Block 06 — Real Sources and Source Equivalents

Student Group

First Name	Surname	Matrikel Nr.

Table of Contents

Block 06 — Real sources and source equivalents

Learning objectives

- Model **real (linear) sources** with an internal resistance/conductance; read and draw their U–I characteristics.
- Determine **open-circuit voltage** $U_{\rm OC}$ and **short-circuit current** $I_{\rm SC}$; relate them by $R_{\rm i}=U_{\rm OC}/I_{\rm SC}$.
- Convert between **Thevenin (voltage)** and **Norton (current)** equivalents and apply the duality formulas.
- Use **deactivation rules** (ideal U-source → short, ideal I-source → open) to find **internal resistance** via superposition.
- Reduce a sub-network to a two-terminal (**one-port**) and use it (e.g., **loaded divider**) to predict $U_{\rm L}$, $I_{\rm L}$.

90-minute plan

1. Warm-up (8–10 min):
    1. Spot the difference: ideal vs. real source (show U–I lines).
    2. Quick quiz: “How to **deactivate** an ideal voltage/current source?”
2. Core concepts & derivations (60–65 min):
    1. Linear source model; $U_{\rm OC}$, $I_{\rm SC}$; load line and operating point.
    2. Thevenin/Norton duality & conversion; internal resistance by deactivation.
    3. Efficiency η and utilization ε; maximum power transfer.
    4. Two-terminal equivalents; **loaded voltage divider** as a Thevenin source.
3. Guided practice (10–15 min): 2 short numericals + 1 sim task (see “Exercises”).
4. Wrap-up (5 min): Summary + pitfalls.

Conceptual overview

1. A **real (linear) source** is an ideal source plus an **internal resistance** $R_{\rm i}$ (or conductance $G_{\rm i}$). Its output follows a **straight load line** between $U_{\rm OC}$ and $I_{\rm SC}$.
2. **Thevenin** (ideal U in series $R_{\rm i}$) and **Norton** (ideal I in parallel $G_{\rm i}$) are **equivalent** seen from the terminals; they are related by $U_{\rm OC}=I_{\rm SC}\,R_{\rm i}$ and $G_{\rm i}=1/R_{\rm i}$.
3. **Deactivate** ideal sources to find **internal resistance** of a network: ideal U-sources → short; ideal I-sources → open. Then combine resistors.
4. **Efficiency** favors $R_{\rm L}\gg R_{\rm i}$ (power systems), **maximum power** favors $R_{\rm L}=R_{\rm i}$ (communications matching).

Core content

It is known from everyday life that battery voltages drop under heavy loads. This can be seen, for example, when turning the ignition key in winter: The load from the starter motor is sometimes so large that the car lights or radio briefly cuts out.
Another example is a $1.5~\rm V$ battery: If such a battery is short-circuited by a piece of wire, the current flows does not let the wire glow. It is noticeably less cutrrent.

So it makes sense here to develop the ideal voltage source concept further. In addition, we will see that this also opens up the possibility of converting and simplifying more complicated circuits.

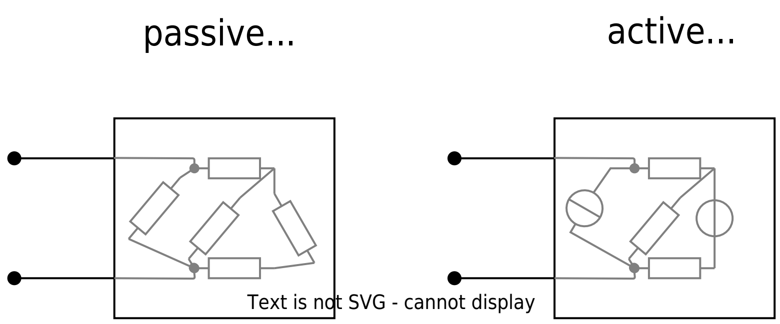


Fig. 1: passive two-terminal network

First, the concept of the two-terminal from the chapter basics and basic concepts is to be expanded (figure 1).

1. As **passive two-terminal network** which acts exclusively as a consumer. Thus it is valid for the passive two-terminal network that the current-voltage-characteristic always runs through the origin (see also chapter simple circuits).
2. **Active two-terminal networks**, on the other hand, also act as generators of electrical energy. Thus, the current-voltage characteristic there does not pass through the origin. Active two-terminal networks always contain at least one source (i.e. at least one current or voltage source).

