

# Block 16 - Ampère's Law and Magnetomotive Force (MMF)

## Student Group

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# Block 16 - Ampère's Law and Magnetomotive Force (MMF)

## Learning objectives

After this 90-minute block, you can

- ...

## Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- ...

## 90-minute plan

1. Warm-up (x min):
  1. ....
2. Core concepts & derivations (x min):
  1. ...
3. Practice (x min): ...
4. Wrap-up (x min): Summary box; common pitfalls checklist.

## Conceptual overview

1. ...

## Core content

### Generalization of the Magnetic Field Strength

So far, only the rotational symmetric problem on a single wire was considered in formula, when the current  $I$  and the length  $s$  of a magnetic field line around it is given:

$$\begin{aligned} \quad H_{\varphi} &= \frac{I}{s} = \frac{I}{2 \cdot \pi \cdot r} \quad \Leftrightarrow \end{aligned}$$

$\oint \mathbf{H} \cdot d\mathbf{s} = I_{\text{enc}} \quad | \quad \text{applies only to the long, straight conductor}$

Now, this shall be generalized. For this purpose, we will look back at the electric field.

For the electric field strength  $E$  of a capacitor with two plates at a distance of  $s$  and the potential difference  $U$  holds:

$U = E \cdot s \quad | \quad \text{applies to capacitor only}$

In words: The potential difference is given by adding up the field strength along the path of a probe from one plate to the other.

This was extended to the voltage between two points  $1$  and  $2$ . Additionally, we know by Kirchhoff's voltage law that the voltage on a closed path is "0".

$U_{12} = \int_1^2 \mathbf{E} \cdot d\mathbf{s} \quad | \quad U = \oint \mathbf{E} \cdot d\mathbf{s} = 0$

We can now try to look for similarities. Also for the magnetic field, the magnitude of the field strength is summed up along a path to arrive at another field-describing quantity.

Because of the similarity the so-called **magnetic potential difference  $V_m$**  between point  $1$  and  $2$  is introduced:

$V_m = \int_1^2 \mathbf{H} \cdot d\mathbf{s} \quad | \quad \text{applies to rotational symmetric problems only}$

$V_m = \int_1^2 \mathbf{H} \cdot d\mathbf{s} \quad | \quad V_m = \oint \mathbf{H} \cdot d\mathbf{s} = \theta$

We need to take a closer look here. Any closed path in the static electric field leads to a potential difference of  $U = \oint \mathbf{E} \cdot d\mathbf{s} = 0$ .

BUT: closed paths in the static magnetic field leads to a magnetic potential difference which is **not mandatorily**  $0$ !  $V_m = \oint \mathbf{H} \cdot d\mathbf{s} = \theta$

Another new quantity is introduced: the **magnetic voltage  $\theta$** :

1. The magnetic voltage  $\theta$  is the magnetic potential difference on a closed path.
2. Since the magnetic voltage  $\theta$  is valid for exactly one turn along our single wire,  $\theta$  is also equal to the current through the wire:  
 $\theta = I \quad | \quad \text{applies only to the long, straight conductor}$
3. The magnetic potential difference can take a fraction or a multiple of one turn and is therefore **not mandatorily** equal to  $I$ .
4. The magnetic voltage is generalized in the following box.

### Notice:

The path integral of the magnetic field strength along an arbitrary closed path is equal to the free currents (= current density) through the surface enclosed by the path.

The magnetic voltage  $\theta$  (and therefore the current) is the cause of the magnetic field strength.

This leads to the **Ampere's Circuital Law**

$\boxed{\oint_S \vec{H} \cdot d\vec{s} = \theta}$	The magnetic voltage $\theta$ can be given as <ul style="list-style-type: none"> <li><math>\theta = I</math> for a single conductor</li> <li><math>\theta = N \cdot I</math> for a coil</li> <li><math>\theta = \sum_n I_n</math> for multiple conductors</li> <li><math>\theta = \iint_A \vec{S} \cdot d\vec{A}</math> for any spatial distribution (see <a href="#">block15</a>)</li> </ul>
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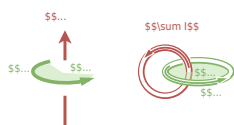
The unit of the magnetic voltage  $\theta$  is **Ampere** (or **Ampere-turns**).

**Notice:**

$\oint_S \vec{s}$  and  $\oint_S \vec{A}$  in  $\oint_S \vec{H} \cdot d\vec{s} = \theta = \iint_A \vec{S} \cdot d\vec{A}$  build a right-hand system.

