Block 22 — Negative-feedback Op-Amp Circuits

Student Group

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Table of Contents

Block 22 — Negative-feedback Op-Amp Circuits ... 2
Learning objectives ... 2
Preparation at Home ... 2
90-minute plan ... 2
Conceptual overview ... 2
Core content ... 2
3.3 Voltage follower ... 2
Remember: steps to the goal ... 4
3.4 Non-inverting amplifier ... 5
Notice: non-inverting amplifier ... 9
3.5 Inverting Amplifier ... 9
Notice: virtual ground ... 11
Notice: Inverting Amplifier ... 13
Common pitfalls ... 14
Exercises ... 14
Worked examples ... 14
Embedded resources ... 14

Block 22 — Negative-feedback Op-Amp Circuits

Learning objectives

> After this 90-minute block, you can
>
> - …

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- …

90-minute plan

1. Warm-up (x min):
    1. ….
2. Core concepts & derivations (x min):
    1. …
3. Practice (x min): …
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

> 1. …

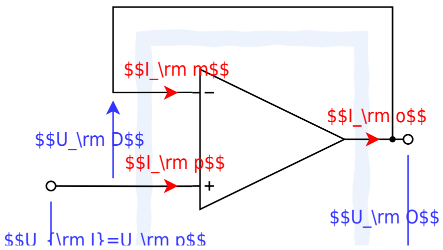
Core content

3.3 Voltage follower

In the Block21 it was described that an amplifier with high open-loop gain can be "tamed" by feeding back a part of the output signal with a negative sign.
In the simplest case, the output signal could be fed directly to the negative input of the operational

amplifier. The input signal $U_\rm I$ of the entire circuit is applied to the positive input. In figure 1 this circuit is shown.

Fig. 1: voltage follower



Using this circuit, the procedure for solving amplifier circuits is now to be illustrated.

1. The aim is always to create a relation between output voltage $U_\rm O$ and input voltage $U_\rm I$.
   Thus, the goal here is the voltage gain $A_{\rm V}=\frac{U_{\rm O}}{U_\rm I}$.

2. Before calculating, it should be checked how many equations describe the system and thus have to be set up.
   This can be determined by the **number of variables**. This is done by counting through the currents and voltages of the circuit.
   In this case, there are 3 currents and 3 voltages. So the **number of equations** needed is 6.

3. Now **equations are set up** that can be used. These are:
    1. **Basic equation**: (1) $U_{\rm O} = U_{\rm D} \cdot A_\rm D$
    2. **Golden rules**: $R_\rm D \rightarrow \infty$ so that (2+3) $I_{\rm p} = I_{\rm m} = 0$, $A_{\rm D} \rightarrow \infty$, $R_\rm O = 0$
    3. Consideration of the existing **loops**: in this example, there is only one loop (4) $-U_{\rm I} + U_{\rm D} + U_{\rm O} = 0$.
       Caution: loops can not enter the amplifier through input and exit through the output! Also to be noted is the direction of $U_{\rm D}$.
    4. Consideration of the existing **nodes**: in this example, there is only one node (5) $I_{\rm o} = I_m$.

4. There appears to be a missing equation. However, this is not correct, because there is still an equation hidden in the objective: (0) $A_{\rm V}=\frac{U_{\rm O}}{U_\rm I}$

5. Now, to **solve the equations**, the equations must be cleverly inserted into each other in such a way that there are no dependencies on the variables left at the end.

The calculation is done here once in detail (clicking on the arrow to the right "►" leads to the next step, alternative representation):

So the voltage gain is $A_\rm V = 1$. This would also have been seen in chapter Block21. There it was derived that for $A_\rm D \rightarrow \infty$ the voltage gain just results from k: $A_{\rm V}=\frac{1}{k}$.
Since the entire output voltage is fed back here, $k=1$ and thus also $A_\rm V=1$.

The output voltage $U_\rm O$ is therefore equal to the input voltage $U_\rm I$. This is where the name "voltage follower" comes from. Now one could assume, that this amplifier is of little help because also a direct connection would deliver $U_{\rm O}=U_{\rm I}$.
But the important thing here is: because of the operational amplifier, there is no feedback from U_O to U_I. This means that a resistor on the output side will not load the input side. In the simulation, the "Resistance" slider (on the right) can be used to change the load resistance. This changes the current flow, but not the voltage.