From ideal to linear sources: load line, $U_{\rm OC}$ and $I_{\rm SC}$

Example / micro-exercise

Practical Example of a realistic Source: For the ideal voltage source, it was defined that it always supplies the same voltage independent of the load. In figure 2, in contrast, an example of a "realistic" voltage source is shown as an active two-terminal network.

1. This active two-terminal network generates a voltage of $1.5~\rm V$ and a current of $0~\rm A$ when the circuit is open.
2. If a resistor is added, the voltage decreases, and the current increases. For example, a voltage of $1.2~\rm V$ is applied to the resistor of $2 ~\Omega$, and a current of $0.6~A$ flows.
3. The terminals of the active two-terminal network can be directly connected via the outer switch. Then a current of $3~\rm A$ flows at a voltage of $0~\rm V$.

Fig. 2: Battery model with load resistor

This realization shall now be described with some technical terms:

- It is called **open circuit** when no current is drawn from an active two-terminal network: $I_{\rm SC}=0$.
  The voltage corresponds to the **open circuit voltage** $U=U_{\rm OC}$ (German: *Leerlaufspannung* $U_{\rm LL}$).
  The open circuit power is $P_{\rm OC}=U_{\rm OC} \cdot I_{\rm OC} = 0$.
- The term **short circuit** is used when the terminals of the two-terminal network are bridged without resistance. The current then flowing is called the **short-circuit current** $I=I_{\rm SC}$ (German: *Kurzschlussstrom* $I_{\rm KS}$).
  The short-circuit voltage is $U_{\rm SC}=0~\rm V$.
  Also, the short-circuit power is $P_{\rm SC}=U_{\rm SC} \cdot I_{\rm SC} = 0$.
- The active two-terminal network outputs power to a connected load in the region between no-load and short-circuit.

Important: As seen in the following, the short-circuit current can cause considerable power loss inside the two-terminal network and thus a lot of waste heat. Not every real two-terminal network is designed for this.

Fig. 3: Current-voltage characteristic of a linear voltage source

$$I$$

What is interesting now is the current-voltage characteristic of the circuit in figure 2. This can be seen in the simulation below. The result is a linear curve (see figure 3).

From a purely mathematical point of view, the course can be represented by the basic equation of linear graphs with the y-axis intercept $I_{\rm SC}$ and a slope of $-{{I_{\rm SC}}\over{U_{\rm OC}}}$:

$$\begin{align*} I = I_{\rm SC} - {{I_{\rm SC}}\over{U_{\rm OC}}}\cdot U \tag{3.1.1} \end{align*}$$

On the other hand, the formula can also be resolved to U:

$$\begin{align*} U = U_{\rm OC} - {{U_{\rm OC}}\over{I_{\rm SC}}}\cdot I \tag{3.1.2} \end{align*}$$

Remember:

If a two-terminal network results in a linear curve between $U_{\rm OC}$ and $I_{\rm SC}$, it is called a **linear source**. This curve describes in good approximation the behavior of many

> real sources. Often one finds synonymous to the term 'linear source' and also the term 'real (voltage) source'. However, this is somewhat misleading as it is a simplified model of reality.

So what does the inside of the linear source look like? In figure 4 two possible linear sources are shown, which will be considered in the following.

Fig. 4: equivalent circuit images of linear sources

Linear Voltage Source

The linear voltage source consists of a series connection of an ideal voltage source with the source voltage U_0 (English: EMF for ElectroMotive Force) and the internal resistance $R_\rm i$. To determine the voltage outside the active two-terminal network, the system can be considered as a voltage divider. The following applies:

$$\begin{align*} U = U_0 - R_{\rm i} \cdot I \end{align*}$$

The source voltage U_0 of the ideal voltage source will be measured at the terminals of the two-terminal network if this is unloaded. Then no current flows through the internal resistor R_i and there is no voltage drop there. Therefore: The source voltage is equal to the open circuit voltage $U_0 = U_{\rm OC}$.

$$\begin{align*} U = U_{\rm OC} - R_{\rm i} \cdot I \end{align*}$$

When the external voltage $U=0$, it is the short circuit case. In this case, $0 = U_{\rm OC} - R_{\rm i} \cdot I_{\rm SC}$ and transform $R_{\rm i} = {{U_{\rm OC}}\over{I_{\rm SC}}}$. Thus, equation $(3.1.2)$ is obtained: $$\begin{align*} U = U_{\rm OC} - {{U_{\rm OC}}\over{I_{\rm SC}}} \cdot I \end{align*}$$

Is this the structure of the linear source we are looking for? To verify this, we will now look at the second linear source.

