Block 13 - Capacitor Circuits and Energy

Student Group

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Learning objectives

After this 90-minute block, you can

- Recognize a series connection of capacitors and distinguish it from a parallel connection,
- Calculate the resulting total capacitance of a series or parallel circuit,
- Know how the total charge is distributed among the individual capacitors in a parallel circuit,
- Determine the voltage across a single capacitor in a series circuit.

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- …

90-minute plan

1. Warm-up (x min):
    1. ….
2. Core concepts & derivations (x min):
    1. …
3. Practice (x min): …
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

1. …

Core content

Series Circuit of Capacitor

If capacitors are connected in series, the charging current I into the individual capacitors $C_1 ... C_n$ is equal. Thus, the charges absorbed ΔQ are also equal: \begin{align*} \Delta Q = \Delta

Q_1 = \Delta Q_2 = ... = \Delta Q_n \end{align*}

Furthermore, after charging, a voltage is formed across the series circuit, which corresponds to the source voltage U_q. This results from the addition of partial voltages across the individual capacitors. \begin{align*} U_q = U_1 + U_2 + ... + U_n = \sum_{k=1}^n U_k \end{align*}

It holds for the voltage $U_k = \Large{{Q_k}\over{C_k}}$.
If all capacitors are initially discharged, then $U_k = \Large{{\Delta Q}\over{C_k}}$ holds. Thus \begin{align*} U_q &= &U_1 &+ &U_2 &+ &... &+ &U_n &= \sum_{k=1}^n U_k \\ U_q &= &{{\Delta Q}\over{C_1}} &+ &{{\Delta Q}\over{C_2}} &+ &... &+ &{{\Delta Q}\over{C_3}} &= \sum_{k=1}^n {{1}\over{C_k}}\cdot \Delta Q \\ {{1}\over{C_{ \rm eq}}}\cdot \Delta Q &= &&&&&&&&\sum_{k=1}^n {{1}\over{C_k}}\cdot \Delta Q \end{align*}

Thus, for the series connection of capacitors $C_1 ... C_n$:

\begin{align*} \boxed{ {{1}\over{C_{ \rm eq}}} = \sum_{k=1}^n {{1}\over{C_k}} } \end{align*} \begin{align*} \boxed{ \Delta Q_k = {\rm const.}} \end{align*}

For initially uncharged capacitors, (voltage divider for capacitors) holds: \begin{align*} \boxed{Q = Q_k} \end{align*} \begin{align*} \boxed{U_{ \rm eq} \cdot C_{ \rm eq} = U_{k} \cdot C_{k} } \end{align*}

In the simulation below, besides the parallel connected capacitors C_1, C_2,C_3, an ideal voltage source U_q, a resistor R, a switch S, and a lamp are installed.

- The switch S allows the voltage source to charge the capacitors.
- The resistor R is necessary because the simulation cannot represent instantaneous charging. The resistor limits the charging current to a maximum value.
- The capacitors can be discharged again via the lamp.

Parallel Circuit of Capacitors

If capacitors are connected in parallel, the voltage U across the individual capacitors $C_1 ... C_n$ is equal. It is therefore valid:

\begin{align*} U_q = U_1 = U_2 = ... = U_n \end{align*}

Furthermore, during charging, the total charge ΔQ from the source is distributed to the individual capacitors. This gives the following for the individual charges absorbed: \begin{align*} \Delta Q = \Delta Q_1 + \Delta Q_2 + ... + \Delta Q_n = \sum_{k=1}^n \Delta Q_k \end{align*}

If all capacitors are initially discharged, then $Q_k = \Delta Q_k = C_k \cdot U$
Thus \begin{align*} \Delta Q &= & Q_1 &+ & Q_2 &+ &... &+ & Q_n &= \sum_{k=1}^n Q_k \\ \Delta Q &= &C_1 \cdot U &+ &C_2 \cdot U &+ &... &+ &C_n \cdot U &= \sum_{k=1}^n C_k \cdot U \\ C_{ \rm eq} \cdot U &= &&&&&&&& \sum_{k=1}^n C_k \cdot U \\ \end{align*}

Thus, for the parallel connection of capacitors $C_1 ... C_n$:

\begin{align*} \boxed{ C_{ \rm eq} = \sum_{k=1}^n C_k } \end{align*} \begin{align*} \boxed{ U_k

= {\rm const.}} \end{align*}

For initially uncharged capacitors, (charge divider for capacitors) holds: \begin{align*} \boxed{\Delta Q = \sum_{k=1}^n Q_k} \end{align*}

\begin{align*} \boxed{ {{Q_k}\over{C_k}} = {{\Delta Q}\over{C_{ \rm eq}}} } \end{align*}

In the simulation below, again, besides the parallel connected capacitors C_1, C_2,C_3, an ideal voltage source U_q, a resistor R, a switch S, and a lamp are installed.