1. Once the thumb of the right hand is pointing along  $\oint_S \vec{A}$ , the fingers of the right hand show the correct direction for  $\oint_S \vec{s}$  for positive  $\vec{H}$  and  $\vec{S}$
2. Currents into the direction of the right hand's thumb count positive. Currents antiparallel to it count negative.

Fig. 1: Right hand rule



# Common pitfalls

- ...

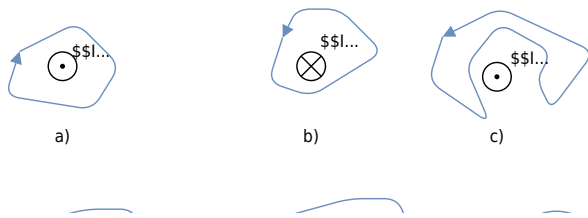
# Exercises

## Task 3.2.3 Magnetic Potential Difference

Fig. 3: different trajectories around current-carrying conductors

Result **b)**

$$b) \int_C \vec{H} \cdot d\vec{s} = 2 \cdot 2 \text{ A} = 4 \text{ A} \Rightarrow I_{enc} = 4 \text{ A} \Rightarrow \oint_C \vec{H} \cdot d\vec{s} = 4 \text{ A} \Rightarrow H \cdot 2\pi r = 4 \text{ A} \Rightarrow H = \frac{4 \text{ A}}{2\pi r} = \frac{2 \text{ A}}{\pi r} \Rightarrow V_m = - \int_C \vec{H} \cdot d\vec{s} = - \int_C \frac{2 \text{ A}}{\pi r} \cdot dr = - \frac{2 \text{ A}}{\pi} \ln r + C$$



Given are the adjacent closed trajectories in the magnetic field of current-carrying conductors (see figure 3). Let  $I_1 = 2 \text{ A}$  and  $I_2 = 4.5 \text{ A}$  be valid.

In each case, the magnetic potential difference  $V_m$  along the drawn path is sought.

## Path

- The magnetic potential difference is given as the **sum of the current through the area within a closed path**.
- The direction of the current and the path have to be considered with the righthand rule.

**Exercise E7 Magnetic Voltage****(written test, approx. 6 % of a 120-minute written test, SS2021)**

The following images show cross-sections of electrical cables.

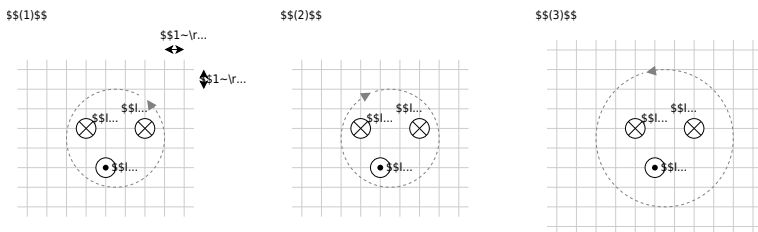
A closed path is shown as a dashed line. The magnetic voltage  $\oint \vec{H} \cdot d\vec{s}$  on these paths shall be analyzed.

The following values are given for the currents:

- ```

\begin{align*}
I_1 &= 5 \text{ A} \\
I_2 &= 2 \text{ A} \\
I_3 &= 1 \text{ A} \\
I_4 &= 4 \text{ A}
\end{align*}

```
- $I_1 = 5 \text{ A}$
  - $I_2 = 2 \text{ A}$
  - $I_3 = 1 \text{ A}$
  - $I_4 = 4 \text{ A}$



Specify which magnetic voltages  $\theta_{(1)}$ ,  $\theta_{(2)}$ , and  $\theta_{(3)}$  result. Note the direction of the path in each case!

Path

For the resulting current the direction of the path has to be considered with the right-hand rule:

- $I_{(1)} = +I_2 - I_1 - I_3 \quad \rightarrow \quad \theta_{(1)} = 2 \text{ A} - 5 \text{ A} - 1 \text{ A}$
- $I_{(2)} = +I_3 + I_4 - I_1 \quad \rightarrow \quad \theta_{(2)} = 1 \text{ A} + 4 \text{ A} - 5 \text{ A}$
- $I_{(3)} = +I_3 - I_4 - I_2 \quad \rightarrow \quad \theta_{(3)} = 1 \text{ A} - 4 \text{ A} - 2 \text{ A}$