Linear Current Source

The linear current source now consists of a parallel circuit of an ideal current source with source current I_0 and internal resistance $R_{\rm i}$, or internal conductance $G_{\rm i} = {{1}\over{R_{\rm i}}}$. To determine the voltage outside the active two-terminal, the system can be considered as a current divider. Here, the following holds:

$$\begin{align*} I = I_0 - G_{\rm i} \cdot U \end{align*}$$

Here, the source current can be measured at the terminals in the event of a short circuit. The following therefore applies: $I_{\rm SC}= I_0$

$$\begin{align*} I = I_{SC} - G_{\rm i} \cdot U \end{align*}$$

When the external current $I=0$, it is the no-load case. In this case, $0 = I_{\rm SC} - G_{\rm i} \cdot U_{\rm OC}$ and transform $G_{\rm i} = {{I_{\rm SC}}\over{U_{\rm OC}}}$.
Thus, equation $(3.1.1)$ is obtained: \begin{align*} I = I_{\rm SC} - {{I_{\rm SC}}\over{U_{\rm

OC}}} \cdot U \end{align*}

So it seems that the two linear sources describe the same thing.

Duality of Linear Sources

Through the previous calculations, we came to the interesting realization that both the linear voltage source and the linear current source provide the same result. It is true: For a linear source, both a linear voltage source and a linear current source can be specified as an equivalent circuit! As already in the case of the star-delta transformation, this not only provides two explanations for a black box. Also, here linear voltage sources can be transformed into linear current sources and vice versa.

The figure 5 compares again the two linear sources and their characteristics:

1. The linear voltage source is given by the source voltage U_0, or the open circuit voltage $U_{\rm OC}$ and the internal resistance $R_{\rm i}$.
2. The linear current source is given by the source current I_0, or the short-circuit current $I_{\rm SC}$ and the internal conductance $G_{\rm i}$.

Fig. 5: duality of linear sources

linear voltage source

The conversion is now done in such a way that the same characteristic curve is obtained:

From linear voltage source to linear current source:
Given: Source voltage U_0, resp. open circuit voltage $U_{\rm OC}$, internal resistance $R_\rm i$
in question: source current I_0, resp. short circuit current $I_{\rm SC}$, internal conductance $G_\rm i$
$\boxed{I_{\rm SC} = {{U_{\rm OC}}\over{R_\rm i}}}$, $\boxed{G_\rm i = {{1}\over{R_\rm i}}}$

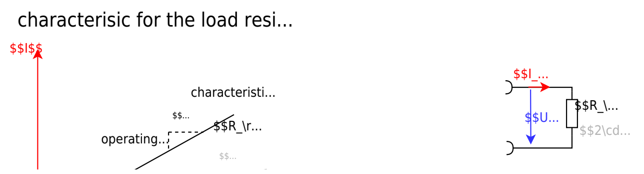
From linear current source to linear voltage source:
Given: Source current I_0, resp. short-circuit current $I_{\rm SC}$, internal resistance $G_\rm i$
in question: source voltage U_0, resp. open-circuit voltage $U_{\rm OC}$, internal resistance $R_\rm i$
$\boxed{U_{\rm OC} = {{I_{\rm SC}}\over{G_\rm i}}}$, $\boxed{R_\rm i = {{1}\over{G_\rm i}}}$

Operating Point of a real Voltage Source

figure 6 shows the characteristics of the linear voltage source (left) and a resistive resistor (right). For this purpose, both are connected to a test system in the simulation: In the case of the source with a variable ohmic resistor, and in the case of the load with a variable source. The characteristic curves formed in this way were described in the previous chapter.

Fig. 6: Source and consumer characteristics

Fig. 7: Determining the operating point



The operating point can be determined from both characteristic curves. This is assumed when both the linear voltage source is connected to the ohmic resistor (without the respective test systems). In figure 7 both characteristic curves are drawn in a current-voltage diagram. The point of intersection is just the operating point that sets in. If the load resistance is varied, the slope changes in inverse proportion, and a new operating point is established (light grey in the figure).

The derivation of the working point is also here explained again in a video.

Conversion of any linear two-terminal Network

In figure 8, it can be seen that the internal resistance of the linear current source measured by the ohmmeter (resistance meter) is exactly equal to that of the linear voltage source.

Fig. 8: Resistance of linear sources

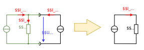
In the simulation, a measuring current I_Ω is used to determine the resistance value. [1] Let us have a look at the properties of the ohmmeter in the simulation by double-clicking on the ohmmeter. Here, a very large measuring current of $I_\Omega=1~\rm A$ is used. This could lead to high voltages or the destruction of components in real setups.

