Block 20 — Inductance and Energy

Student Group

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Block 20 — Electromagnetic Induction and Energy

Learning objectives

After this 90-minute block, you can

- …

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- …

90-minute plan

1. Warm-up (x min):
    1. ….
2. Core concepts & derivations (x min):
    1. …
3. Practice (x min): …
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

1. …

Core content

…

Common pitfalls

- …

Exercises

Exercise E12 Self-Induction
(written test, approx. 8 % of a 120-minute written test, SS2024)

A coil with a length of $30 ~\rm cm$ and a radius of $2 ~\rm cm$ has 500 turns.
The current through the coil changes linearly from 0 to $3 ~\rm A$ in $0.02 ~\rm ms$.
The arrangement is located in air ($\mu_{\rm r}=1$).
$\mu_0= 4\pi \cdot 10^{-7} ~\rm Vs/Am$

1. Calculate the (self-)inductance of the coil.

Path

The formula for the induction of a long coil is: \begin{align*} L &= \mu_0 \mu_{\rm r} \cdot N^2 \cdot {{A}\over{l}} \\ &= 4\pi \cdot 10^{-7} {~\rm Vs/Am} \cdot (500)^2 \cdot {{\pi \cdot (2\cdot 10^{-2} ~\rm m)^2}\over{ 2 \cdot 10^{-2} ~\rm m}} \\ \end{align*}

Result

$ L = 1.32 ~\rm mH$

2. Determine the induced voltage $u_{\rm ind}$ during the change in current.

Path

For the linear change of the current the formula of the induced voltage can also be linearized: \begin{align*} u_{\rm ind} &= - L \cdot {{ {\rm d} i }\over{ {\rm d} t }} \\ &\rightarrow - L \cdot {{ {\Delta} i }\over{ {\Delta} t }} \\ &= - 1.32 \cdot 10^{-3} \cdot {{3 A}\over{0.02 \cdot 10^{-3} s}} \end{align*}

Result

$ u_{\rm ind} = -197 ~\rm V$

Exercise E9 Self Induction
(written test, approx. 8 % of a 120-minute written test, SS2022)

A motor with an inductance of $L=50 ~\rm mH$ is connected to a DC voltage source, which is fused with a circuit breaker.
Accidentally, the circuit breaker opens at $t_0=0$ with a current of $I=63 ~\rm A$. The current is reduced linearly down to $0 ~\rm A$ within $1 ~\rm \mu s$.
(The inner resistance of the motor shall be neglected.)

1. Draw the circuit (the circuit breaker can be drawn as a switch), with all voltage and current arrows.

2. What is the maximum magnitude of the voltage, which the circuit breaker has to withstand?
Sketch the diagrams $i(t)$ and $u_{\rm ind}(t)$ with the calculated value of the induced voltage.

Path

For the maximum voltage on the circuit breaker one has to consider the following:

- external voltage of the voltage source $U_\rm s$
- voltage $u_{\rm ind} (t)$ induced by the change of the current

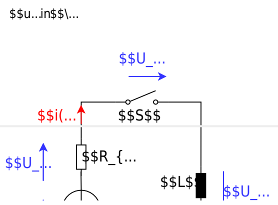
The first one is not given in the exercise, and therefore not considered here.

Result

$u_{\rm ind} (t) = 3150 ~\rm V$

The induced voltage can be calculated by linearizing the following: \begin{align*} u_{\rm ind} (t) &= - L {{ {\rm d} i}\over{ {\rm d} t }}\\ \rightarrow u_{\rm ind} (t) &= - L {{ \Delta i}\over{ \Delta t }}\\ \end{align*}

With the given details: \begin{align*} u_{\rm ind} (t) &= - L {{ 0 - I}\over{ t_1 - t_0 }}\\ &= 50 \cdot 10^{-6} {~\rm H} {{ 63 ~\rm A}\over{ 1 \cdot 10^{-6} ~\rm s}}\\ &= 3150 {{ ~\rm Vs}\over{ ~\rm A}} \; {{ ~\rm A}\over{ ~\rm s}}\\ \end{align*}

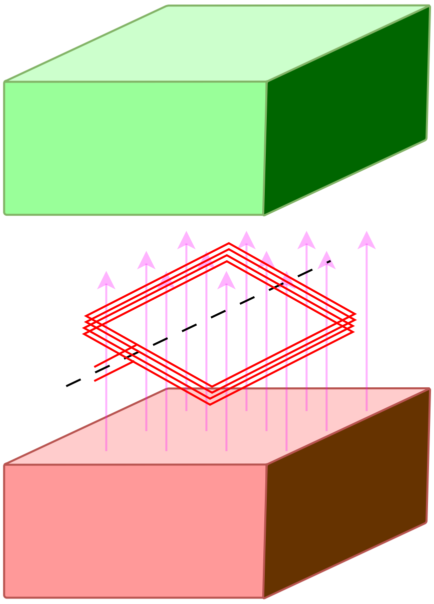


Exercise E1 Coil in a magnetic Field
(written test, approx. 4 % of a 120-minute written test, SS2021)

A coil with $n = 300$ turns and a cross-sectional area $A = 600 ~\rm cm^2$ is located in a homogeneous magnetic field.
The rotation of the coil causes a sinusoidal change in the magnetic field in the coil with the frequency $f = 80~\rm Hz$.
The maximum value of the magnetic flux density in the coil is $\hat{B} = 2 \cdot 10^{-6} ~\rm {{Vs}\over{cm^2}}$.