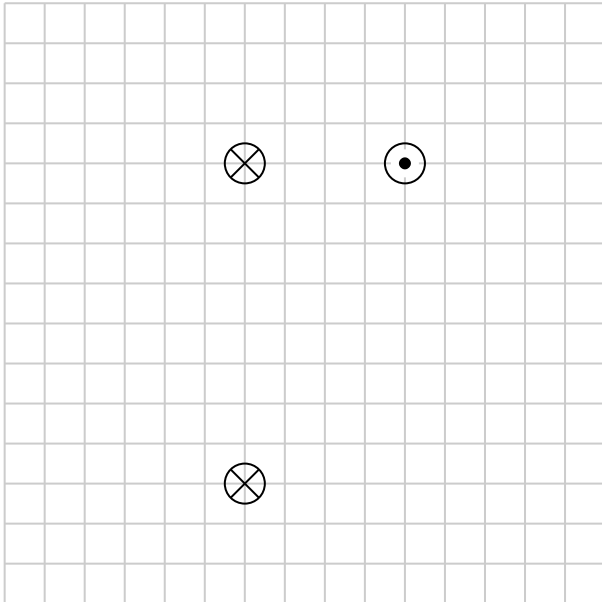
### Exercise E2 Magnetic Field Lines

**(written test, approx. 4 % of a 120-minute written test, SS2021)**

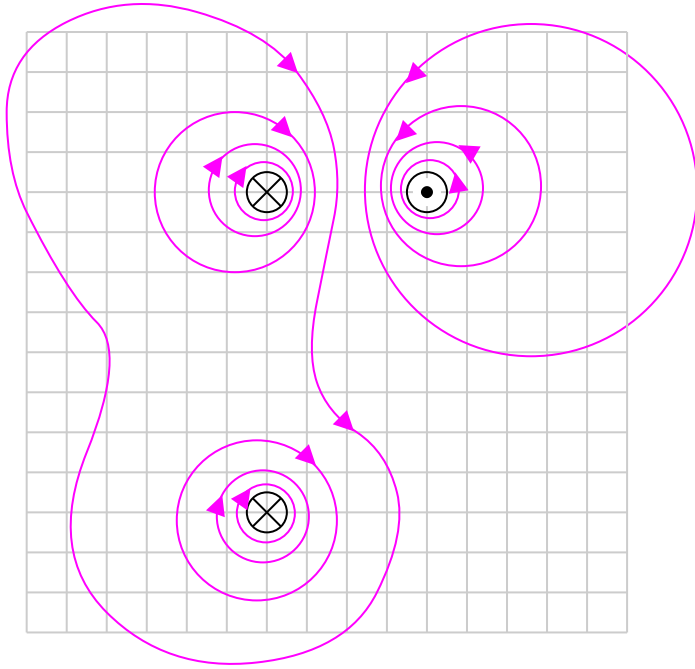
Several parallel conductors are projecting out of the plane.

The same current  $I$  flows through all the conductors in different directions (see image below).

Sketch at least 10 field lines of the magnetic field strength  $\vec{H}$  in such a way that the different properties of the field lines (e.g. direction and density) can be seen.

**Result**

- high density of field lines near the conductors
- direction of the field lines given by the right-hand rule
- magnetic field has closed field lines
- resulting field given by superposition of field lines



