

Block 09 - Force on Charges and electric Field Strength

Student Group

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Block 09 – Force on charges and electric field strength

Learning objectives

By the end of this section, you will be able to:

1. Know that an electric field is formed around a charge.
2. Sketch the field lines of electric fields.
3. Represent the field vectors in a sketch when given several charges.
4. Determine the resulting field vector by superimposing several field vectors using vector calculus.
5. Determine the force on a charge in an electrostatic field by applying Coulomb's law.
Specifically:
 1. The force vector in coordinate representation
 2. The magnitude of the force vector
 3. The angle of the force vector
6. Determine the direction of the forces using the given charges.
7. Represent the acting force vectors in a sketch.
8. Determine a force vector by superimposing several force vectors using vector calculus.
9. State the following quantities for a force vector:
 1. The force vector in coordinate representation
 2. The magnitude of the force vector
 3. The angle of the force vector

90-minute plan

1. Warm-up (5-10 min):
 1. Recall / Quick quiz ...
2. Core concepts & derivations (60-70 min):
 1. ...
3. Practice (10-20 min): ...
4. Wrap-up (5 min): ...

Conceptual overview

1. ...

Core content

Electric Effects

Every day life teaches us that there are various charges and their effects. The image [figure 1](#) depicts a chargeable body that can be charged through charge separation between the sole and the floor. The movement of the foot generates a negative surplus charge in the body, which progressively spreads throughout the body. A current can flow even through the air if a pointed portion of the body (e.g., a finger) is brought into close proximity to a charge reservoir with no extra charges.

Fig. 1: John Tra-Voltage

We had already considered the charge as the central quantity of electricity in the first chapter of the previous semester and recognized it as a multiple of the elementary charge. There was already a mutual force action ([the Coulomb-force](#)) derived. This will be more fully explained.

First, we shall define certain terms:

1. **Electricity** is a catch-all term for any occurrences involving moving and resting charges.
2. **Electrostatics** is the study of charges at rest and consequently electric fields that do not vary over time. As a result, the electrical quantities have no temporal dependence. For any function of the electric quantities, $\frac{df}{dt} = 0$ holds mathematically.
3. **Electrodynamics** describes the behavior of moving charges. Hence, electrodynamics covers both changing electric fields and magnetic fields. For the time being, the simple explanation will be that magnetic fields are dependent on current or charge flow. It is no longer true in electrodynamics that the derivative is always necessary for any function of electric values.

Only electrostatics is discussed in this chapter. For the time being, magnetic fields are thus excluded. Furthermore, electrodynamics is not covered in this chapter and is provided in further detail in subsequent chapters.

Fields

The concept of a field will now be briefly discussed in more detail.

1. The introduction of the field distinguishes the cause from the effect.
 1. The field in space is caused by the charge Q .
 2. As a result of the field, the charge q in space feels a force.
 3. This distinction is brought up again in this chapter. It is also fairly obvious in electrodynamics at high frequencies: the field corresponds to photons, i.e. to a transmission of effects with a finite (light) speed c .
2. There are different-dimensional fields, just like physical quantities:
 1. In a **scalar field**, each point in space is assigned a single number. For example,
 1. a temperature field $T(\vec{x})$ on a weather map or in an object

2. a pressure field $p(\vec{x})$
2. Each point in space in a **vector field** is assigned several numbers in the form of a vector. This reflects the action as it occurs along the spatial coordinates. As an example.
 1. gravitational field $\vec{g}(\vec{x})$ pointing to the object's center of mass.
 2. electric field $\vec{E}(\vec{x})$
 3. magnetic field $\vec{H}(\vec{x})$
3. A tensor field is one in which each point in space is associated with a two- or more-dimensional physical quantity - that is, a tensor. Tensor fields are useful in mechanics (for example, the stress tensor), but they are not required in electrical engineering.

Vector fields are defined as follows:

1. Effects along spatial axes x , y and z (Cartesian coordinate system).
2. Effect in magnitude and direction vector (polar coordinate system)

educational Task

Place a negative charge Q in the middle of the simulation and turn off the electric field. The latter is accomplished by using the hook on the right. The situation is now close to reality because a charge appears to have no effect at first glance.