Energy in the electric Field

Now we want to see how much energy is stored in a capacitor during charging. When we want to charge a capacior charges have be separated (see figure 1). This gets more and more difficult as more charges were moved, since these already moved charges create an electric field.

Fig. 1: summary of electrostatics

We already had a first look onto the energy in the electric field in block09.
There, we got:

\begin{align*} \Delta W &= \int \vec{F} d\vec{r} \\ &= q \int \vec{E} d\vec{r} \\ &= q \cdot U \\ dW &= dq \cdot U \end{align*}

Now, For a capacitor we include the formula for the capacitor $C = {{q}\over{U}}$, or better its rearranged version $U = {{q}\over{C}}$:

\begin{align*} dW &= dq \cdot {{q}\over{C}} \\ \int dW &= \int {{q}\over{C}} dq \end{align*}

Here we again see, that the needed energy portion dW to move a portion dq is also related to the already moved charges q.
To get the energy $Delta W$ needed to move all of the charges $$Q = \int dq$$ we have to integrate from 0 to Q:

\begin{align*} \Delta W &= \int_0^Q dW \\ &= \int_0^Q {{q}\over{C}} dq \\ &= {{1}\over{2}}{{Q^2}\over{C}} \\ \end{align*} \begin{align*} \boxed{ \Delta W = {{1}\over{2}}{{Q^2}\over{C}} = {{1}\over{2}}QU = {{1}\over{2}}CQ^2 } \end{align*}

Summary on the Electric Field

Fig. ##: summary of electrostatics

Common pitfalls

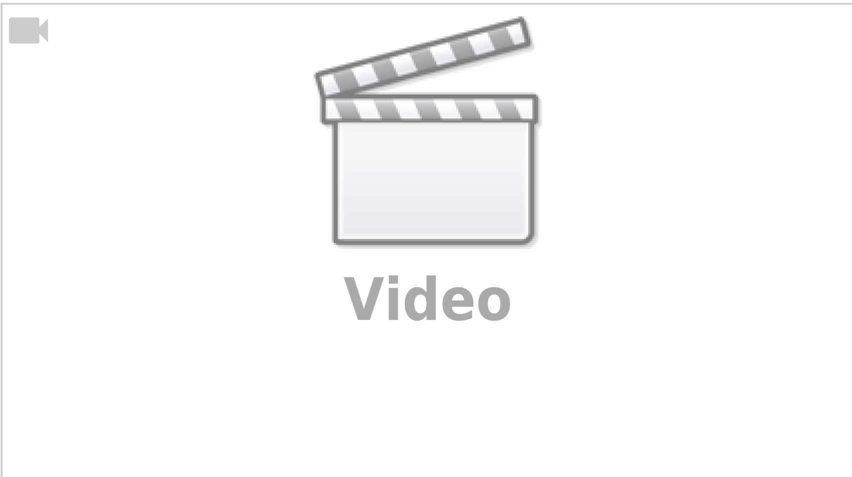
- …

Exercises

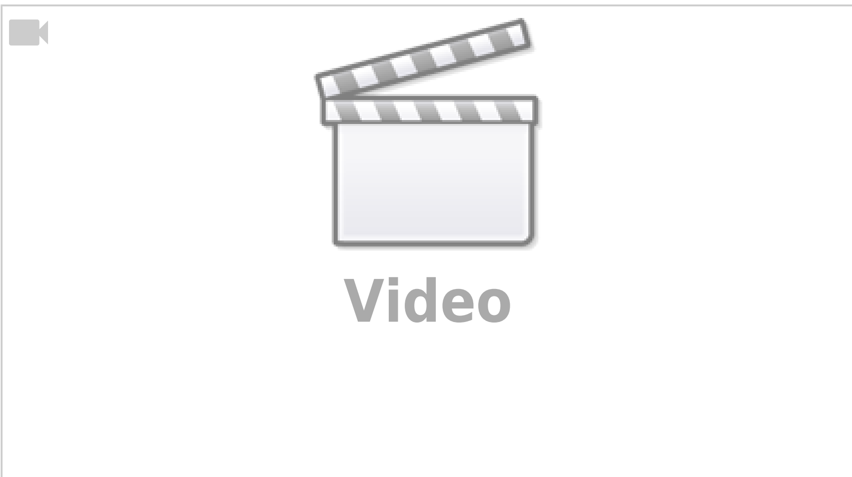
Task 5.8.1 Calculating a circuit of different capacitors