In order to understand why is this nevertheless chosen so high in the simulation, do the following: Set the measuring current for both linear sources to (more realistic) $1~\rm mA$. What do you notice?

The circuit in figure 9 shows this circuit again. The ohmmeter is replaced by a current source and a voltmeter since only the electrical properties are important in the following. In this setup, it can be seen that the current through $G_{\rm i}$ is just given by $I_{\rm i} = I_0 + I_\Omega$ (node theorem). Thus, the two sources in the circuit can be reduced.

This should make the situation clear with a measuring current of $1~\rm mA$. The voltage at the resistor is now given by $U_\Omega = R \cdot (I_0 + I_\Omega)$. Only when I_Ω is very large does I_0 become negligible. The current of a conventional ohmmeter cannot guarantee this for every measurement.

Fig. 9: circuit with two current sources



Note:

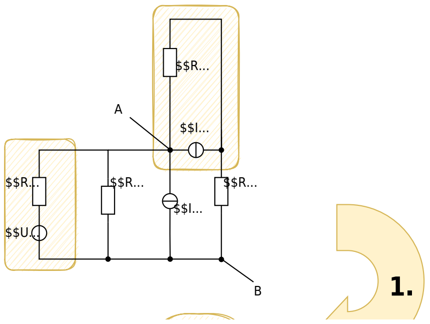
If resistors are to be measured in a circuit, at least one terminal of the resistor must be disconnected from the circuit. Otherwise, other sources or resistors may falsify the measurement result.

Example / micro-exercise

This knowledge can now be used for more complicated circuits. In figure 11 such a circuit is drawn. This is to be converted into a searched equivalent conductance $G_{\rm eq}$ and a searched equivalent current source with $I_{\rm eq}$.

Important here: Only two-terminal networks can be converted via source duality. This means that only 2 nodes may act as output terminals for selected sections of the circuit. If there are more nodes the conversion is not possible.

Fig. 11: circuit with multiple sources



1. As a first step, sources are to be converted in such a way that resistors can be combined after the conversion. In this example, this is done by:
    1. converting the linear voltage source R_1 and U_1 into a linear current source with $I_1={{U_1}\over{R_1}}$ and R_1 (or $G_1={{1}\over{R_1}}$)
    2. converting the linear current source R_4 and I_4 into a linear voltage source with $U_4={{I_4}\cdot{R_4}}$ and R_4
2. In the second step, the linear voltage source U_4 formed in 1. with R_4 can be connected to the resistor R_3. From this again a linear current source can be created. This now has a resistance of $R_5 = R_3+R_4$ and an ideal current source with $I_5 = {{U_4}\over{R_3+R_4}}= {{I_4}\cdot{R_4}\over{R_3+R_4}} $.
3. The circuit diagram that now emerges is a parallel circuit of ideal current sources and resistors. This can be used to determine the values of the ideal equivalent current source and the equivalent resistance:
    1. ideal equivalent current source $I_{\rm eq}$: \begin{align*} I_{\rm eq} = I_1 + I_3 + I_5 = I_1 + I_3 + I_4\cdot{{R_4}\over{R_3+R_4}} \end{align*}
    2. Substitute conductance $G_{\rm eq}$: \begin{align*} G_{\rm eq} = \Sigma G_i = {{1}\over{R_1}}+{{1}\over{R_2}}+{{1}\over{R_5}}={{1}\over{R_1}}+{{1}\over{R_2}}+{{1}\over{R_3+R_4}} \end{align*}

Any interconnection of linear voltage sources, current sources, and ohmic resistors can be.

- as a single, linear voltage source
  (Thévenin theorem) or
- as a single, linear current source
  (Norton theorem)

In figure 10 it can be seen that the three circuits give the same result (voltage/current) with the same load. This is also true when an (AC) source is used instead of the load.

Fig. 10: Equivalent voltage and current source

Simplified Determination of the internal Resistance

Note:
If only the equivalent resistance of a more complex circuit is sought, the following approach to deactivate the sources can be used:

1. Replace all ideal voltage sources with a short circuit
   (= internal resistance of the ideal voltage source).
2. Replace all ideal current sources with an open contact
   (= internal resistance of the ideal current source)
3. Add the remaining resistors to an equivalent resistance using the rules for parallel and series connection.

The equivalent circuits for the ideal sources can be seen via the circuit diagrams (see figure 12).