### Exercise E1 Magnetic Field Lines (written test, approx. 6 % of a 120-minute written test, SS2024)

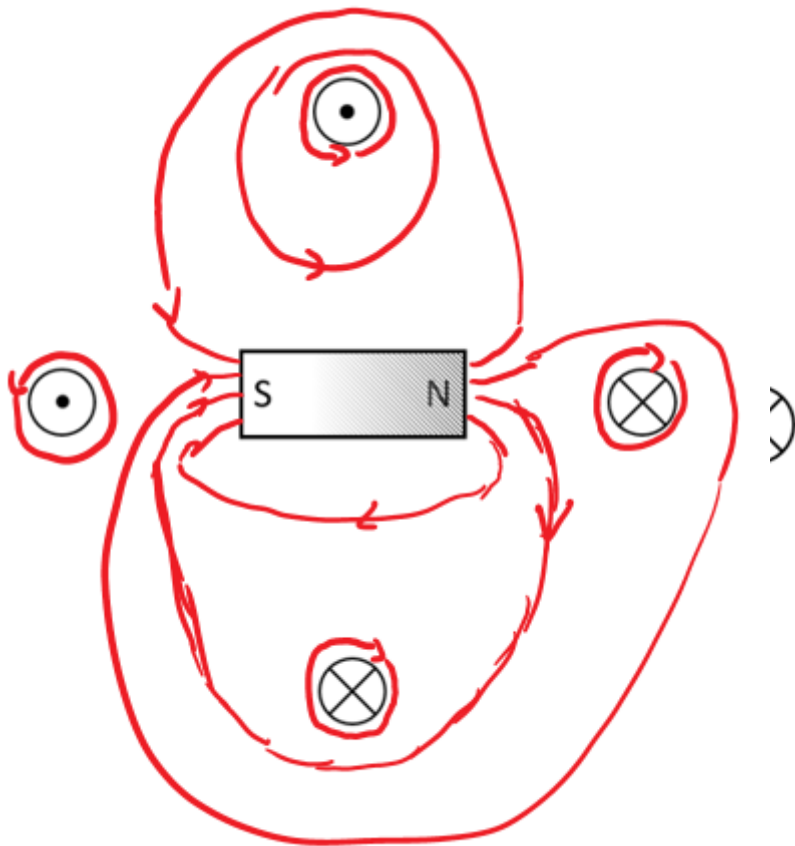
The following setup of a permanent magnet affects the H-field, based on the fundamental definition of the H-field.

- Four conductors are located perpendicular to the plane of the diagram

Result: All of them conduct a current with the same magnitude, but not in the same direction.

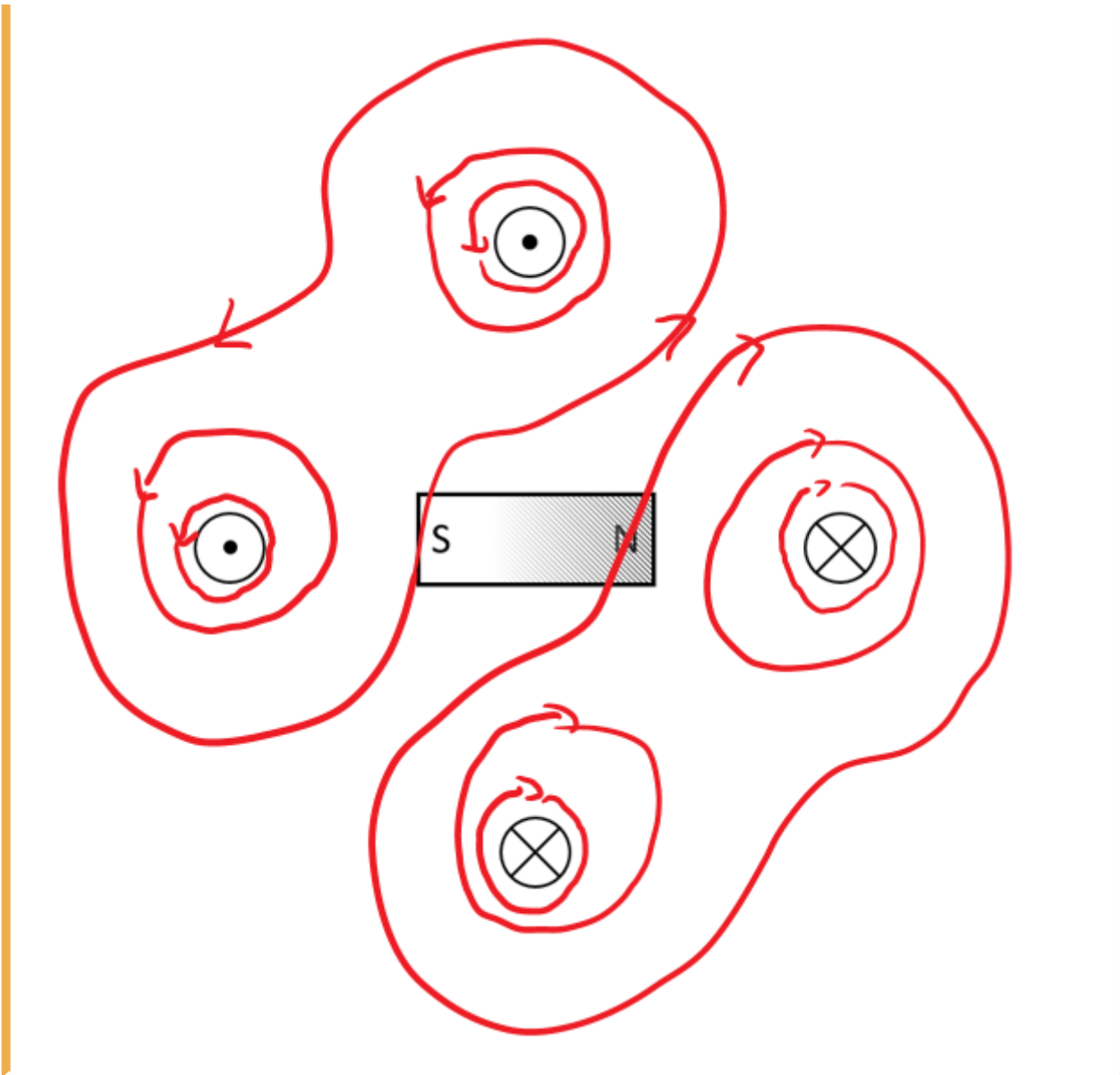
- A permanent magnet is located in between the conductors.