This behavior can also be explained in another way: The input signal usually comes from a voltage source, which can only produce low currents.
That means the input signals are high impedance ($\text{high impedance}=\frac{\text{voltage}}{\text{low current}}$).
However, a load of arbitrary impedance can be applied to the output. That is, to keep the output signal constant, a large current must be provided depending on the load.
As the output resistance of the amplifier approaches 0, the signal is low impedance ($\text{low impedance}=\frac{\text{voltage}}{\text{(likely) large current}}$). This is where the second name of the circuit "**impedance converter**" comes from.

Remember: steps to the goal

To solve tasks, the following procedure helps:

1. Where to? Clarification of the goal (here: always the relation between output and input signal)
2. What to? Clarification of what is needed (here: always equations. The number of needed equations can be determined by the number of variables)
3. With what? Clarification of what is already available (here: known equations: voltage amplification equation, basic equation, golden rules, loop/node theorem, relationships of voltages and currents of components).
4. Go. Work out the solution (here: inserting the equations) It helps to rearrange the equation so that $1/A_\rm D$ appears without a prefactor. It is valid: $1/A_{\rm D} \xrightarrow{A_{\rm D} \rightarrow \infty} 0$

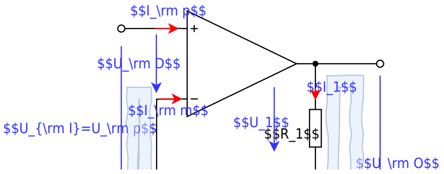
3.4 Non-inverting amplifier

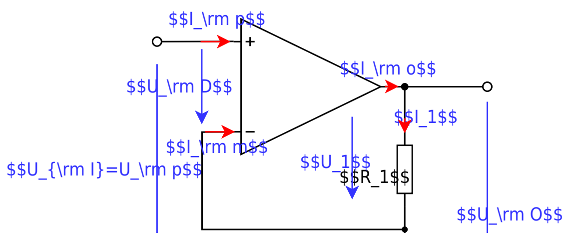
So far, the entire output voltage has been negative-feedback. Now only a part of the voltage is to be fed back.
To do this, the output voltage can be reduced using a voltage divider R_1+R_2. The circuit for this can be seen in figure 2.

By considering the feedback, the result can be quickly derived here as well: only $\frac{R_2}{R_1+R_2}\cdot U_{\rm O}$ is fed back from the output voltage $U_{\rm O}$.
So the feedback factor is $k=\frac{R_2}{R_1+R_2}$ and thus the voltage gain becomes $A_{\rm V}=\frac{R_1+R_2}{R_2}$.

This “trick” via $A_{\rm V}=\frac{1}{k}$ is no longer possible for some of the following circuits. Accordingly, a possible solution via network analysis is to be derived here as well.

Fig. 2: Non-inverting amplifier


$$I_\rm p$$
$$U_\rm D$$
$$I_\rm n$$
$$U_{\rm I}=U_\rm p$$
$$I_1$$
$$U_1$$
$$R_1$$
$$U_\rm O$$



Step	Description	Implementation
1	What is wanted?	$A_{\rm V} = \frac{U_{\rm O}}{U_{\rm I}}=?$
2	Counting the variables $\rightarrow$ Number of equations needed	5 voltages + 5 currents $\rightarrow$ Number of equations needed: 10

Step	Description	Implementation
3	Setting up the equations	always usable equations (1) Basic equation: $U_{\rm O} = A_{\rm D} \cdot U_{\rm D}$ Golden rules: $R_\rm D \rightarrow \infty$ so that (2+3) $I_\rm p \rightarrow 0$ and $I_\rm m \rightarrow 0$ $R_{\rm O} = 0$ $A_\rm D \rightarrow \infty$ Loops and nodes (see) (4) Loop I: $-U_{\rm I} + U_{\rm D} + U_2 = 0$ (5) Loop II: $-U_2 -U_1 + U_{\rm O} = 0$ (6) Node I: $I_{\rm o} = I_1$ (7) Node II / voltage divider: $I_1 - I_2 - I_\rm m = 0$ U, I relationships across components (8) Resistor $R_1= \frac{U_1}{I_1}$ (9) Resistor $R_2= \frac{U_2}{I_2}$

