Equivalent Linear Source
(written test, approx. 10 % of a 60-minute written test, SS2023)

The circuit below has to be simplified. Use equivalent linear sources for simplification. Calculate the internal resistance $R_\rm i$ and the source voltage $U_\rm s$ of an equivalent linear voltage source.

- $R_1 = 5 ~\Omega$
- $U_0 = 10 ~\rm V$
- $R_2 = 7.5 ~\Omega$
- $I_3 = 0.5 ~\rm A$
- $R_4 = 10 ~\Omega$
- $U_5 = 4 ~\rm V$

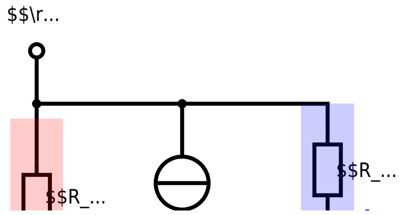
Result

$$\begin{align*} U_\rm s &= U_{\rm AB} =1.11...~\rm V \\ R_{\rm i} &= 5.55... ~\Omega \end{align*}$$



Solution

The principle idea here is to find parts of the circuit which are already a linear (voltage or current) source. Then this can be transformed into the equivalent other source, as shown in the next picture.



In order to get the currents one has to calculate it by $I_x = {{U_x}\over{R_x}}$ \begin{align*} I_0 &= {{U_0}\over{R_1}} = {{10~\rm V}\over{5 ~\Omega}} = 2 ~\rm A\\ I_5 &= {{U_5}\over{R_4}} = {{4 ~\rm V}\over{10~\Omega}} = 0.4 ~\rm A \\ \end{align*}

I_3 and I_0 can be combined to $I_{03}=I_0 - I_3$ facing upwards: \begin{align*} I_{03}=1.5~\rm A \end{align*}

Then, the linear current source I_{03} with R_1 gets transformed into a linear voltage source with $U_{03}=R_1\cdot I_{03}$ facing down. \begin{align*} U_{03}=7.5~\rm V \end{align*}

Then, the resistors R_1 and R_2 can be combined to $R_{12}= R_1 + R_2$.

After this, the next step is to make a linear current source out of U_{03} and R_{12}. The current will be $I_{0123}={{U_{03}}\over{R_{12}}}$, facing up again. \begin{align*} I_{0123}=0.6~\rm A \end{align*}

The second-last step is the sum up of the current sources I_{0123} and I_5 as $I_{01235}=I_{0123}-I_5$ and the resistors as $R_{124}=R_{12}||R_4$. \begin{align*} I_{01235} &=0.2~\rm A \\ R_{124} &=5.55... ~\Omega \end{align*}

The final step is the back-transformation to a linear voltage source, with $U_{\rm AB} = R_{124} \cdot I_{01235}$.

The simplest and fastest (= for exams) is to work with interim results in the calculation.
Here, there there is also a full final formula given:

$$\begin{align*} U_\rm s &= U_{\rm AB} = I_{01235} \cdot R_{124} \\ &= (I_{0123}-I_5) \cdot (R_{12}||R_4) \\ &= \left({{U_{03}}\over{R_{12}}}-I_5\right) \cdot \left((R_1 + R_2)||R_4\right) \\ &= \left({{R_1\cdot I_{03}}\over{R_1+R_2}}-I_5\right) \cdot \left((R_1 + R_2)||R_4\right) \\ &= \left({{R_1\cdot \left({{U_0}\over{R_1}} - I_3\right)}\over{R_1+R_2}}-I_5\right) \cdot \left((R_1 + R_2)||R_4\right) \\ \end{align*}$$

Exercise E4 (Dis)Charging Capacities
(written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit below shall be analyzed. The values are given in the drawing below.

- $U = 10 ~\rm V$
- $C_1 = 200 ~\rm nF$
- $R_1 = 8.0 ~\rm k\Omega$
- $R_2 = 17 ~\rm k\Omega$
- $R_3 = 7.0 ~\rm k\Omega$
- $I = 2 ~\rm mA$

Solution

Solution

$\begin{align*} \tau_1 ... 3.36 ~\rm ms \end{align*}$

$\begin{align*} U_C &= 50 ~\rm V \end{align*}$

$\begin{align*} u_C(t) &= U_0 \cdot(1-e^{-t/\tau}) \\ &= 10~\rm V \cdot(1-e^{-4 ~ms/3 ~ms}) \end{align*}$

Result

At which point in time t_2 ... S_1 switches to the situation shown in the drawing. What is the new time constant?

Again, the time constant is generally given as: $\begin{align*} \tau &= R\cdot C \end{align*}$

The current of the source flows through the circuit consisting of C in parallel with R_1+R_2. Without the parallel resistors, the current source would charge the capacitor “to infinity” ($U_C \rightarrow \infty$). This is here limited by the parallel resistors R_1+R_2.

