

# Block 16 - Ampère's Law and Magnetomotive Force (MMF)

## Student Group

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# Block 16 - Ampère's Law and Magnetomotive Force (MMF)

## Learning objectives

After this 90-minute block, you can

- ...

## Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- ...

## 90-minute plan

1. Warm-up (x min):
  1. ....
2. Core concepts & derivations (x min):
  1. ...
3. Practice (x min): ...
4. Wrap-up (x min): Summary box; common pitfalls checklist.

## Conceptual overview

1. ...

## Core content

### Generalization of the Magnetic Field Strength

So far, only the rotational symmetric problem on a single wire was considered in formula, when the current  $I$  and the length  $s$  of a magnetic field line around it is given:

$$\begin{aligned} \quad H_{\varphi} &= \frac{I}{s} = \frac{I}{2 \cdot \pi \cdot r} \quad \Leftrightarrow \end{aligned}$$

$\oint \mathbf{H} \cdot d\mathbf{s} = I_{\text{enclosed}}$  | applies only to the long, straight conductor

Now, this shall be generalized. For this purpose, we will look back at the electric field.

For the electric field strength  $E$  of a capacitor with two plates at a distance of  $s$  and the potential difference  $U$  holds:

$U = E \cdot s$  | applies to capacitor only

In words: The potential difference is given by adding up the field strength along the path of a probe from one plate to the other.

This was extended to the voltage between two points  $1$  and  $2$ . Additionally, we know by Kirchhoff's voltage law that the voltage on a closed path is "0".

$U_{12} = \int_1^2 \mathbf{E} \cdot d\mathbf{s}$  |  $\oint \mathbf{E} \cdot d\mathbf{s} = 0$

We can now try to look for similarities. Also for the magnetic field, the magnitude of the field strength is summed up along a path to arrive at another field-describing quantity.

Because of the similarity the so-called **magnetic potential difference  $V_m$**  between point  $1$  and  $2$  is introduced:

$V_m = \int \mathbf{H} \cdot d\mathbf{s}$  | applies to rotational symmetric problems only

$V_m = \int_{m,1}^2 \mathbf{H} \cdot d\mathbf{s}$  |  $V_m = \oint \mathbf{H} \cdot d\mathbf{s} = \theta$

We need to take a closer look here. Any closed path in the static electric field leads to a potential difference of  $U = \oint \mathbf{E} \cdot d\mathbf{s} = 0$ .

BUT: closed paths in the static magnetic field leads to a magnetic potential difference which is **not mandatorily**  $0$ !  $V_m = \oint \mathbf{H} \cdot d\mathbf{s} = \theta$

Another new quantity is introduced: the **magnetic voltage  $\theta$** :

1. The magnetic voltage  $\theta$  is the magnetic potential difference on a closed path.
2. Since the magnetic voltage  $\theta$  is valid for exactly one turn along our single wire,  $\theta$  is also equal to the current through the wire:  
 $\theta = H \cdot s = I$  | applies only to the long, straight conductor
3. The magnetic potential difference can take a fraction or a multiple of one turn and is therefore **not mandatorily** equal to  $I$ .
4. The magnetic voltage is generalized in the following box.

### Notice:

The path integral of the magnetic field strength along an arbitrary closed path is equal to the free currents (= current density) through the surface enclosed by the path.

The magnetic voltage  $\theta$  (and therefore the current) is the cause of the magnetic field strength.

This leads to the **Ampere's Circuital Law**

$$\boxed{\oint_{\mathcal{S}} \vec{H} \cdot \mathrm{d}\vec{s} = \theta}$$

The magnetic voltage  $\theta$  can be given as

- $\theta = I$  for a single conductor
- $\theta = N \cdot I$  for a coil
- $\theta = \sum_n I_n$  for multiple conductors
- $\theta = \int_A \vec{S} \cdot \mathrm{d}\vec{A}$  for any spatial distribution (see [block15](#))

The unit of the magnetic voltage  $\theta$  is **Ampere** (or **Ampere-turns**).