The calculation is done here again in detail (clicking the right arrow "►" leads to the next step, alternative representation):

So the voltage gain of the non-inverting amplifier is $A_{\rm V}=\frac{R_1+R_2}{R_2}$ or $A_{\rm V}=1+\frac{R_1}{R_2}$. Thus, the numerical value $A_{\rm V}$ can only become larger than 1. This is shown again in the simulation. In real circuits, the resistors R_1 and R_2 will be in the range between a few $100 ~\Omega$ and a few $\rm M\Omega$.
If the sum of the resistors is too small, the operational amplifier will be heavily loaded. However, the output current must not exceed the maximum current.
If the sum of the resistors is too large, the current $I_1 = I_2$ can come into the range of the current $I_\rm m$, which is present in the real operational amplifier.

The **input and output resistance of the entire circuit** should also be considered here. Both resistors are marked here with a superscript 0 to distinguish them from the input and output resistance of the operational amplifier.
The input resistance $R_{\rm I}^0$ is given by $R_{\rm I}^0=\frac{U_\rm I}{I_\rm I}$ with $I_{\rm I}=I_\rm p$. Thus, for the ideal operational amplifier, it is also true that the input resistance $R_{\rm I}^0=\frac{U_\rm I}{I_\rm p} \rightarrow \infty$ becomes when $I_\rm p \rightarrow 0$.

In the **real case** it is important in how far the total input resistance depends on the input resistance of the operational amplifier $R_{\rm I}^0(R_\rm D)$.
This can be derived as follows: (clicking on the right arrow "►" leads to the next step, alternative representation):

So it can be assumed simplistically, that the input resistance of the whole circuit is many times higher than the input resistance of the operational amplifier. The output resistance $R_\rm O^0$ of the whole circuit with real operational amplifiers shall only be sketched: In this case, the output resistance $R_\rm O$ of the operational amplifier is in parallel with $R_1 + R_2$. Thus the output resistance

$R_\rm O^0$ will be somewhat smaller than $R_\rm O$.

Notice: non-inverting amplifier

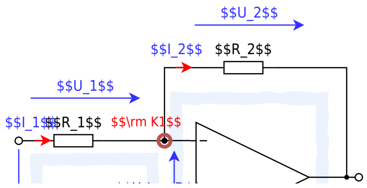
For the non-inverting amplifier, the following holds:

- The input voltage $U_\rm I$ is at the non-inverting input of the operational amplifier.
- The feedback is done by a voltage divider $R_1 + R_2$
- The voltage gain is $A_{\rm V}=\frac{R_1+R_2}{R_2}$ or $A_{\rm V}=1+\frac{R_1}{R_2}$ and is always greater than 1.
- Both input and output resistances of the overall circuit are smaller than those for the (real) operational amplifier used.

3.5 Inverting Amplifier

The circuit of the inverting amplifier can be derived from that of the non-inverting amplifier (see figure 4). To do this, first consider the noninverting amplifier as a system with 3 connections (or as a quadripole): $U_\rm I$, $\rm GND$, and $U_\rm O$. These terminals can be rearranged - while keeping the output terminal $U_\rm O$.

Fig. 3: Inverting Amplifier



Thus the voltage divider $R_1 + R_2$ is no longer between $U_\rm O$ and $\rm GND$, but between $U_\rm O$ and $U_\rm O$, see figure 3. In this circuit, the resistor R_2 is also called the negative feedback resistor.