A sample charge q is placed near the existing charge Q for impact analysis (in the simulation, the sample charge is called "sensors"). The charge Q is observed to affect a force on the sample charge. At any point in space, the magnitude and direction of this force can be determined. In space, the force behaves similarly to gravity. A field serves to describe the condition space changed by the charge Q .

Fig. 2: setup for own experiments

Take a charge ($+1 \sim \{ \text{rm nC} \}$) and position it.

Measure the field across a sample charge (a sensor).

Note:

1. Fields describe a physical state of space.
2. Here, a physical quantity is assigned to each point in space.
3. The electrostatic field is described by a vector field.

The electric Field

To determine the electric field, a measurement of its magnitude and direction is now required. The Coulomb force between two charges Q_1 and Q_2 is known from the first chapter of the previous semester:

$$\begin{aligned} F_C = & \frac{1}{4\pi \cdot \epsilon_0} \cdot \frac{Q_1 \cdot Q_2}{r^2} \end{aligned}$$

The force on a (fictitious) sample charge q is now considered to obtain a measure of the magnitude of the electric field.

$$F_C = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q_1 \cdot q}{r^2} \quad \&= \underbrace{\frac{1}{4\pi\epsilon_0}} \cdot \frac{Q_1}{r^2} \quad \text{_text{=independent of } q} \cdot q$$

As a result, the left part is a measure of the magnitude of the field, independent of the size of the sample charge q . Thus, the magnitude of the electric field is given by

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q_1}{r^2} \quad \text{with} \\ E = \frac{F}{q} = 1 \sim \frac{N}{As} = 1 \sim \frac{N \cdot m}{As \cdot m} = 1 \sim \frac{V \cdot A \cdot s}{As \cdot m} = 1 \sim \frac{V}{m}$$

The result is therefore $F_C = E \cdot q$

Note:

1. The test charge q is always considered to be positive (mnemonic: t = +). It is only used as a thought experiment and has no retroactive effect on the sampled charge Q .
2. The sampled charge here is always a point charge.

Note:

At a measuring point P , a charge Q produces an electric field $\vec{E}(Q)$. This electric field is given by

1. the magnitude $|\vec{E}| = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q_1}{r^2}$ and
2. the direction of the force \vec{F}_C experienced by a sample charge on the measurement point P . This direction is indicated by the unit vector $\vec{e}_r = \frac{\vec{F}_C}{|F_C|}$ in that direction.

Be aware that in English courses and literature \vec{E} is simply referred to as the electric field, and the electric field strength is the magnitude $|\vec{E}|$. In German notation, the *Elektrische Feldstärke* refers to \vec{E} (magnitude and direction), and the *Elektrische Feld* denotes the general presence of an electrostatic interaction (often without considering exact magnitude).

The direction of the electric field is switchable in [figure 2](#) via the “Electric Field” option on the right.

Electric Field Lines

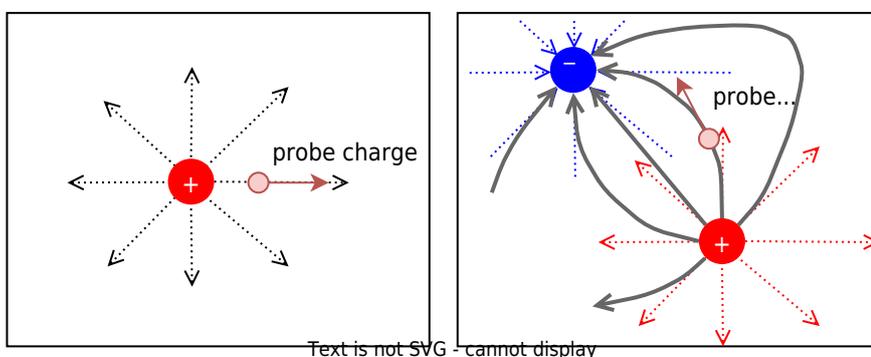
Electric field lines result from the (fictitious) path of a sample charge. Thus, also electric field lines of several charges can be determined. However, these also result from a superposition of the individual effects - i.e., electric field - at a measuring point P .