See https://www.youtube.com/watch?v=vSeSHAmpd4Y

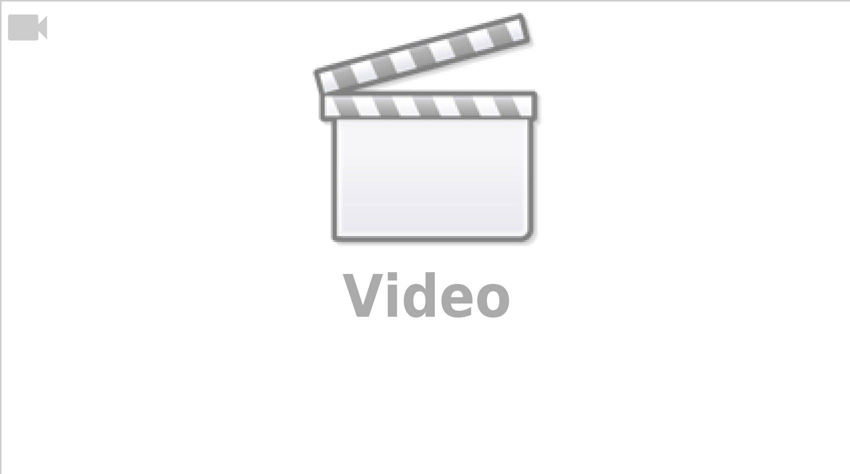
Task 5.9.1 Layered Capacitor Task



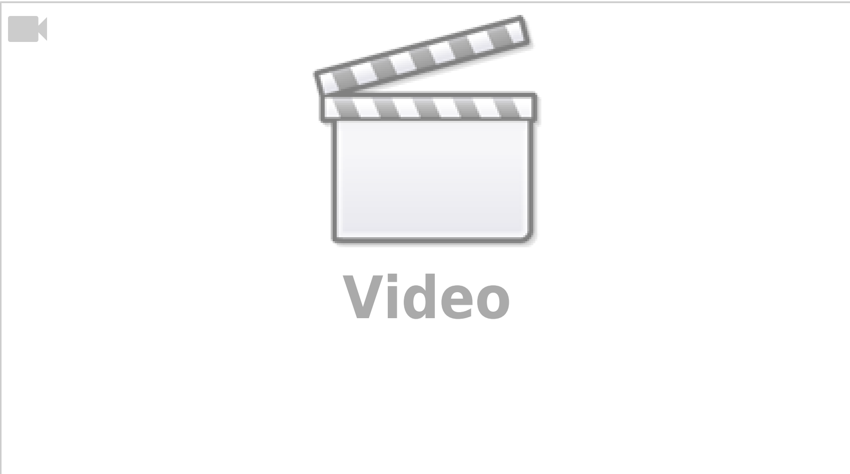
Exercise 5.9.2 Further capacitor charging/discharging practice Exercise



Exercise 5.9.3 Further practice charging the capacitor

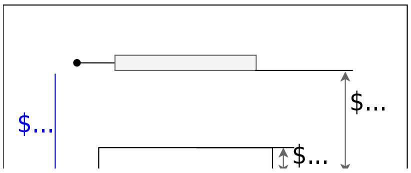


Exercise 5.9.4 Charge balance of two capacitors



Exercise 5.9.5 Capacitor with glass plate

Fig. 2: Structure of a capacitor with glass plate

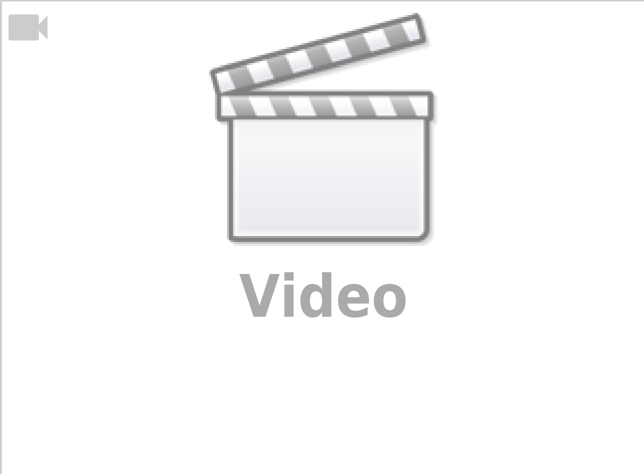


Two parallel capacitor plates face each other with a distance $d_{ \rm K} = 10~{ \rm mm}$. A voltage of $U = 3'000~{ \rm V}$ is applied to the capacitor. Parallel to the capacitor plates, there is a glass plate ($\varepsilon_{ \rm r, G}=8$) with a thickness $d_{ \rm G} = 3~{ \rm mm}$ in the capacitor.

1. Calculate the partial voltages $U_{ \rm G}$ in the glass and $U_{ \rm A}$ in the air gap.
2. What is the maximum thickness of the glass pane if the electric field $E_{ \rm 0, G} =12 ~{ \rm kV/cm}$ must not exceed?

Embedded resources

The equivalent capacitor for series of parallel configuration is well explained here


Video