In the english literature the magnetic voltage is called **Magnetomotive force**

### Notice:

$\oint_{\mathcal{S}} \vec{S} \cdot \mathrm{d}\vec{A}$  and  $\oint_{\mathcal{S}} \vec{A} \cdot \mathrm{d}\vec{s}$  in  $\oint_{\mathcal{S}} \vec{H} \cdot \mathrm{d}\vec{s} = \theta = \int_A \vec{S} \cdot \mathrm{d}\vec{A}$  build a right-hand system.

1. Once the thumb of the right hand is pointing along  $\oint_{\mathcal{S}} \vec{A} \cdot \mathrm{d}\vec{s}$ , the fingers of the right hand show the correct direction for  $\oint_{\mathcal{S}} \vec{S} \cdot \mathrm{d}\vec{A}$  for positive  $\oint_{\mathcal{S}} \vec{H} \cdot \mathrm{d}\vec{s}$  and  $\int_A \vec{S} \cdot \mathrm{d}\vec{A}$
2. Currents into the direction of the right hand's thumb count positive. Currents antiparallel to it count negative.

Fig. 1: Right hand rule



## Common pitfalls

- ...

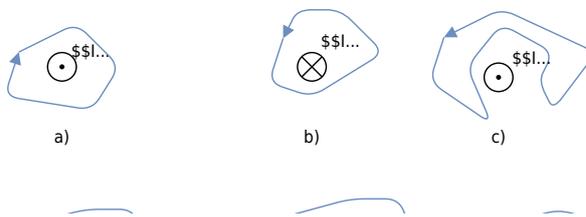
## Exercises

### Task 3.2.3 Magnetic Potential Difference

Fig. 3: different trajectories around current-carrying conductors

Result  $\vec{H}$ )

$$e) \oint_{\Gamma} \vec{H} \cdot d\vec{s} = 20 \text{ A} \cdot \text{m} = 20 \text{ A} \cdot \text{m} \cdot \frac{1}{2} = 10 \text{ A} \cdot \text{m} = 10 \text{ A} \cdot \text{m} \cdot \frac{1}{2} = 5 \text{ A} \cdot \text{m} = 5 \text{ A} \cdot \text{m} \cdot \frac{1}{2} = 2.5 \text{ A} \cdot \text{m}$$



Given are the adjacent closed trajectories in the magnetic field of current-carrying conductors (see figure 3). Let  $I_1 = 2 \text{ A}$  and  $I_2 = 4.5 \text{ A}$  be valid.

In each case, the magnetic potential difference  $V_{\text{m}}$  along the drawn path is sought.

Path

- The magnetic potential difference is given as the **sum of the current through the area within a closed path**.
- The direction of the current and the path have to be considered with the righthand rule.

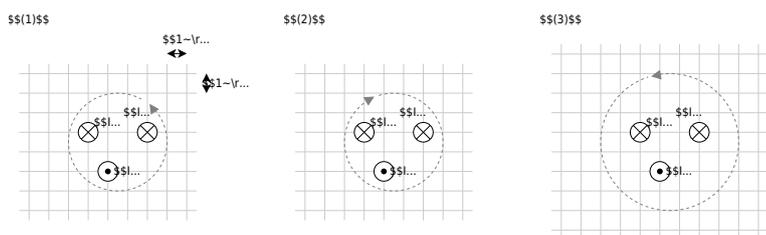
**Exercise E7 Magnetic Voltage**  
**(written test, approx. 6 % of a 120-minute written test, SS2021)**

The following images show cross-sections of electrical cables.

A closed path is shown as a dashed line. The magnetic voltage  $\theta$  on these paths shall be analyzed.

The following values are given for the currents:

$I_1 = 5 \text{ A}$      $I_2 = 5 \text{ A}$      $I_3 = 1 \text{ A}$      $I_4 = 4 \text{ A}$   
 $I_1$  and  $I_2$  are directed into the page,  $I_3$  and  $I_4$  are directed out of the page.



Specify which magnetic voltages  $\theta_{(1)}$ ,  $\theta_{(2)}$ , and  $\theta_{(3)}$  result. Note the direction of the path in each case!