The superposition is sketched in [figure ##](#): Two charges Q_1 and Q_2 act on the test charge q with the forces F_1 and F_2 . Depending on the positions and charges, the forces vary, and so does the resulting force. The simulation also shows a single field line.

Fig. ##: examples of field lines

For a full picture of the field lines between charges, one has to start with a single charge. The in- and outgoing lines on this charge are drawn equidistant from the charge. This is also true for the situation with multiple charges. However, there, the lines are not necessarily run radially anymore. The test charge is influenced by all the single charges, and therefore, the field lines can get bent.

Fig. ##: examples of field lines



In [figure ##](#) the field lines are shown. The additional “equipotential lines” will be discussed later and can be deactivated by clearing the checkmark **Show Equipotentials**. Try the following in the simulation:

- Get accustomed to the simulation. You can...
 - ... move the charges by drag and drop.
 - ... add another Charge with **Add » Add Point Charge**.
 - ... delete components with a right click on them and **delete**
- Where is the density of the field lines higher?
- How does the field between two positive charges look? How does it look between two different charges?

Fig. ##: examples of field lines

Note:

1. The electrostatic field is a source field. This means there are sources and sinks.
2. From the field line diagrams, the following can be obtained:
 1. Direction of the field ($\hat{=}$) parallel to the field line).

2. Magnitude of the field ($\hat{=}$ number of field lines per unit area).
3. The magnitude of the field along a field line is usually not constant.

Note:

Field lines have the following properties:

- The electric field lines have a beginning (at a positive charge) and an end (at a negative charge).
- The direction of the field lines represents the direction of a force onto a positive test charge.
- There are no closed field lines in electrostatic fields. The reason for this can be explained by considering the energy of the moved particle (see later subchapters).
- Electric field lines cannot cut each other: This is based on the fact that the direction of the force at a cutting point would not be unique.
- The field lines are always perpendicular to conducting surfaces. This is also based on energy considerations; more details later.
- The inside of a conducting component is always field-free. Also, this will be discussed in the following.

Electric Charge and Coulomb Force (reloaded)

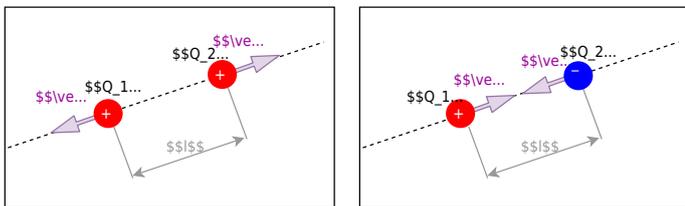
The electric charge and Coulomb force have already been described last semester. However, some points are to be caught up here.

Direction of the Coulomb force and Superposition

In the case of the force, only the direction has been considered so far, e.g., direction towards the sample charge. For future explanations, it is important to include the cause and effect in the naming. This is done by giving the correct labeling of the subscript of the force. In [figure 3](#) (a) and (b), the convention is shown: A force \vec{F}_{21} acts on charge Q_2 and is caused by charge Q_1 . As a mnemonic, you can remember “tip-to-tail” (first the effect, then the cause).

Furthermore, several forces on a charge can be superimposed, resulting in a single, equivalent force. Strictly speaking, it must hold that ε_0 is constant in the structure. For example, the resultant force in [figure 3](#) Fig. (c) on Q_3 becomes equal to: $\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32}$.

Fig. 3: direction of coulomb force



Geometric Distribution of Charges

In previous chapters, only single charges (e.g., Q_1 , Q_2) were considered.

- The charge Q was previously reduced to a **point charge**. This can be used, for example, for the elementary charge or for extended charged objects from a large distance. The distance is sufficiently large if the ratio between the largest object extent and the distance to the measurement point P is small.

