Block 16 - Ampère's Law and Magnetomotive Force (MMF)

Student Group

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Learning objectives

After this 90-minute block, you can

- …

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- …

90-minute plan

1. Warm-up (x min):
    1. ….
2. Core concepts & derivations (x min):
    1. …
3. Practice (x min): …
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

1. …

Core content

Generalization of the Magnetic Field Strength

So far, only the rotational symmetric problem on a single wire was considered in formula, when the current I ant the length s of a magnetic field line around it is given:

\begin{align*} \quad H_\varphi ={I\over{s}} = {{I}\over{2 \cdot \pi \cdot r}} \quad \Leftrightarrow

\quad I = H_\varphi \cdot {s} \quad \quad \quad | \quad \text{applies only to the long, straight conductor} \end{align*}

Now, this shall be generalized. For this purpose, we will look back at the electric field.
For the electric field strength E of a capacitor with two plates at a distance of s and the potential difference U holds:

\begin{align*} U = E \cdot s \quad \quad | \quad \text{applies to capacitor only} \end{align*}

In words: The potential difference is given by adding up the field strength along the path of a probe from one plate to the other.
This was extended to the vltage between to points 1 and 2. Additionally, we know by Kirchhoff's voltage law that the voltage on a closed path is “0”.

\begin{align*} U_{12} &= \int_1^2 \vec{E} \; {\rm d}\vec{s} \\ U &= \oint \vec{E} \; {\rm d}\vec{s} =0 \\ \end{align*}

We can now try to look for similarities. Also for the magnetic field, the magnitude of the field strength is summed up along a path to arrive at another field-describing quantity.
Because of the similarity the so-called **magnetic potential difference V_m** is introduced:

\begin{align*} V_m &= H \cdot s \quad \quad | \quad \text{applies to rotational symmetric problems only} \\ \end{align*}

\begin{align*} \boxed{ V_m = V_{m, 12} = \int_1^2 \vec{H} \; {\rm d}\vec{s} \\ V_m = \oint \vec{H} {\rm d}\vec{s} = \theta } \end{align*}

We need to take a loser look here. Any closed path in the static electric field leads to a potential difference of $U = \oint \vec{E} \; {\rm d}\vec{s} =0$.
BUT: closed paths in the static magnetic field leads to a magnetic potential difference which is **not mandatorily** 0! $ V_m = \oint \vec{H} {\rm d}\vec{s} = \theta$

Another new quantity is introduced: the **magnetic voltage θ**. The magnetic voltage is the magnetic potential difference on a closed path.

Now, what is the difference between the magnetic potential difference V_m and the magnetic voltage θ?

1. The first equation of the toroidal coil ($\theta = H \cdot l$) is valid for exactly <u>one turn</u> along a field line with the length l. In addition, the magnetic voltage was equal to the current times the number of windings: $\theta = N \cdot I$.
2. The second equation ($V_{\rm m} = H \cdot s$) is independent of the length of the field line l. Only if $s = l$ is chosen, the magnetic voltage equals the magnetic potential difference. The path length s can be a fraction or multiple of a single revolution l for the magnetic potential difference.

Thus, for each infinitesimally small path ${\rm d}s$ along a field line, the resulting infinitesimally small magnetic potential difference ${\rm d}V_{\rm m} = H \cdot {\rm d}s$ can be determined. If now along the field line the magnetic field strength $H = H(\vec{s})$ changes, then the magnetic potential difference from point $\vec{s_1}$ to point $\vec{s_2}$ results to:

\begin{align*} V_{\rm m12} = V_{\rm m}(\vec{s_1}, \vec{s_2}) = \int_\vec{s_1}^\vec{s_2} H(\vec{s}) {\rm d}s \end{align*}

Up to now, only the situation was considered that one always walks along one single field line. $\vec{s}$ therefore always arrived at the same spot of the field line. If one wants to extend this to arbitrary directions (also perpendicular to field lines), then only that part of the magnetic field strength $\vec{H}$ may be used in the formula, which is parallel to the path ${\rm d} \vec{s}$. This is made possible by scalar multiplication. Thus, it is generally valid:

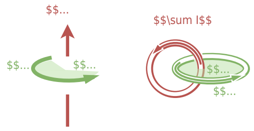
\begin{align*} \boxed{V_{\rm m12} = \int_\vec{s_1}^\vec{s_2} \vec{H} \cdot {\rm d} \vec{s}} \end{align*}

The magnetic voltage θ (and therefore the current) is the cause of the magnetic field strength. From the chapter The stationary Electric Flow the general representation of the current through a surface is known. This leads to the **Ampere's Circuital Law**

\begin{align*} \boxed{\oint_{s} \vec{H} \cdot {\rm d} \vec{s} = \iint_A \; \vec{S} {\rm d}\vec{A} = \theta} \end{align*}

- The path integral of the magnetic field strength along an arbitrary closed path is equal to the free currents (= current density) through the surface enclosed by the path.
- The magnetic voltage θ can be given as
  - for a single conductor: $\theta = I$
  - for a coil: $\theta = N \cdot I$
  - for multiple conductors: $\theta = \sum_n \cdot I_n$
  - for spatial distribution: $\theta = \iint_A \; \vec{S} {\rm d}\vec{A}$
- ${\rm d}\vec{s}$ and ${\rm d}\vec{A}$ build a right-hand system: once the thumb of the right hand is pointing along ${\rm d}\vec{A}$, the fingers of the right hand show the correct direction for ${\rm d}\vec{s}$ for positive $\vec{H}$ and $\vec{S}$

Fig. 1: Right hand rule



Common pitfalls

- ...

Exercises

Worked examples

...

Embedded resources

Explanation (video): ...