Block 14 - The steady Conduction Field

Student Group

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Block 14 - The steady Conduction Field

14.0 Intro

14.0.1 Learning objectives

> After this 90-minute block, you can
>
> - explain what a **steady (stationary) conduction field** is and relate it to the electrostatic field (cause/effect view: $\vec{E}$ vs. $\vec{D}$; conduction uses $\vec{E}$ and material σ).
> - calculate **conductance** G and **resistance** R for key geometries (parallel plates , coaxial conductor).

14.0.2 Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- 2.2.2

14.0.3 90-minute plan

1. Warm-up (10 min):
    1. Quick recap of Block 11 field pictures (parallel plates, coax) → link to resistance by replacing ε with σ.
    2. Mini check: which vector integrates over length/area? ($\vec{E}$ along paths, $\vec{J}$ across areas)
2. Core concepts (20 min):
    1. Definitions: steady conduction, $\vec{j}=\sigma\vec{E}$, current I.
    2. From **potential drop** to **Ohm’s law** in fields.
3. Guided derivations (25 min):
    1. Parallel-plate bar
    2. Coaxial conductor
4. Practice (30 min):
    1. Short exercises: compute R for a busbar, and for a coax segment; compare materials (copper vs. aluminum).
    2. “What-if” variations: halve l, double A, change σ; predict R qualitatively before computing.
5. Wrap-up (5 min):
    1. Summary box (key formulas, units); **Common pitfalls** checklist and Q&A.

14.0.4 Conceptual overview

1. **Analogy:** Replace *displacement flow* in dielectrics ($\vec{D}=\varepsilon\vec{E}$, charge storage) by **flow density** in conductors ($\vec{J}=\sigma\vec{E}$, charge transport).
   Driving cause is still the electric field $\vec{E}$; the material parameter changes from ε to $\sigma=\dfrac{1}{\rho}$.
2. **Global relations:** Voltage is a line integral $U=\int \vec{E}\cdot{\rm d}\vec{s}$; current is a flux integral $I=\iint_A \vec{J}\cdot{\rm d}\vec{A}$.
   Their ratio defines $G=\dfrac{I}{U}$ and $R=\dfrac{U}{I}$ for a given geometry and material.
3. **Geometry matters:** Uniform fields (parallel plates) give $E=\text{const}$ and simple $G=\dfrac{\sigma A}{l}$.
   Curved fields (coax) spread with radius → logarithmic dependence.
4. **Checks:** Units (σ in $\rm S/m$, G in $\rm S$, R in Ω). Limits:
   $A\!\to\!\infty \Rightarrow R\!\to\!0$
   $l\!\to\!0 \Rightarrow R\!\to\!0$
   $r_a\!\downarrow r_i \Rightarrow R\!\to\!0$.

14.1 Core content

In the discussion of the electrostatic field in principle, no charges in motion were considered. This lead to multiple fomulas, which are aggregated in the following diagram:

Fig. 1: summary of electro static field

One outcome was, that the capacitance is defined as:

\begin{align*} C &= {{Q}\over{U}} \\ &= {{ \rlap{\Large \rlap{\int_A} \int} \, \LARGE \circ} \;\; \vec{D} \, {\rm d} \vec{A}\over{\int \vec{E} \,{\rm d} \vec{s} }} \end{align*}

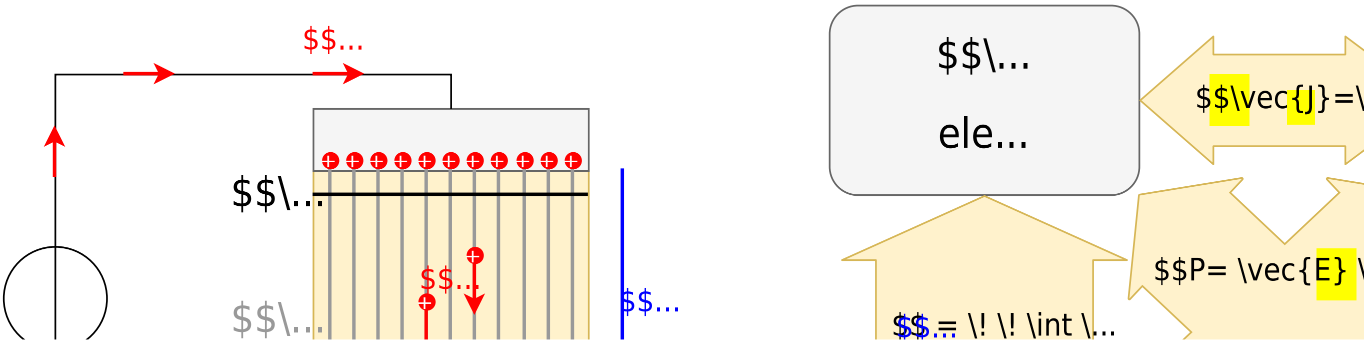
Now the motion of charges shall be considered explicitly.
With the knowledge of th electrostatic field, we want to see, whether we can calulate the resistance of more complicated geometries.