- If the charges are lined up along a line, this is referred to as a **line charge**. Examples of this are a straight trace on a circuit board or a piece of wire. Furthermore, this also applies to an extended charged object, which has exactly an extension that is no longer small in relation to the distance. For this purpose, the charge Q is considered to be distributed over the line. Thus, a (line) charge density ρ_l can be determined:

$$\rho_l = \frac{Q}{l}$$

or, in the case of different charge densities on subsections:

$$\rho_l = \frac{\Delta Q}{\Delta l} \rightarrow \rho_l(l) = \frac{dQ}{dl} Q(l)$$

- It is spoken of as an **area charge** when the charge is distributed over an area. Examples of this are the floor or the plate of a capacitor. Again, an extended charged object can be considered when two dimensions are no longer small in relation to the distance (e.g. surface of the earth). Again, a (surface) charge density ρ_A can be determined:

$$\rho_A = \frac{Q}{A}$$

or if there are different charge densities on partial surfaces:

$$\rho_A = \frac{\Delta Q}{\Delta A} \rightarrow \rho_A(A) = \frac{dQ}{dA} Q(A) = \frac{dQ}{dx} \frac{dQ}{dy} Q(A)$$

- Finally, a **space charge** is the term for charges that span a volume. Here, examples are plasmas or charges in extended objects (e.g., the doped volumes in a semiconductor). As with the other charge distributions, a (space) charge density ρ_V can be calculated here:

$$\rho_V = \frac{Q}{V}$$

or for different charge density in partial volumes:

$$\rho_V = \frac{\Delta Q}{\Delta V} \rightarrow \rho_V(V) = \frac{dQ}{dV} Q(V) = \frac{dQ}{dx} \frac{dQ}{dy} \frac{dQ}{dz} Q(V)$$

In the following, area charges and their interactions will be considered.

Types of Fields depending on the Charge Distribution

There are two different types of fields:

In **homogeneous fields**, magnitude and direction are constant throughout the field range. This field form is idealized to exist within plate capacitors. e.g., in the plate capacitor (figure ##), or the vicinity of widely extended

For **inhomogeneous fields**, the magnitude and/or direction of the electric field changes from place to place. This is the rule in real systems, even the field of a point charge is inhomogeneous (figure ##).

bodies.

Fig. ##: Field lines of an inhomogeneous field

Fig. ##: Field lines of a homogeneous field

Common pitfalls

- ...

...

Exercises

Task 1.1.1 simple task with charges



Task 1.1.2 Field lines

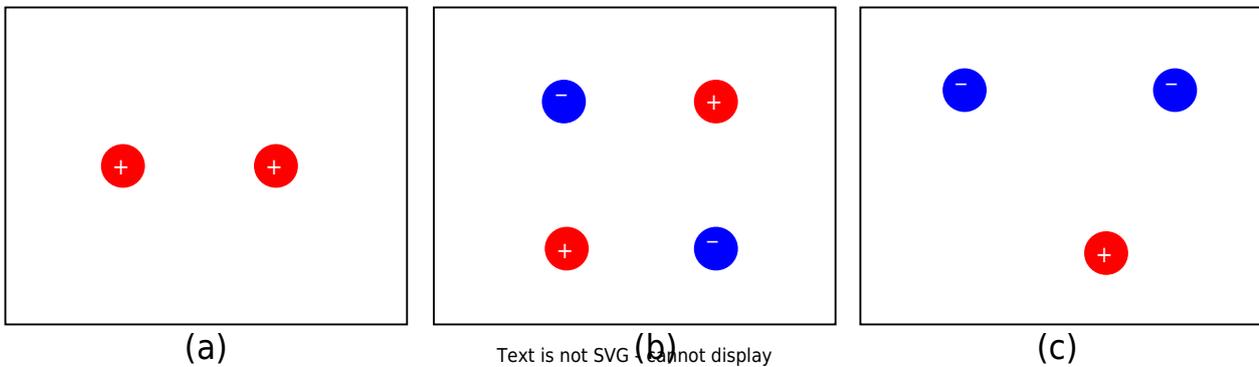
Sketch the field line plot for the charge configurations given in figure ##.

Note:

- The overlaid picture is requested.
- Make sure that it is a source field.