Path

For the resulting current the direction of the path has to be considered with the right-hand rule:

- $I_{(1)} = +I_2 - I_1 - I_3 \quad \rightarrow \quad \theta_{(1)} = 2 \text{ A}$

- $I_{(2)} = +I_3 + I_4 - I_1 \quad \rightarrow \quad \theta_{(2)} = 1 \text{ A} + 4 \text{ A} - 5 \text{ A}$
- $I_{(3)} = +I_3 - I_4 - I_2 \quad \rightarrow \quad \theta_{(3)} = 1 \text{ A} - 4 \text{ A} - 2 \text{ A}$

**Exercise E1 Magnetic Field Lines**  
**(written test, approx. 6 % of a 120-minute written test, SS2024)**

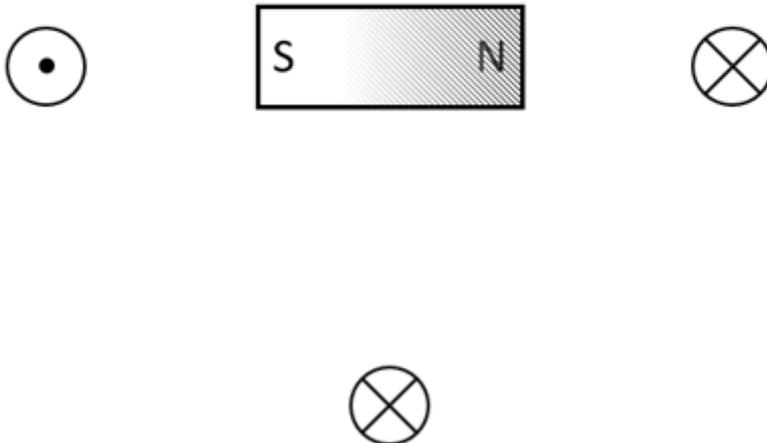
The following setup of a permanent magnet affects the H-field, based on the fundamental definition of the H-field.

- Four conductors are located perpendicular to the plane of the diagram

All of them conduct a current with the same magnitude, but not in the same direction.