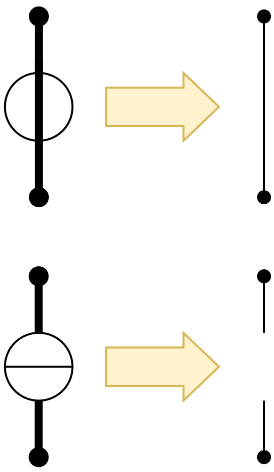
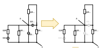


Fig. 12: equivalent resistance of ideal sources

Thus also the equivalent resistance of the complex circuit above can be derived quickly.
For the source current I_0 ideal equivalent current source resp. the source voltage U_0 ideal equivalent voltage source this derivation can not be used.

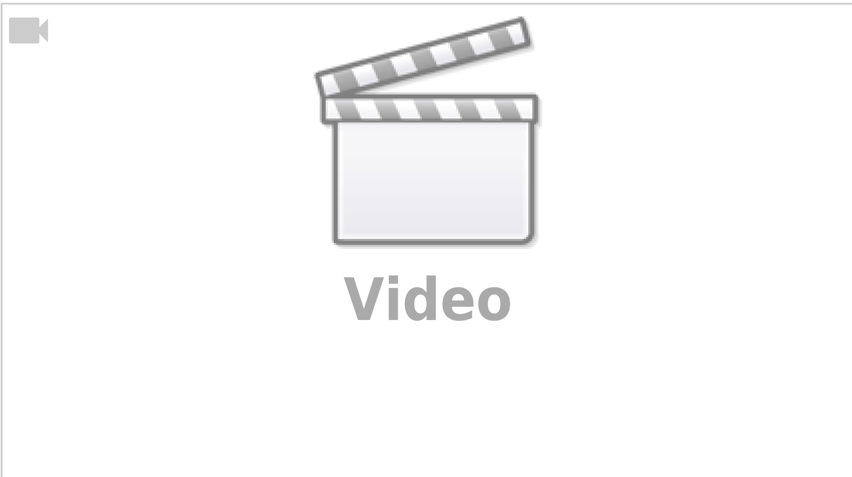
Example / micro-exercise

In the following, a simplification is shown. Fig. 13: Simplified determination of internal resistance

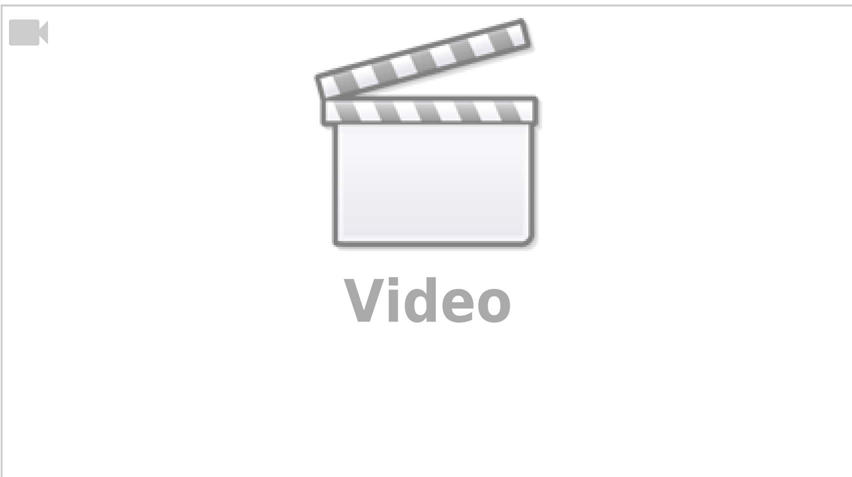


Exercises

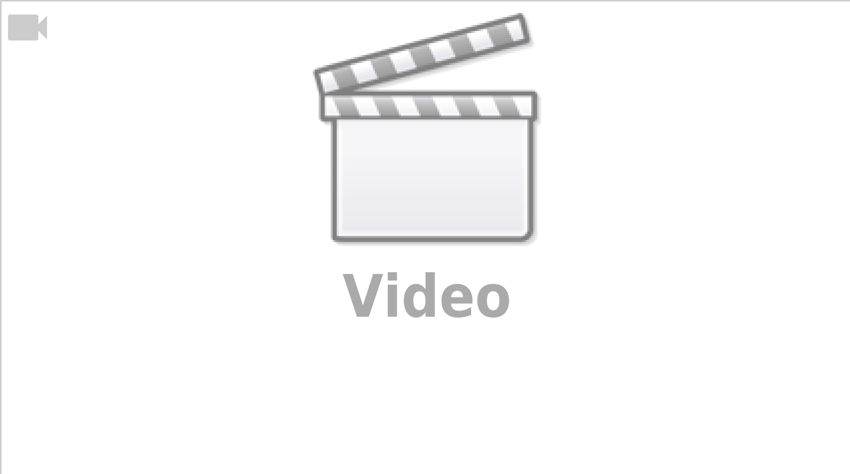
Exercise 3.2.1 Solving a circuit simplification I