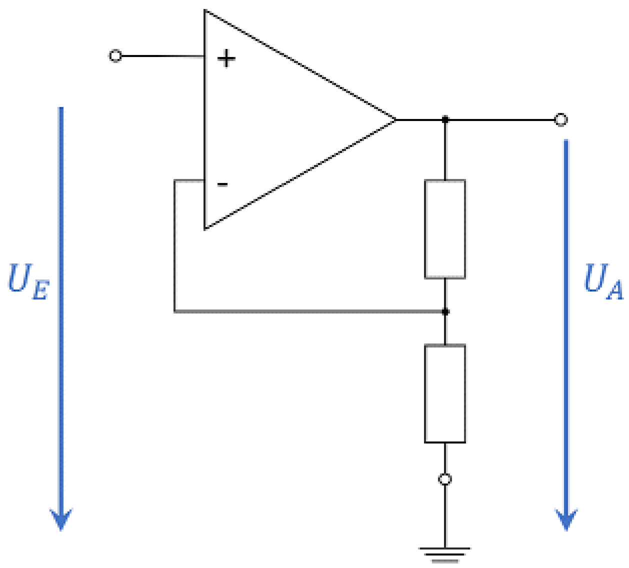


Fig. 4: Converting non-inverting amplifier to inverting amplifier. U_E is the input voltage (Eingangsspannung), U_A is the output voltage

(Ausgangspannung).

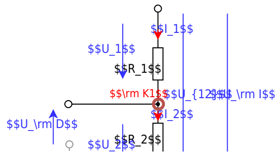
Before the voltage gain is determined, the node $\rm K1$ in figure 3 is to be considered first. This is just larger than the ground potential by the voltage $U_\rm D$; thus, it lies on the potential difference $U_\rm D$. For a feedback amplifier with finite voltage supply, $U_\rm O$ can only be finite, and thus $U_{\rm D}= U_{\rm O} / A_{\rm D} \rightarrow 0$ (cf. basic_equation of the operational amplifier), since $A_D \rightarrow \infty$ holds. Thus it can be seen that the node $\rm K1$ is always at ground potential in the ideal operational amplifier. This property is called **virtual ground** because there is no direct short to ground. The op-amp regulates its output voltage $U_\rm O$ in such a way that the voltage divider sets a potential of $0~\rm V$ at node $\rm K1$. This can also be seen in the simulation by the voltage curve at $\rm K1$.

The following diagram shows again the interactive simulation.
$R_{\rm 1}$ and $R_{\rm 2}$ can be manipulated by the sliders. Hit Run / STOP to run

Notice: virtual ground

For the ideal feedback amplifier, $U_\rm D \rightarrow 0$ holds. This means that the same voltage is always present at both inputs. if one of the two voltages is fixed, for example by connecting ground potential or even by a fixed voltage source, this property is called **virtual ground**.

Fig. 5: Voltage divider in inverting amplifier



For the determination of the voltage gain, the consideration of the feedback $A_{\rm V}=\frac{1}{k}$ seems to be of little use at first. Instead, however, the determination via network analysis is possible. figure 3 shows a possible variant to choose the loops for this purpose. However, network analysis is not to be done here, but is given in Exercise 3.5.1 below.

Instead, two other ways of derivation will be shown here to bring further approaches closer. For the first derivation, the **voltage divider** $\boldsymbol{R_1 + R_2}$ is considered. For the unloaded voltage divider, the general rule is:

\begin{align*} U_2 = U_{12} \cdot \frac{R_2}{R_1 + R_2} \end{align*}

This equation is now to be adapted for concrete use. First figure 3, the voltages of the voltage divider can be read as given in figure 5. From this, using the general voltage divider formula:

\begin{align*} U_2 = (U_I - U_O) \cdot \frac{R_2}{R_1 + R_2} \end{align*}

With the virtual mass at node $\rm K1$ in figure 5, it holds that U_2 points away from the (virtual) mass and thus $U_2 = U_\rm O$. Similarly, $U_{\rm I} = U_1$ holds. Thus it follows:

\begin{align*} - U_{\rm O} = (U_{\rm I} - U_{\rm O}) \cdot \frac{R_2}{R_1 + R_2} \end{align*}

And from that:

\begin{align*} - U_O &= U_I \cdot \frac{R_2}{R_1 + R_2} - U_O \cdot \frac{R_2}{R_1 + R_2} \\ - U_O + U_O \cdot \frac{R_2}{R_1 + R_2} &= U_I \cdot \frac{R_2}{R_1 + R_2} \\ U_O \cdot (\frac{R_2}{R_1 + R_2}-1) &= U_I \cdot \frac{R_2}{R_1 + R_2} \\ \frac{U_O}{U_I} &= \frac{\frac{R_2}{R_1 + R_2}}{\frac{R_2}{R_1 + R_2}-1} \\

