Block 10 - Field Patterns of key Geometries

Student Group

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Geometric Distribution of Charges

In previous chapters, only single charges (e.g., Q_1, Q_2) were considered.

- The charge Q was previously reduced to a **point charge**.
  This can be used, for example, for the elementary charge or for extended charged objects from a large distance. The distance is sufficiently large if the ratio between the largest object extent and the distance to the measurement point P is small.
- If the charges are lined up along a line, this is referred to as a **line charge**.
  Examples of this are a straight trace on a circuit board or a piece of wire. Furthermore, this also applies to an extended charged object, which has exactly an extension that is no longer small in relation to the distance. For this purpose, the charge Q is considered to be distributed over the line. Thus, a (line) charge density ρ_l can be determined:

  $$\rho_l = {{Q}\over{l}}$$

  or, in the case of different charge densities on subsections:

  $$\rho_l = {{\Delta Q}\over{\Delta l}} \rightarrow \rho_l(l)={{\rm d}\over{{\rm d}l}} Q(l)$$
- It is spoken of as an **area charge** when the charge is distributed over an area.
  Examples of this are the floor or the plate of a capacitor. Again, an extended charged object can be considered when two dimensions are no longer small in relation to the distance (e.g. surface of the earth). Again, a (surface) charge density ρ_A can be determined:

  $$\rho_A = {{Q}\over{A}}$$

  or if there are different charge densities on partial surfaces:

  $$\rho_A = {{\Delta Q}\over{\Delta A}} \rightarrow \rho_A(A) ={{\rm d}\over{{\rm d}A}} Q(A)={{\rm d}\over{{\rm d}x}}{{\rm d}\over{{\rm d}y}} Q(A)$$
- Finally, a **space charge** is the term for charges that span a volume.
  Here, examples are plasmas or charges in extended objects (e.g., the doped volumes in a semiconductor). As with the other charge distributions, a (space) charge density ρ_V can be calculated here:

  $$\rho_V = {{Q}\over{V}}$$

  or for different charge density in partial volumes:

  $$\rho_V = {{\Delta Q}\over{\Delta V}} \rightarrow \rho_V(V) ={{\rm d}\over{{\rm d}V}} Q(V)={{\rm d}\over{{\rm d}x}}{{\rm d}\over{{\rm d}y}}{{\rm d}\over{{\rm d}z}} Q(V)$$

Electric Field Lines

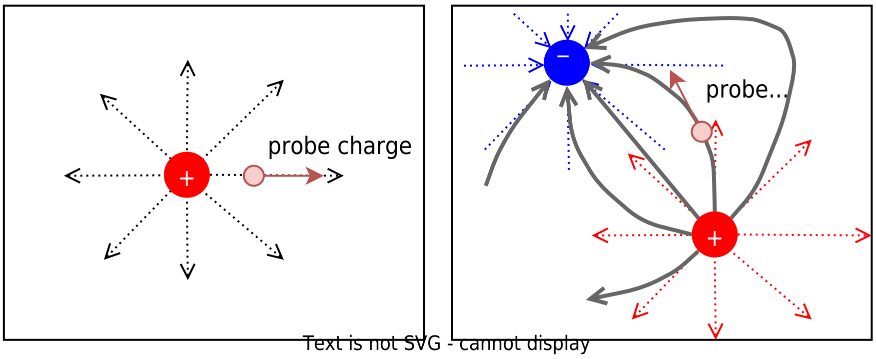
Electric field lines result from the (fictitious) path of a sample charge. Thus, also electric field lines of several charges can be determined. However, these also result from a superposition of the individual effects - i.e., electric field - at a measuring point P.

The superposition is sketched in figure 4: Two charges Q_1 and Q_2 act on the test charge q with the forces F_1 and F_2. Depending on the positions and charges, the forces vary, and so does the resulting force. The simulation also shows a single field line.

Fig. 1: examples of field lines

For a full picture of the field lines between charges, one has to start with a single charge. The in- and outgoing lines on this charge are drawn equidistant from the charge. This is also true for the situation with multiple charges. However, there, the lines are not necessarily run radially anymore. The test charge is influenced by all the single charges, and therefore, the field lines can get bent.

Fig. 2: examples of field lines



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In figure 3 the field lines are shown. The additional “equipotential lines” will be discussed later and can be deactivated by clearing the checkmark `Show Equipotentials`. Try the following in the simulation:

- Get accustomed to the simulation. You can…
  - … move the charges by drag and drop.
  - … add another Charge with Add » `Add Point Charge`.
  - … delete components with a right click on them and `delete`
- Where is the density of the field lines higher?
- How does the field between two positive charges look? How does it look between two different charges?

Fig. 3: examples of field lines

Fig. 4: examples of field lines in 3D

Note:

1. The electrostatic field is a source field. This means there are sources and sinks.
2. From the field line diagrams, the following can be obtained:
    1. Direction of the field ($\hat{=}$ parallel to the field line).
    2. Magnitude of the field ($\hat{=}$ number of field lines per unit area).
3. The magnitude of the field along a field line is usually not constant.