- The H-field is defined by currents  $\sum I = \int H \{ \text{r m d} \} s$  .
- In the permanent magnet, there are no free currents.
- The bound currents (of the permanent magnet) create also an H field.
- This exits on the north pole and enters the magnet on the south pole (similar to the B-field)\_
- $H = B/\mu$
- The H-field from task 1 gets distracted



... Do not consider the permanent magnet at first. Draw at least 10 field lines of the H-field qualitatively. Give a correct representation of their direction, and density for the shown area.

Result



**Exercise E10 Magnetic Potential**  
**(written test, approx. 8 % of a 120-minute written test, SS2024)**

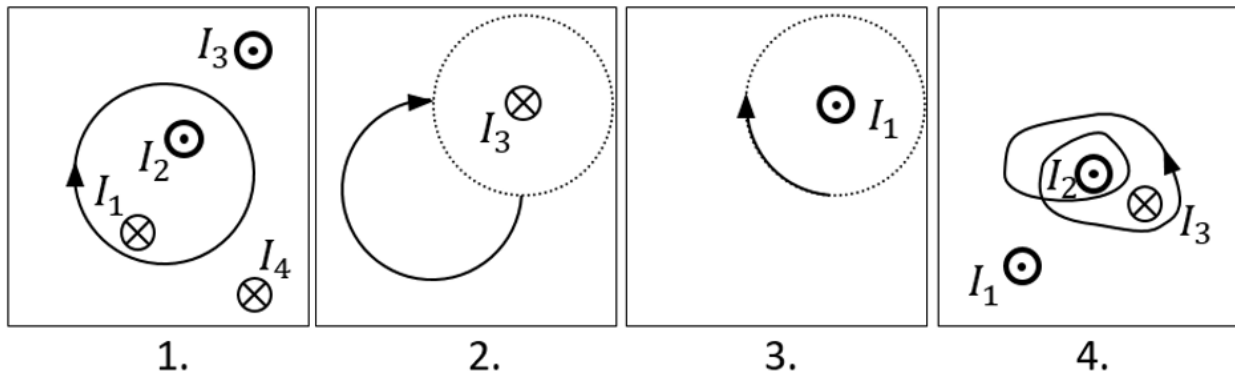
Calculate the magnetic potential difference  $V_{\text{m}}$  for the following paths as shown by the solid lines.

Dotted lines are only for there for symmetry aspects!

The wires conduct the following currents:

- $|I_1|=2 \text{ A}$
- $|I_2|=5 \text{ A}$
- $|I_3|=11 \text{ A}$
- $|I_4|=7 \text{ A}$

Pay attention to the signs of the currents (given by the diagrams) and of the results!



## Result

Based on the right-hand rule and the part of a full revolution the following results:

1. Task:  $+I_1 - I_2 = -3 \text{ ~\rm A}$
2. Task:  $+{\frac{1}{4}} I_3 = 11/4 \text{ ~\rm A}$  (it does not matter which way the path goes from the startpoint to the endpoint, as long as it has the same direction and number of revolutions)
3. Task:  $-{\frac{1}{4}} I_1 = -0.5 \text{ ~\rm A}$
4. Task:  $+2 \cdot I_2 - 1 \cdot I_3 = -1 \text{ ~\rm A}$

## Embedded resources

Explanation (video): ...

From:

<https://mexle.te.hs-heilbronn.de/> - MEXLE Wiki

Permanent link:

[https://mexle.te.hs-heilbronn.de/electrical\\_engineering\\_and\\_electronics\\_1/block16?rev=1763836316](https://mexle.te.hs-heilbronn.de/electrical_engineering_and_electronics_1/block16?rev=1763836316)

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