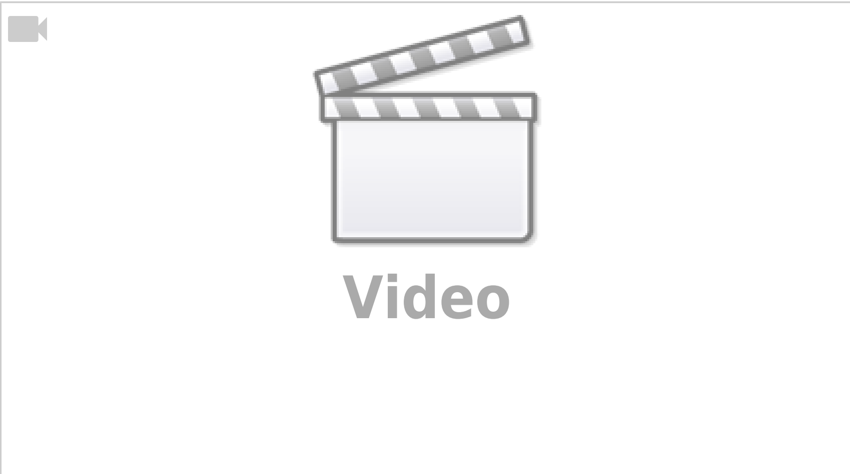
Exercise 3.2.2 Solving a circuit simplification II



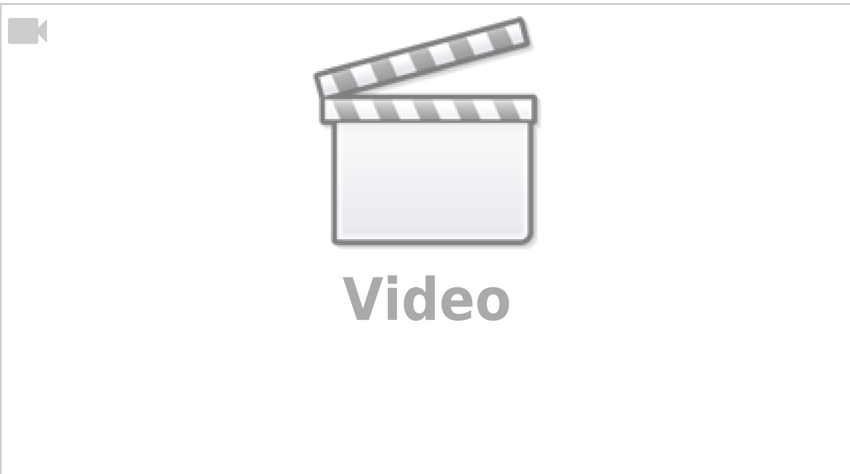
Exercise 3.2.3 Solution sketch for a more difficult circuit simplification



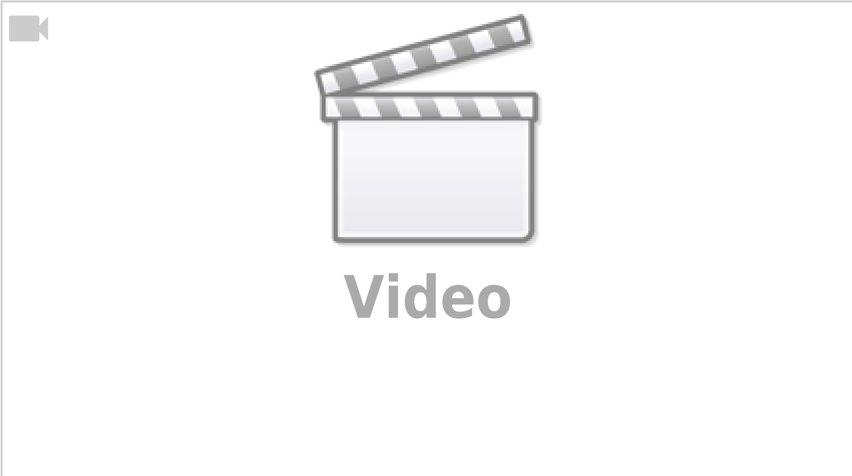
Exercise 3.2.4 Interesting circuit tasks