Result

\begin{align*} u_{\rm ind} = - 181 {~\rm V} \cdot \cos(503 {{1}\over{\rm s}} \cdot t) \\ \end{align*}



Derive the formula for the voltage induced in the coil and calculate the voltage amplitude.

Path

The induced voltage $u_{\rm ind}$ is given by:

$$\begin{align*} u_{\rm ind} &= - {{{\rm d}\Psi(t)}\over{{\rm d}t}} \\ &= - n{{{\rm d}\Phi(t)}\over{{\rm d}t}} \\ \end{align*}$$

With $\Phi(t)= B(t) \cdot A$, where A is the constant area of a single winding and $B(t)$ is the changing field through this winding.
Due to the rotation, the field changes as:

$$\begin{align*} B(t) &= \hat{B} \cdot \sin(\omega t + \varphi) \\ &= \hat{B} \cdot \sin(2\pi f \cdot t + \varphi) \\ \end{align*}$$

This leads to: $$\begin{align*} u_{\rm ind} &= - n{{{\rm d}}\over{{\rm d}t}}A \hat{B} \cdot \sin(2\pi f \cdot t + \varphi) \\ &= - n \cdot A \hat{B} \cdot 2\pi f \cdot \cos(2\pi \cdot f t + \varphi) \\ \end{align*}$$

The absolute value of the factor in front of the $\cos$ is the maximum induced voltage $\hat{U}_{\rm ind}$: $$\begin{align*} \hat{U}_{\rm ind} &= n \cdot A \hat{B} \cdot 2\pi f \\ &= 300 \cdot 0.06{~\rm m^2} \cdot 2 \cdot 10^{-2} ~\rm {{Vs}\over{m^2}} \cdot 2\pi \cdot 80 {{1}\over{\rm s}} \\ &= 180.95... {~\rm m^2} \cdot {{\rm Vs}\over{\rm m^2}} \cdot {{1}\over{\rm s}} \\ &= 180.95... {~\rm V} \\ \end{align*}$$

Exercise E2 effect of induction
(written test, approx. 5 % of a 120-minute written test, SS2021)

A single conductor loop is penetrated by a changing magnetic flux.
Result
The following figure shows the variation of the flux $\Phi(t)$ over time.

Calculate the variation of the induced voltage $u_{\rm ind}(t)$ over time and draw it in a separate diagram.

$$u...in$$\...

$$\..in$$\...

Path

Based on Faraday's Law of Induction the induced voltage is given by: \begin{align*} u_{\rm ind} =& - {{ {\rm d} }\over{ {\rm d}t}} \Psi(t) \bigg\rvert_{n=1}\\ =& - {{ {\rm d} }\over{ {\rm d}t}} \Phi(t) \\ \end{align*}

For a linear function, the derivative can be substituted by Deltas ($\rm d \rightarrow \Delta$):
\begin{align*} u_{\rm ind} = - {{ \Delta \Phi(t)}\over{ \Delta t}} = - { { \Phi(t_{\rm n+1}) - \Phi(t_{\rm n}) } \over { t_{\rm n+1} - t_{\rm n} } } \\ \end{align*}

For a piece-wise linear function, the induced voltage can be calculated for each interval.
Here, there are 5 different intervals - in the following called $\rm I$ to $\rm V$ from left to right:

$$\...in$$\...

- For the intervals $\rm I$, $\rm III$, and $\rm V$, the flux $\Phi(t)$ is constant. Therefore, $\Delta \Phi(t)=0$ and $u_{\rm ind}(t)=0{~\rm V}$

$$\..in$$\...

- For the interval $\rm II$:
  - The change in the flux is: $ \Delta \Phi(t) = 1.5 \cdot 10^{-4} {~\rm Vs} - 4.5 \cdot 10^{-4} {~\rm Vs}= - 3.0 \cdot 10^{-4} {~\rm Vs}$
  - The time span is: $0.2 ~\rm s$
  - Conclusively, the induced voltage is: $u_{\rm ind}(t) = + {{3.0 \cdot 10^{-4} {~\rm Vs}} \over {0.2 ~\rm s} } = 1.5 {~\rm mV}$

- For the interval $\rm IV$:
  - The change in the flux is: $ \Delta \Phi(t) = 0 \cdot 10^{-4} {~\rm Vs} - 1.5 \cdot 10^{-4} {~\rm Vs}= - 1.5 \cdot 10^{-4} {~\rm Vs}$
  - The time span is: $0.2 ~\rm s$
  - Conclusively, the induced voltage is: $u_{\rm ind}(t) = + {{1.5 \cdot 10^{-4} {~\rm Vs}} \over {0.2 ~\rm s} } = 0.75 {~\rm mV}$

$$\..in$$\...

Embedded resources

Explanation (video): …