For the second derivation, the current flow through the resistors R_1 and R_2 of the unloaded voltage divider

\frac{U_O}{U_I} &= \frac{R_2}{R_2 - (R_1 + R_2)} = \frac{R_2}{-R_1} \\ \\ \boxed{ A_V = - \frac{R_2}{R_1}} \end{align*}

is to be considered. These two currents I_1 and I_2 are just equal. Thus:

\begin{align*} I_\boxed{}=\frac{U_\boxed{}}{R_\boxed{}}=const. \quad \text{with} \: \boxed{}=\{1,2\} \end{align*}

respectively

\begin{align*} \frac{U_1}{R_1}=\frac{U_2}{R_2} \end{align*}

This can also be converted into a “seesaw” or mechanical analog via **like triangles**. In the mechanical analog, the potentials are given by height.
As in the electrical case with the ground potential, a height reference plane must be chosen in the mechanical picture. The electric currents correspond to forces (i.e., a momentum flux) - but the consideration of forces is not necessary here. [1).

Fig. 6: Inverting Amplifier - Animation

Now, if a certain height (voltage $U_\rm I$) is set, a certain height on the right side (voltage $U_\rm O$) is obtained via the force arm (resistor R_1) and load arm (resistor R_2). This is shown in figure 6 above. In the figure, all points marked in red () can be manipulated. Accordingly, the input voltage $U_{\rm I} = U_{\rm in}$ is adjustable and automatically results in a voltage $U_{\rm O}=U_{\rm out}$. In the circuit (figure below), the resistors R_1 and R_2 can be changed.

The **input resistance of the entire circuit** $R_{\rm I}^0=\frac{U_{\rm I}}{I_{\rm I}}$ is easily obtained by considering the input side: since $\rm K1$ is at $0 ~\rm V$, $U_1 = U_\rm I$. The complete current flowing into the input passes through resistor R_1. So, it is then true that the input resistance is $R_{\rm I} = R_1$.
At the **output resistance of the whole circuit** $R_{\rm O}^0$, there is again a parallel connection between the output resistance of the operational amplifier $R_\rm O$ and the resistor R_2. So the output resistance will be slightly smaller than the output resistance of the operational amplifier $R_\rm O$.

Notice: Inverting Amplifier

In the case of the inverting amplifier:

- The input voltage $U_\rm I$ is at the inverting input of the operational amplifier.
- The feedback is done by a voltage divider of R_1 and R_2.
- The voltage gain is $A_{\rm V}= - \frac{R_2}{R_2}$ and is always less than or greater than 0. However, the magnitude of the voltage gain can be greater than or less than 1.
- The input resistance of the whole circuit is defined by R_1 and is usually smaller than the input resistance of the used (real) operational amplifier.
  The output resistance is smaller than that of the used (real) operational amplifier.

Common pitfalls

- ...

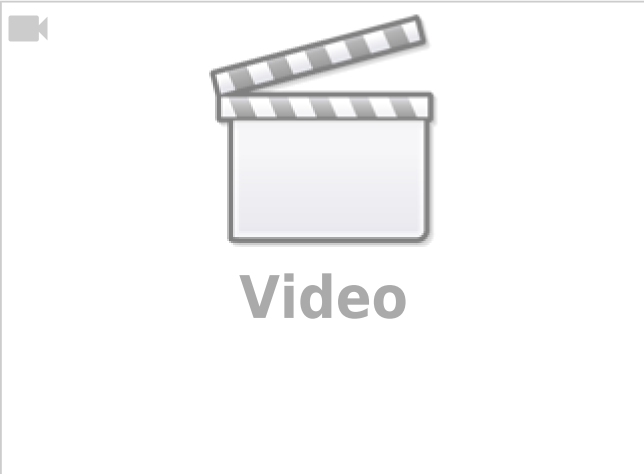
Exercises

Worked examples

...

Embedded resources

Non-inverting operation amplifier circuit



[1] To complete the mechanical analogue of the setup, one can assume that there is an external "force source". This always acts in such a way that it always lands on the height reference surface at the point corresponding to the virtual mass

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