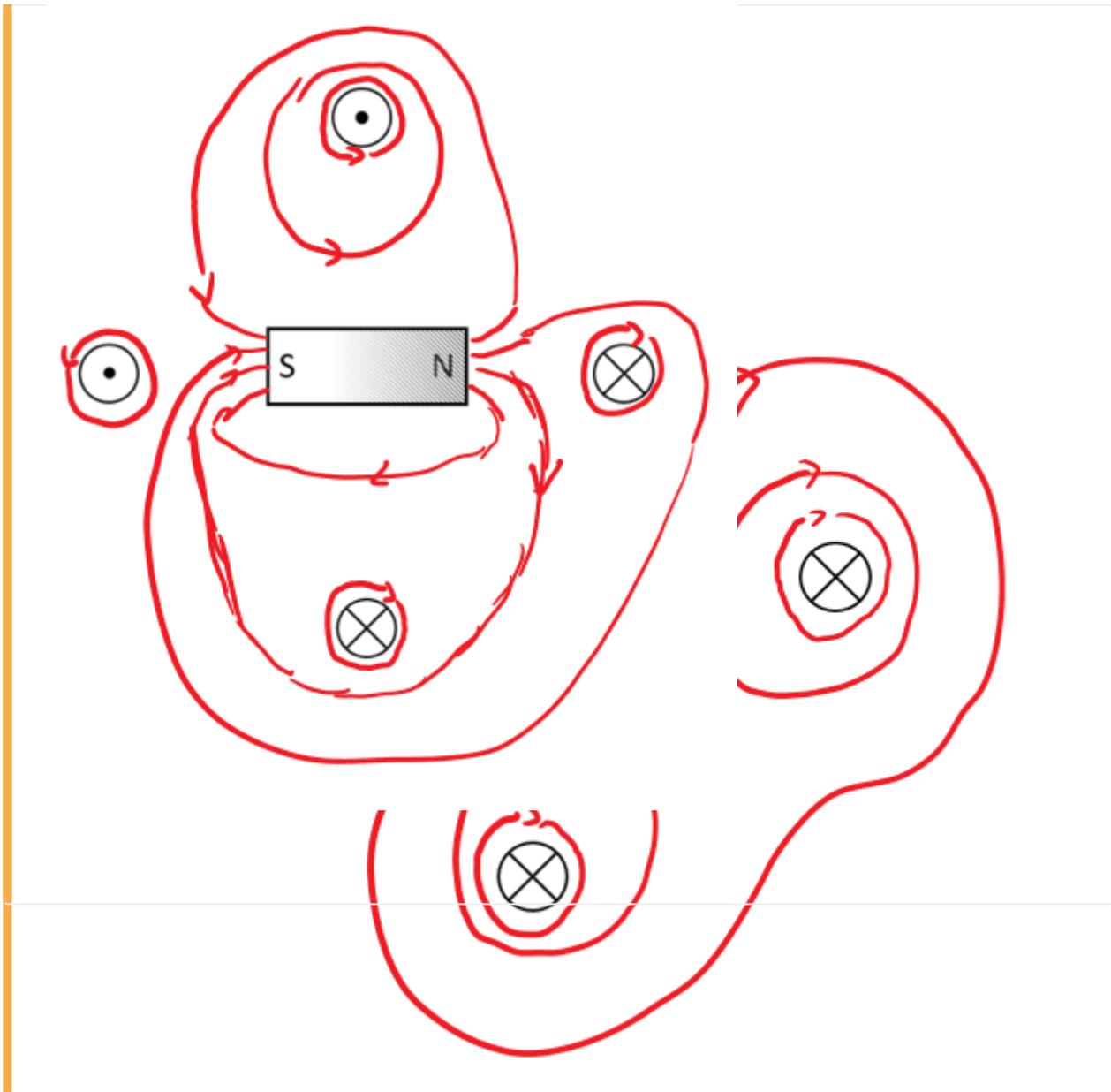
- A permanent magnet is located in between the conductors.

- The H-field is defined by currents  $\sum I = \int H \, ds$ .
- In the permanent magnet, there are no free currents.
- The bound currents (of the permanent magnet) create also an H field.
- This exits on the north pole and enters the magnet on the south pole (similar to the B-field)
- $H = B/\mu$
- The H-field from task 1 gets distracted



Do not consider the permanent magnet at first. Draw at least 10 field lines of the H-field qualitatively. Give a correct representation of their direction, and density for the shown area.

Result



**Exercise E10 Magnetic Potential**  
**(written test, approx. 8 % of a 120-minute written test, SS2024)**

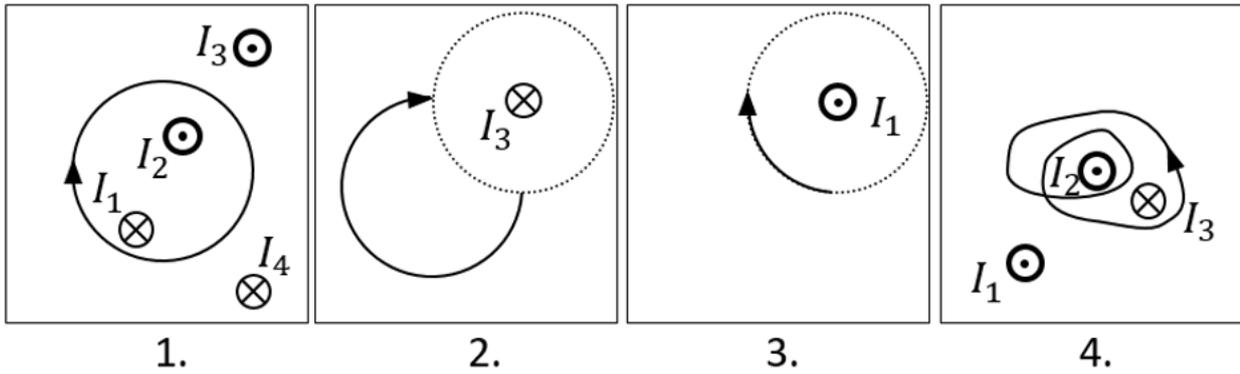
Calculate the magnetic potential difference  $V_{\text{m}}$  for the following paths as shown by the solid lines.