Exercise 3.1.1 Convert current source to voltage source



Exercise 3.1.2 Convert voltage source to current source



Common pitfalls

- **Wrong deactivation:** do **not** set an ideal voltage source to open or an ideal current source to short; the rules are: U-source→**short**, I-source→**open**.
- **Confusing goals: max power** ($R_{\rm L}=R_{\rm i}$, $\eta=50\%$) vs. **high efficiency** ($R_{\rm L}\gg R_{\rm i}$). Don’t equate them.
- **Ignoring ratings:** not every real source is short-circuit-proof—$I_{\rm SC}$ is a **model parameter**, not a recommended experiment.
- **Mixed conventions:** keep the **passive sign convention** for loads; use conventional current ($+$ to $-$).

Exercises

Quick checks

Exercise E1.1 From $U_{\rm OC}$, $I_{\rm SC}$ to $R_{\rm i}$ and $U_{\rm L}$

A source has $U_{\rm OC}=12.0~\rm V$, $I_{\rm SC}=3.0~\rm A$. Find $R_{\rm i}$ and, for $R_{\rm L}=9.0~\Omega$, compute $U_{\rm L}$ and η.

Result

> $R_{\rm i}=U_{\rm OC}/I_{\rm SC}=4.00~\Omega$. $U_{\rm L}=U_{\rm OC}\dfrac{R_{\rm L}}{R_{\rm L}+R_{\rm i}}=12.0~{\rm V}\cdot \dfrac{9.0~\Omega}{13.0~\Omega}=8.31~{\rm V}$. $\eta=\dfrac{R_{\rm L}}{R_{\rm L}+R_{\rm i}}=\dfrac{9.0}{13.0}=0.692\;(-)$.

Exercise E2.2 Thevenin ↔ Norton conversion

Given a Thevenin source $U_{\rm OC}=5.00~\rm V$ and $R_{\rm i}=250~\Omega$, find the Norton pair $(I_{\rm SC},G_{\rm i})$.

Result

> $I_{\rm SC}=U_{\rm OC}/R_{\rm i}=20.0~\rm mA$, $G_{\rm i}=1/R_{\rm i}=4.00~\rm mS$.

Longer exercises

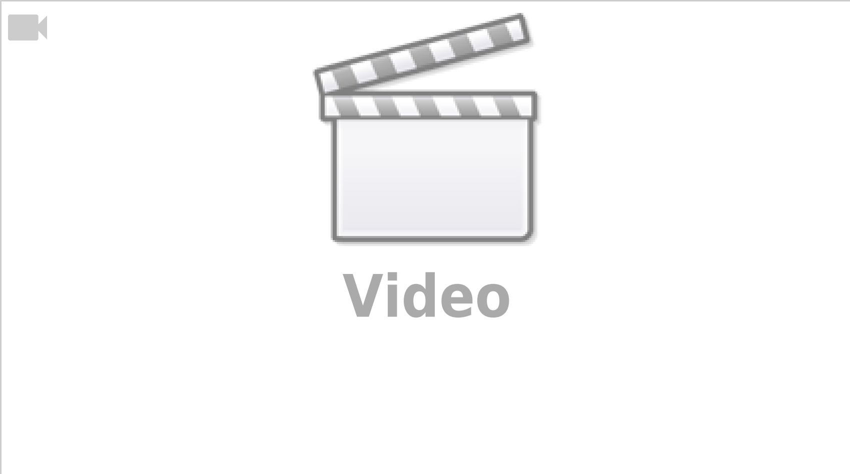
Exercise E3.1 Loaded divider as Thevenin

A divider $R_1=3.3~\rm k\Omega$, $R_2=6.8~\rm k\Omega$ is fed from $U=10.0~\rm V$ and loaded by $R_{\rm L}=10.0~\rm k\Omega$. Replace the divider by its Thevenin equivalent, then compute $U_{\rm L}$ and the **loading error** relative to the ideal (no-load) divider output.