You can prove your result with the simulation [figure ##](#).

Fig. ##: Task on field lines



Task 1.2.1 Multiple Forces on a Charge I (exam task, ca 8% of a 60-minute exam, WS2020)



Given is the arrangement of electric charges in the picture on the right.
 The following force effects result:

$$F_{01} = -5 \text{ N}$$

$$F_{02} = -6 \text{ N}$$

$$F_{03} = +3 \text{ N}$$

Calculate the magnitude of the resulting force.

Tips for the Solution

- How have the forces be prepared, to add them correctly?

Solution

$$F_0 = |\vec{F}_0| \quad \text{with } \vec{F}_0 = \left(\begin{matrix} F_{x,0} \\ F_{y,0} \end{matrix} \right) = \left(\sum_{n=1}^3 F_{x,0n} \quad \sum_{n=1}^3 F_{y,0n} \right) \quad F_0 = \sqrt{\left(\sum_{n=1}^3 F_{x,0n} \right)^2 + \left(\sum_{n=1}^3 F_{y,0n} \right)^2}$$

The forces have to be resolved into coordinates. Here, it is recommended to use an orthogonal coordinate system (x and y).

The coordinate system shall be in such a way, that the origin lies in Q_0 , the x -axis is directed towards Q_3 and the y -axis is orthogonal to it.

For the resolution of the coordinates, it is necessary to get the angles α_{0n} of the forces with respect to the x -axis.

In the chosen coordinate system this leads to: $\alpha_{0n} = \arctan(\frac{\Delta y}{\Delta x})$

$$\alpha_{01} = \arctan(\frac{3}{1}) = 1.249 = 71.6^\circ$$

$$\alpha_{02} = \arctan(\frac{4}{3}) = 0.927 = 53.1^\circ$$

$$\alpha_{03} = \arctan(\frac{0}{3}) = 0 = 0^\circ$$

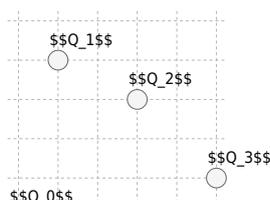
Consequently, the resolved forces are:

$$\begin{aligned} F_{x,0} &= F_{x,01} + F_{x,02} + F_{x,03} \quad | \quad \text{with } F_{x,0n} \\ &= F_{0n} \cdot \cos(\alpha_{0n}) \quad | \quad F_{x,0} = (-5 \text{ N}) \cdot \cos(71.6^\circ) + (-6 \text{ N}) \\ &\quad \cdot \cos(53.1^\circ) + (+3 \text{ N}) \cdot \cos(0^\circ) \quad | \quad F_{x,0} = -9.54 \text{ N} \\ F_{y,0} &= F_{y,01} + F_{y,02} + F_{y,03} \quad | \quad \text{with } F_{y,0n} = F_{0n} \cdot \sin(\alpha_{0n}) \\ F_{y,0} &= (-5 \text{ N}) \cdot \sin(71.6^\circ) + (-6 \text{ N}) \cdot \sin(53.1^\circ) \\ &\quad + (+3 \text{ N}) \cdot \sin(0^\circ) \quad | \quad F_{y,0} = -2.18 \text{ N} \end{aligned}$$

Result

$$F_0 = \sqrt{(-9.54 \text{ N})^2 + (-2.18 \text{ N})^2} = 9.79 \text{ N} \rightarrow 9.8 \text{ N}$$

Task 1.2.2 Variation: Multiple Forces on a Charge II (exam task, ca 8% of a 60 minute exam, WS2020)



Given is the arrangement of electric charges in the picture on the right. The following force effects result:

$$F_{01} = -5 \text{ N}$$

$$F_{02} = -6 \text{ N}$$

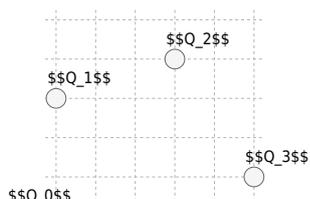
$$F_{03} = +3 \text{ N}$$

Calculate the magnitude of the resulting force.