Dotted lines are only for there for symmetry aspects!

The wires conduct the following currents:

- $|I_1| = 2 \text{ A}$
- $|I_2| = 5 \text{ A}$
- $|I_3| = 11 \text{ A}$
- $|I_4| = 7 \text{ A}$

Pay attention to the signs of the currents (given by the diagrams) and of the results!



Result

Based on the right-hand rule and the part of a full revolution the following results:

1. Task:  $+I_1 - I_2 = -3 \text{ A}$
2. Task:  $+ \frac{1}{4} I_3 = 11/4 \text{ A}$  (it does not matter which way the path goes from the startpoint to the endpoint, as long as it has the same direction and number of revolutions)
3. Task:  $- \frac{1}{4} I_1 = -0.5 \text{ A}$
4. Task:  $+2 \cdot I_2 - 1 \cdot I_3 = -1 \text{ A}$

**Exercise E6 Fields of an coax Cable**  
 (written test, approx. 12 % of a 120-minute written test, SS2024)

2. On the graph of the magnitude of the electric field  $E$  with the radius  $r$  of the coax cable (0.55 mm) as the x-axis and the electric field intensity  $E$  in  $\text{V/m}$  as the y-axis, the diagram below is shown. The diagram shows the cross-section of the coax cable with the origin  $r=0$  in the center of the inner conductor. The diagram is labeled as follows:

path

- Inner conductor:  $+3.3 \text{ mA}$ ,  $+10 \text{ nC}$  (current into the plane of the diagram)
- for  $(0.1 \text{ mm} | 0)$ :  $E_{\text{in}} = 328.9 \text{ V/m}$
- Outer conductor:  $-3.3 \text{ mA}$ ,  $0 \text{ nC}$  (current out of the plane of diagram)
- for  $(0.55 \text{ mm} | 0)$ :  $E_{\text{out}} = 0.985 \text{ V/m}$

The magnitude of the electric displacement field  $D$  can be calculated by:  $\int D \cdot dA = Q$ .

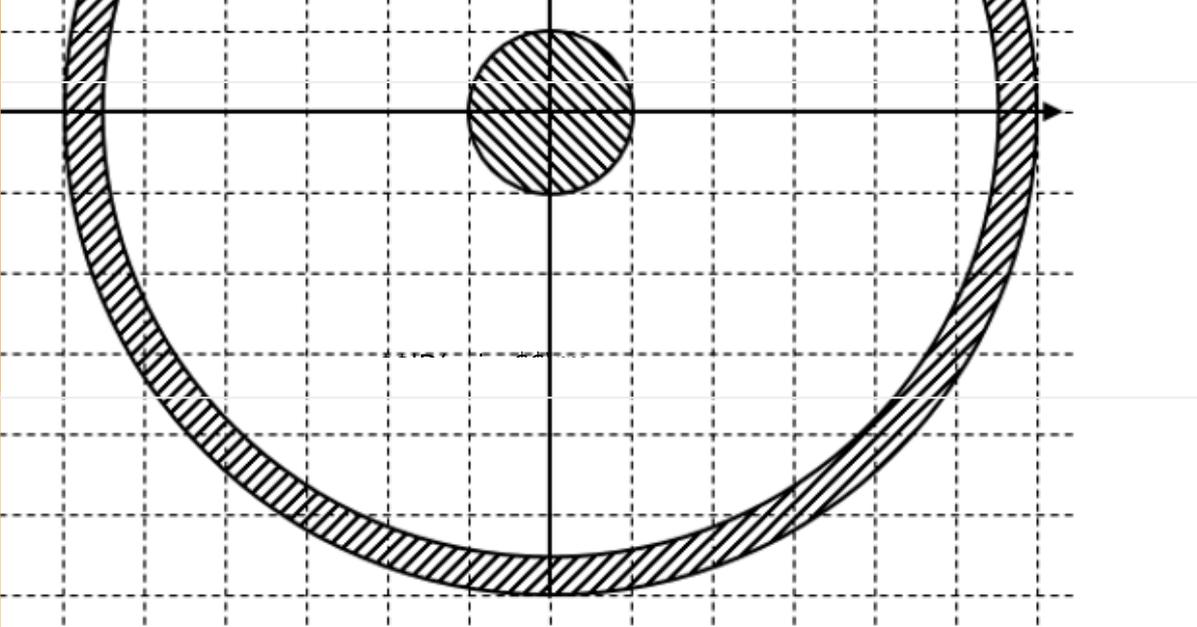
Here, for any position radial to the center, the surrounding area is the surface of a cylindrical shape (here for simplicity without the round endings).