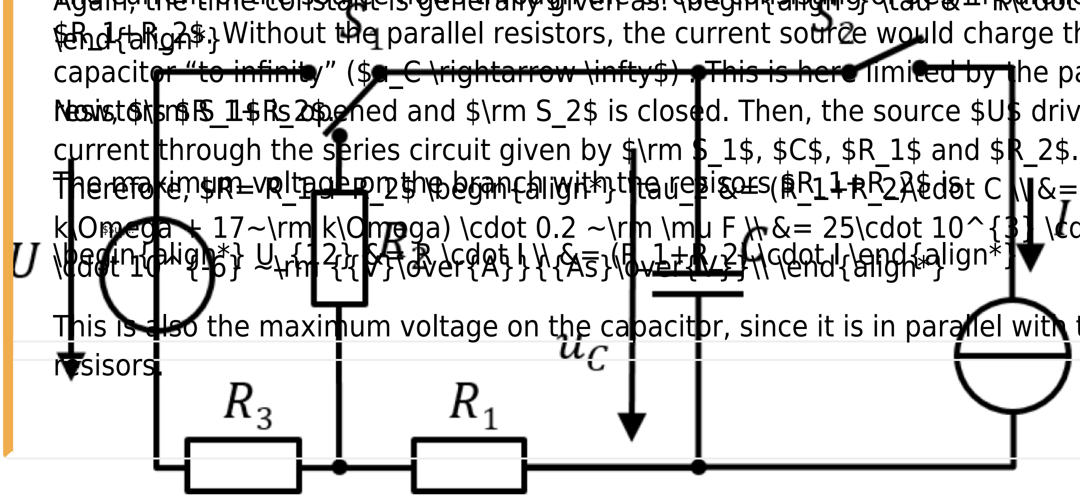
Now, the $\rm S_1$ is opened and $\rm S_2$ is closed. Then, the source U drives the current through the series circuit given by $\rm S_1$, C, R_1 and R_2.

Therefore, $R = R_1 + R_2$ $\begin{align*} \tau_2 &= (R_1+R_2)\cdot C \\ &= (8~\rm k\Omega + 17~\rm k\Omega) \cdot 0.2 ~\rm \mu F \\ &= 25\cdot 10^{3} \cdot 0.2 \cdot 10^{-6} ~\rm {{V}\over{A}}{{As}\over{V}} \\ \end{align*}$

The maximum voltage on the branch with the resisors R_1+R_2 is

$\begin{align*} U_{12} &= R \cdot I \\ &= (R_1+R_2)\cdot I \end{align*}$

This is also the maximum voltage on the capacitor, since it is in parallel with the resisors.



Before t_0 all switches are switched as shown and the capacitor is fully discharged.
At $t_0=0 ~\rm s$ the switch S_1 shall switch to the voltage source.

1. Calculate the time constant for charging the capacitor.

Solution

The time constant is generally given as: \begin{align*} \tau &= R\cdot C \end{align*}

Once S_1 is closed and S_2 is open at t_0, the source U drives the current through the series circuit given by S_1, C, R_1 and R_3.
Therefore, $R= R_1 + R_3$ \begin{align*} \tau_1 &= (R_1+R_3)\cdot C \\ &= (8~\rm k\Omega + 7~\rm k\Omega) \cdot 0.2 ~\rm \mu F \\ &= 15\cdot 10^{3} \cdot 0.2 \cdot 10^{-6} ~\rm {{V}\over{A}}{{As}\over{V}}\\ \end{align*}

Solution

Both courses of the voltage for charging and discharging are described with an exponential function. However, the curve for charging increases first steep and flattens out for longer time scales ($\propto (1-e^{-x})$).

Exercise E5 Impedances at Frequencies
(written test, approx. 14 % of a 60-minute written test, SS2023)

At which frequency with $C_2 = 50~\rm \mu F$, the following, show AC voltage of $U_2 = 6.8 ~\rm V$ the absolute value of inductances with $X_{L1} = 60 ~\rm m\Omega$ and $L_1 = 15.9 ~\rm \mu H$.

Solution

\begin{align*} f_2 = 300000 ~{\rm Hz} \rightarrow f_2 = 300 ~{\rm kHz}\end{align*}

\begin{align*} X_{C1} &= X_{C3} \\ {{1}\over{\omega_2 \cdot C_1}} &= \omega_2 \cdot L_1 \\ C_1 &= {{1}\over{\omega_2^2 \cdot L_1}} \\ &= {{1}\over{(2\pi \cdot 300 \cdot 10^{3} ~\rm 1/s)^2 \cdot 15.9 \cdot 10^{-6} ~\rm Vs/A}} \\ \end{align*}

Exercise E1 Efficiency
(written test, approx. 14 % of a 60-minute written test, SS2023)

A lithium battery has an internal resistance and an open-circuit voltage of $U_\rm S = 3.5 ~\rm V$. The battery shall provide energy for a mobile device with a load resistance of $R_\rm L = 2 ~\Omega$. The following values are from the lithium-ion battery datasheet.

- Internal impedance: $R_\rm i = 0.05 ~\Omega$
- Maximum discharge current: $I_{\rm Dis max} = 3 ~\rm A$
- Typical capacity: $2600 ~\rm mAh$

1. Draw an **equivalent circuit diagram** with the internal resistance and an external load. Label all voltages and currents.

Solution

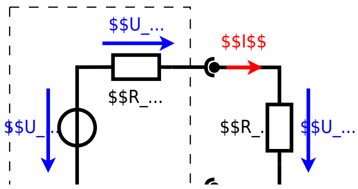
Result

\begin{align*} n_{\rm e^-} ={{Q}\over{q_\rm e}} \\ \\ Q &= 2.6 {~\rm Ah} \\ &= 2.6 \cdot 3600 {~\rm As} \\ &= 9360 {~\rm As} = 9360 {~\rm C} \end{align*}

What is the **lowest possible efficiency**?

The lowest efficiency occurs for the highest current, so for $I_{\rm Dis max}$. In this case, the efficiency is:

\begin{align*} \eta &= {{U_\rm S - R_\rm i \cdot I_{\rm Dis max}}\over{U_\rm S}} \\ &= 1 - R_\rm i \cdot {{I_{\rm Dis max}}\over{U_\rm S}} \\ &= 1 - 0.05 {~\rm \Omega} \cdot {{3~\rm A}\over{3.5 ~\rm V}} \\ \end{align*}


$$U_...
$$I$$
$$R_...
$$U_...
$$R_...
$$U_...