Result

$$|F_0| = \sqrt{(-0.418 \text{ N})^2 + (-9.264 \text{ N})^2} = 9.274 \text{ N} \rightarrow 9.3 \text{ N}$$

Task 1.2.3 Variation: Multiple Forces on a Charge II (exam task, ca 8% of a 60 minute exam, WS2020)



Given is the arrangement of electric charges in the picture on the right. The following force effects result:

$$F_{01} = +2 \text{ N}$$

$$F_{02} = -3 \text{ N}$$

$$F_{03} = +4 \text{ N}$$

Calculate the magnitude of the resulting force.

Result

$$|F_0| = \sqrt{(2.12 \text{ N})^2 + (0.38 \text{ N})^2} = 2.16 \text{ N} \rightarrow 2.2 \text{ N}$$

Task 1.2.4 Superposition of Charges in 1D



Task 1.2.5 Forces on Charges (exam task, ca 8 % of a 60 minute exam, WS2020)



Given is an arrangement of electric charges located in a vacuum (see picture on the right).

The charges have the following values:

$Q_1 = 7 \text{ } \mu\text{C}$ (point charge)

$Q_2 = 5 \text{ } \mu\text{C}$ (point charge)

$Q_3 = 0 \text{ C}$ (infinitely extended surface charge)

$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m}$, $\epsilon_r = 1$

1. calculate the magnitude of the force of Q_2 on Q_1 , without the force effect of Q_3 .

Tips for the solution

- Which equation is to be used for the force effect of charges?
- How can the distance between the two charges be determined?

Solution

$$F_C = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q_1 \cdot Q_2}{r^2} \quad \&\amp; \quad \text{with } r = \sqrt{\Delta x^2 + \Delta y^2} \quad F_C = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q_1 \cdot Q_2}{\Delta x^2 + \Delta y^2} \quad \&\amp; \quad \text{Insert numerical values, read off distances: } \Delta x = 5 \text{ dm}, \Delta y = 3 \text{ dm} \quad F_C = \frac{1}{4\pi \cdot 8.854 \cdot 10^{-12} \text{ F/m}} \cdot \frac{7 \cdot 10^{-6} \text{ C} \cdot 5 \cdot 10^{-6} \text{ C}}{(0.5 \text{ m})^2 + (0.2 \text{ m})^2}$$

Result

$$|\vec{F}_C| = 1.084 \text{ N} \rightarrow 1.1 \text{ N}$$

2. is this force attractive or repulsive?

Tips for the solution

- What force effect do equally or oppositely charged bodies exhibit on each other?

Solution

The force is repulsive because both charges have the same sign.

Now let $Q_2=0$ and the surface charge Q_3 be designed in such a way that a homogeneous electric field with $E_3=100 \text{ kV/m}$ results. What force (magnitude) now results on Q_1 ?

Tips for the solution

- Which equation is to be applied for the force action in the homogeneous field?

Solution

$$F_C = E \cdot Q_1 \quad \&\amp; \quad \text{Insert numerical values} \quad F_C = 100 \cdot 10^3 \text{ V/m} \cdot 7 \cdot 10^{-6} \text{ C}$$

Result

$$|\vec{F}_C| = 0.7 \text{ N}$$

Task 1.2.6 Variation: Forces on Charges (exam task, ca 8% of a 60 minute exam, WS2020)



Given is an arrangement of electric charges located in a vacuum (see picture on the right).

The charges have the following values:

$Q_1 = 5 \mu\text{C}$ (point charge)

$Q_2 = -10 \mu\text{C}$ (point charge)

$Q_3 = 0 \text{ C}$ (infinitely extended surface charge)

$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m}$, $\epsilon_r = 1$

1. calculate the magnitude of the force of Q_2 on Q_1 , without the force effect of Q_3 .

Result

$$|\vec{F}_C| = 1.321 \text{ N} \rightarrow 1.3 \text{ N}$$

2. is this force attractive or repulsive?

Solution

The force is repulsive because both charges have the same sign.