- In general, the  $E$ -field is proportional to  $\frac{1}{r}$  for the situation between both conductors.
- Since the charges are on the surfaces of the conductors, there is no  $D$ -field within the conductors.

This leads to:  $D(x) = \frac{1}{2\pi r} \cdot Q$  within a circle with the radius  $x$ .

So, we get for  $H_{\text{outer}}$  at  $(0.1 \text{ mm} | 0)$  and  $H_{\text{inner}}$  at  $(0.55 \text{ mm} | 0)$   $H = \frac{I}{2 \pi \cdot r}$  gets  $H(x) = \frac{I}{2 \pi \cdot x} \cdot \pi \cdot x^2 \cdot \frac{1}{(0.1 \text{ mm})^2}$ . This leads to a formula proportional to  $x$ .  
 For  $H_{\text{outer}}$  with the outer conductor also gets a formula proportional to  $x$  with a  $\frac{1}{10^9}$  factor.  $H_{\text{outer}} = \frac{I}{2 \pi \cdot \{0.1 \cdot 10^{-3} \text{ m}\}} \cdot \{10 \cdot 10^{-9} \text{ C}\} = \frac{10 \cdot 10^{-9}}{2 \pi \cdot \{0.55 \cdot 10^{-3} \text{ m}\} \cdot 0.5 \text{ mm}}$

Hint: For the direction, one has to consider the sign of the enclosed charge. By this, we see that the  $D$ -field is positive.  
 But here, again only the magnitude was questioned!



.. What is the magnitude of the magnetic field strength  $H$  at  $(0.1 \text{ mm} | 0)$  and  $(0.55 \text{ mm} | 0)$ ?

Path

The magnitude of the magnetic field strength  $H$  can be calculated by:  $H = \frac{I}{2 \pi \cdot r}$   
 So, we get for  $H_{\text{inner}}$  at  $(0.1 \text{ mm} | 0)$ , and  $H_{\text{outer}}$  at  $(0.55 \text{ mm} | 0)$ :  

$$H_{\text{inner}} = \frac{I}{2 \pi \cdot r_{\text{inner}}} = \frac{+3.3 \text{ A}}{2 \pi \cdot \{0.1 \cdot 10^{-3} \text{ m}\}}$$

$$H_{\text{outer}} = \frac{I}{2 \pi \cdot r_{\text{outer}}} = \frac{+3.3 \text{ A}}{2 \pi \cdot \{0.55 \cdot 10^{-3} \text{ m}\}}$$

Hint: For the direction, one has to consider the right-hand rule. By this, we see that the  $H$ -field on the right side points downwards.  
 Therefore, the sign of the  $H$ -field is negative.  
 But here, only the magnitude was questioned!

## Embedded resources

Explanation (video): ...

From:

<https://mexle.te.hs-heilbronn.de/> - MEXLE Wiki

Permanent link:

[https://mexle.te.hs-heilbronn.de/electrical\\_engineering\\_and\\_electronics\\_1/block16?rev=1763838138](https://mexle.te.hs-heilbronn.de/electrical_engineering_and_electronics_1/block16?rev=1763838138)

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