For this we want to introduce the current density J: The current density here describes how charge carriers move together (collectively).
The stationary current density describes the charge carrier movement if a **direct voltage** is the cause of the movement.
Then, a constant direct current flows in the stationary electric flow field. Thus, there is no time dependency on the current:

$\large{{{\rm d}I}\over{{\rm d}t}}=0$

Important: Up to now it was considered, that charges had moved through a field in the past or could be moved in the future. Now, the exact moment of moving the charge is considered.

Fig. 2: summary of conduction field



By comparison, we see now, that the resistance can be defined as:

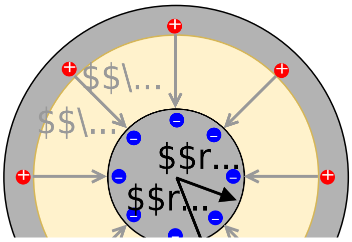
$$\begin{align*} {{1}\over{R}} &= {{I}\over{U}} \\ &= {{ \rlap{\Large \rlap{\int_A} \int} \, \LARGE \circ} \;\; \vec{J} \, {\rm d} \vec{A}\over{\int \vec{E} \,{\rm d} \vec{s} }} \end{align*}$$

Given the results from block 11 we can derive:

Fig. 3: current between parallel plates

- for a current between **parallel plates**
    - The current density is given as: \begin{align*} J = {{I}\over{A}} = \sigma \cdot E = {\rm const.} \end{align*}
    - This leads to the electric field: \begin{align*} E = {{J}\over{\sigma}} \end{align*}
    - The resistance value is given as: \begin{align*} {{1}\over{R}}&= {{ \rlap{\Large \rlap{\int_A} \int} \, \LARGE \circ} \;\; \vec{J} \, {\rm d} \vec{A}\over{\int \vec{E} \,{\rm d} \vec{s} }} = {{J} \cdot \rlap{\int_A}\int \; {\rm d} {A}\over{{E} \cdot \int \,{\rm d} {s} }} \end{align*}\begin{align*} \boxed{ {{1}\over{R}}= {{\sigma A}\over{l}} }_\text{between parallel plates} \end{align*}

Fig. 4: current between coaxial plates



- for a current between **coaxial plates**
    - The current density is given as: \begin{align*} J = {{I}\over{2\pi \cdot l \cdot r}} \end{align*}
    - The resistance value is given as: \begin{align*} \boxed{ {{1}\over{R}}=\dfrac{2\pi\sigma l}{\ln(r_a/r_i)} }_\text{between coaxial plates}\end{align*}

14.2 Common pitfalls

- Mixing **$\vec{D}$** (electrostatics) with **$\vec{j}$** (conduction). Use $\vec{D}=\varepsilon\vec{E}$ for capacitors, $\vec{j}=\sigma\vec{E}$ for resistive flow.
- Forgetting **surface orientation** in $I=\iint_A \vec{j}\cdot{\rm d}\vec{A}$ (normal must align with the chosen current reference arrow).
- Confusing **material parameters**: σ vs. ρ with $\rho=\dfrac{1}{\sigma}$. Writing both in the same formula yields unit errors.
- Using the **wrong area**: for coax, the relevant area element is the *lateral* surface $2\pi r\,l$ (not πr^2).
- Dropping **units** or not checking dimensions; e.g., verify $G=\dfrac{\sigma A}{l}$ gives $\rm S$ and R gives Ω.

14.3 Exercises

Task 2.2.1 Simulation

The simulation program of Falstad can show equipotential surfaces, electric field strength, and current density in different objects.

1. Open the simulation program via the link
2. Select: `Setup: Wire w/ Current` and `Show Current (j)`.
3. You will now see a finite conductor with charge carriers starting at the top end and arriving at the bottom end.
4. We now want to observe what happens when the conductor is tapered.
    1. To do this, select `Mouse = Clear Square`. You can now use the left mouse button to remove parts from the conducting material. The aim should be, that in the middle of the conductor, there is only a one-box wide line, on a length of at least 10 boxes. If you want to add conductive material again, this is possible with `Mouse = Add - Conductor`.
    2. Consider why more equipotential lines are now accumulating as the conductor is tapered.
    3. If you additionally draw in the E-field with `Show E/j`, you will see that it is stronger along the taper. This can be checked with the slider `Brightness`. Why is this?
5. Select `Setup: Current in 2D 1`, `Show E/rho/j`. Why doesn't the cavity behave like a Faraday cage here?

Task 2.2.2 Water Resistor

In transformer stations sometimes water resistors are used as Liquid rheostat. In this resistor, the water works as a (poor) conductor which can handle a high power loss.

The water resistor consists of a water basin. In the given basin two quadratic plates with the edge length of $l = 80 ~{\rm cm}$ are inserted with the distance d between them. The resistivity of the water is $\rho = 0.25 ~\Omega {\rm m}$. The resistor shall dissipate the energy of $P = 4 ~{\rm kW}$ and shall exhibit a homogeneous current field.

1. Calculate the required distance of the plates to get a current density of $J = 25 ~{\rm mA/cm^2}$
2. What are the values of the current I and the voltage U at the resistor, such as the internal electric field strength E in the setup?

Embedded resources

Explanation (video): ...

The online book 'University Physics II' is strongly recommended as a reference for this chapter. Especially the following chapters:

- Chapter 9.3 Model of Conduction in Metals