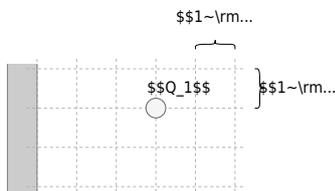
Now let $Q_2 = 0$ and the surface charge Q_3 be designed in such a way that a homogeneous electric field with $E_3 = 500 \text{ kV/m}$ results.

What force (magnitude) now results on Q_1 ?

Result

$$\begin{aligned} |\vec{F}_C| &= 2.5 \text{ N} \end{aligned}$$

Task 1.2.7 Variation: Forces on Charges (exam task, ca 8% of a 60 minute exam, WS2020)



Given is an arrangement of electric charges located in a vacuum (see picture on the right).

The charges have the following values:

$Q_1 = 2 \text{ } \mu\text{C}$ (point charge)

$Q_2 = -4 \text{ } \mu\text{C}$ (point charge)

$Q_3 = 0 \text{ C}$ (infinitely extended surface charge)

$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m}$, $\epsilon_r = 1$

1. calculate the magnitude of the force of Q_2 on Q_1 , without the force effect of Q_3 .

Result

$$\begin{aligned} |\vec{F}_C| &= 0.3595 \text{ N} \rightarrow 0.36 \text{ N} \end{aligned}$$

2. is this force attractive or repulsive?

Solution

The force is attractive because the charges have different signs.

Now let $Q_2=0$ and the surface charge Q_3 be designed in such a way that a homogeneous electric field with $E_3=100 \text{ ~}\text{V/m}$ results.
What force (magnitude) now results on Q_1 ?

Result

$$\begin{aligned} |\vec{F}_C| &= 0.4 \text{ ~}\text{N} \end{aligned}$$

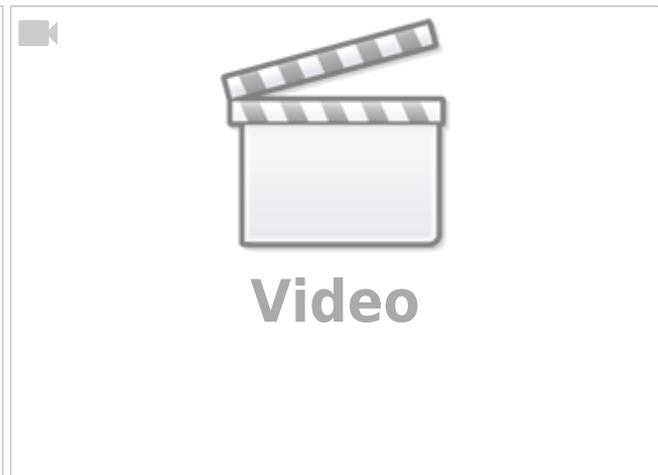
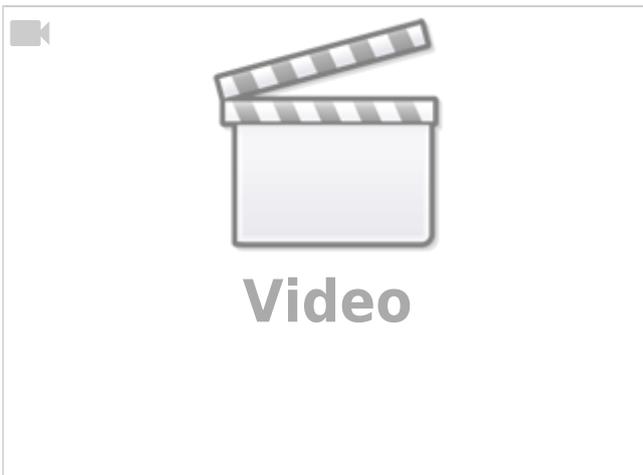
Embedded resources

The online book 'University Physics II' is strongly recommended as a reference for this chapter. Especially the following chapters:

- Chapter 5. [Electric Charges and Fields](#)
- Chapter 6. [Gauss's Law](#)
- Chapter 7. [Electrical Potential](#)
- Chapter 8. [Capacitance](#)

Intro into electric field

Field lines of various extended charged objects



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