Note:

Field lines have the following properties:

- The electric field lines have a beginning (at a positive charge) and an end (at a negative charge).
- The direction of the field lines represents the direction of a **force onto a positive test charge**.
- There are **no closed field lines** in electrostatic fields. The reason for this can be explained by considering the energy of the moved particle (see later subchapters).
- Electric **field lines cannot cut** each other: This is based on the fact that the direction of the force at a cutting point would not be unique.
- The field lines are **always perpendicular to conducting surfaces**
- The **inside of a conducting component is always field-free**.
- The density of the field lines is a measure for the electric field density.

Types of Fields depending on the Charge Distribution

There are two different types of fields:

In **homogeneous fields**, magnitude and direction are constant throughout the field range. This field form is idealized to exist within plate capacitors. e.g., in the plate capacitor (figure 5), or the vicinity of widely extended bodies.

Fig. 5: Field lines of a homogeneous field

For **inhomogeneous fields**, the magnitude and/or direction of the electric field changes from place to place. This is the rule in real systems, even the field of a point charge is inhomogeneous (figure 6).

Fig. 6: Field lines of an inhomogeneous field

Dielectric strength

In Block03 we had a short look on conductivity of matter.
Here, we want to have again a look onto isolators.

- The ability to insulate is dependent on the material.
- If a maximum electric field E_0 is exceeded, the insulating ability is eliminated.
    - One says: The insulator breaks down. This means that above this electric field, a current can flow through the insulator.
    - Examples are: Lightning in a thunderstorm, ignition spark, glow lamp in a phase tester
    - The maximum electric field E_0 is referred to as **dielectric strength** (in German: *Durchschlagfestigkeit* or *Durchbruchfeldstärke*).
    - E_0 depends on the material (see table 1), but also on other factors (temperature, humidity, …).

Material	**Dielectric strength E_0 in ${ \rm kV/mm}$**
air	$\rm 0.1...0.3$
SF6 gas	$\rm 8$
insulating oils	$\rm 5...30$
vacuum	$\rm 20...30$
quartz	$\rm 30...40$
PP, PE	$\rm 50$
PS	$\rm 100$
distilled water	$\rm 70$

Tab. 1: Dielectric strength

Exercises

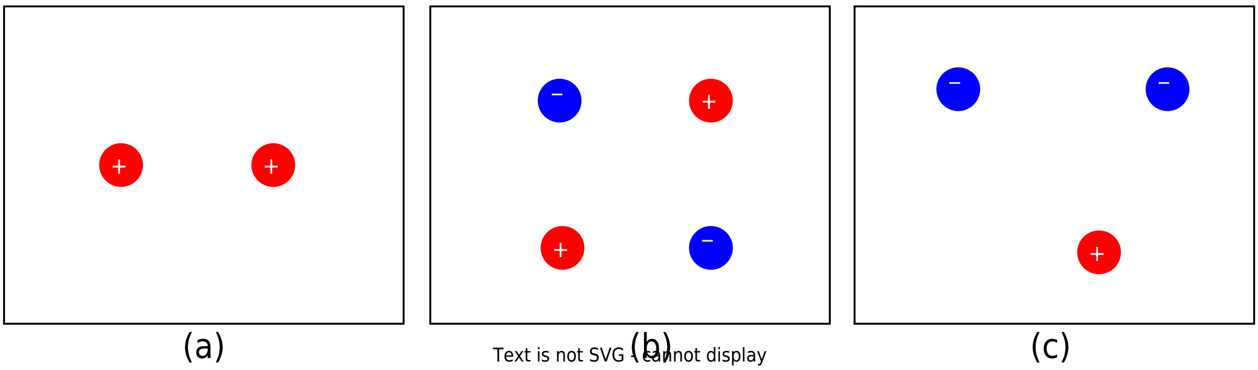
Task 1.1.2 Field lines

Sketch the field line plot for the charge configurations given in figure 10.
Note:

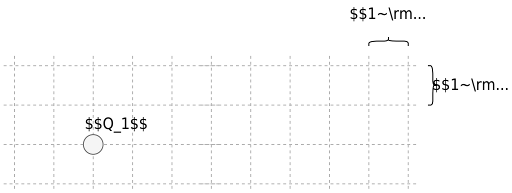
- The overlaid picture is requested.
- Make sure that it is a source field.

You can prove your result with the simulation figure 4.

Fig. 10: Task on field lines



Task 1.2.5 Forces on Charges (exam task, ca 8 % of a 60 minute exam, WS2020)



Given is an arrangement of electric charges located in a vacuum (see picture on the right).
The charges have the following values:
$Q_1=7 ~\rm{µC}$ (point charge)
$Q_2=5 ~\rm{µC}$ (point charge)
$Q_3=0 ~\rm{C}$ (infinitely extended surface charge)

$\varepsilon_0=8.854\cdot 10^{-12} ~\rm{F/m}$, $\varepsilon_r=1$

1. calculate the magnitude of the force of Q_2 on Q_1, without the force effect of Q_3.

Tips for the solution

- Which equation is to be used for the force effect of charges?
- How can the distance between the two charges be determined?

Solution

\begin{align*} F_C &= {{{1} \over {4\pi\cdot\varepsilon}} \cdot {{Q_1 \cdot Q_2} \over {r^2}}} \quad && | \text{with } r=\sqrt{\Delta x^2 + \Delta y^2} \\ F_C &= {{{1} \over {4\pi\cdot\varepsilon}} \cdot {{Q_1 \cdot Q_2} \over {\Delta x^2 + \Delta y^2}}} \quad && | \text{Insert numerical values, read off distances: } \Delta x = 5~\rm{dm}, \Delta y = 3~\rm{dm} \\ F_C &= {{{1} \over {4\pi\cdot 8.854\cdot 10^{-12} ~\rm{F/m}}} \cdot {{7 \cdot 10^{-6} ~\rm{C} \cdot 5 \cdot 10^{-6} ~\rm{C}} \over { (0.5~\rm{m})^2 + (0.2~\rm{m})^2}}} \end{align*}

Result

$$\begin{align*} |\vec{F}_C| = 1.084 ~\rm{N} \rightarrow 1.1 ~\rm{N} \end{align*}$$

2. is this force attractive or repulsive?

Tips for the solution

- What force effect do equally or oppositely charged bodies exhibit on each other?

Solution

The force is repulsive because both charges have the same sign.

Now let $Q_2=0$ and the surface charge Q_3 be designed in such a way that a homogeneous electric field with $E_3=100 ~\rm{kV/m}$ results.
What force (magnitude) now results on Q_1?

Tips for the solution

- Which equation is to be applied for the force action in the homogeneous field?

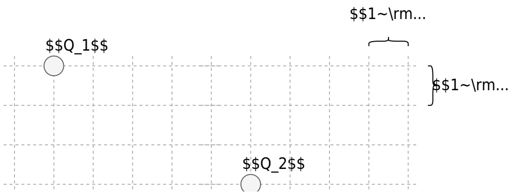
Solution

$$\begin{align*} F_C &= E \cdot Q_1 \quad && | \text{Insert numerical values} \\ F_C &= 100 \cdot 10^3 ~\rm{V/m} \cdot 7 \cdot 10^{-6} ~\rm{C} \end{align*}$$

Result

$$\begin{align*} |\vec{F}_C| = 0.7 ~\rm{N} \end{align*}$$

Task 1.2.6 Variation: Forces on Charges (exam task, ca 8% of a 60 minute exam, WS2020)



Given is an arrangement of electric charges located in a vacuum (see picture on the right).
The charges have the following values:
$Q_1=5 ~\rm{μC}$ (point charge)
$Q_2=-10 ~\rm{μC}$ (point charge)
$Q_3= 0 ~\rm{C}$ (infinitely extended surface charge)

$\varepsilon_0=8.854\cdot 10^{-12} ~\rm{F/m}$, $\varepsilon_r=1$

1. calculate the magnitude of the force of Q_2 on Q_1, without the force effect of Q_3.

Result

\begin{align*} |\vec{F}_C| = 1.321 ~\rm{N} \rightarrow 1.3 ~\rm{N} \end{align*}

2. is this force attractive or repulsive?

Solution

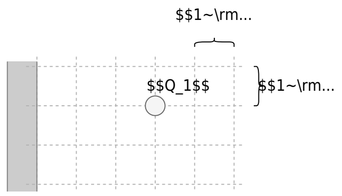
The force is repulsive because both charges have the same sign.

Now let $Q_2=0$ and the surface charge Q_3 be designed in such a way that a homogeneous electric field with $E_3=500 ~\rm{kV/m}$ results.
What force (magnitude) now results on Q_1?

Result

\begin{align*} |\vec{F}_C| = 2.5 ~\rm{N} \end{align*}

Task 1.2.7 Variation: Forces on Charges (exam task, ca 8% of a 60 minute exam, WS2020)



Given is an arrangement of electric charges located in a vacuum (see picture on the right).
The charges have the following values:
$Q_1= 2 ~\rm{µC}$ (point charge)
$Q_2=-4 ~\rm{µC}$ (point charge)
$Q_3= 0 ~\rm{C}$ (infinitely extended surface charge)

$\varepsilon_0=8.854\cdot 10^{-12} ~\rm{F/m}$, $\varepsilon_r=1$

1. calculate the magnitude of the force of Q_2 on Q_1, without the force effect of Q_3.

Result

\begin{align*} |\vec{F}_C| = 0.3595 ~\rm{N} \rightarrow 0.36 ~\rm{N} \end{align*}

2. is this force attractive or repulsive?

Solution

The force is attractive because the charges have different signs.

Now let $Q_2=0$ and the surface charge Q_3 be designed in such a way that a

homogeneous electric field with $E_3=100 ~\rm{kV/m}$ results.
What force (magnitude) now results on Q_1?

Result

\begin{align*} |\vec{F}_C| = 0.4 ~\rm{N} \end{align*}

Embedded resources

Field lines of various extended charged objects