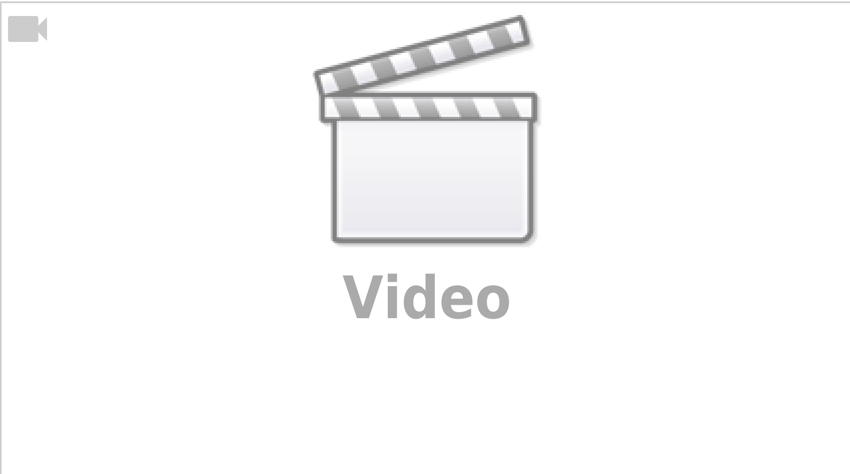
Result

> $R_{\rm ie}=R_1\parallel R_2=\dfrac{(3.3)(6.8)}{3.3+6.8}~\rm k\Omega=2.22~\rm k\Omega$. $U_{0\rm e}=\dfrac{R_2}{R_1+R_2}U=6.8/(3.3+6.8)\cdot 10.0~\rm V=6.80~\rm V$. $U_{\rm L}=U_{0\rm e}\dfrac{R_{\rm L}}{R_{\rm L}+R_{\rm ie}}=6.80~{\rm V}\cdot \dfrac{10.0}{12.22}=5.56~{\rm V}$. Ideal (no-load) output would be $6.80~\rm V$ ⇒ loading error $=1.24~\rm V$.

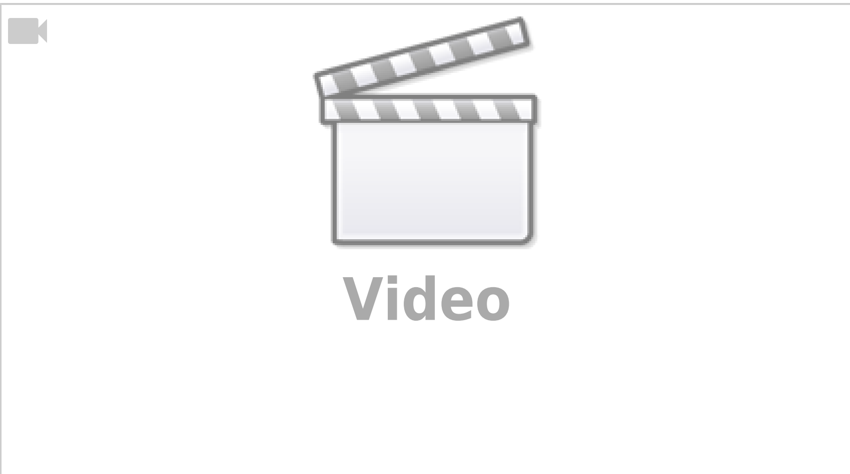
Exercise 3.1.1 Convert current source to voltage source



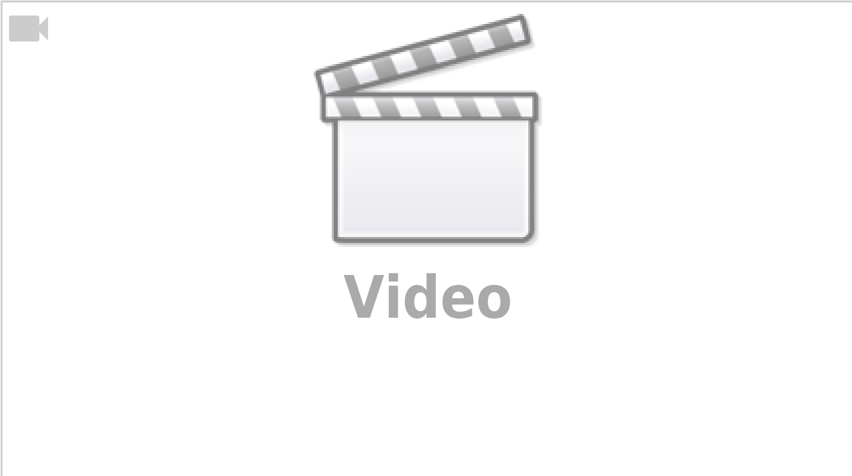
Exercise 3.1.2 Convert voltage source to current source



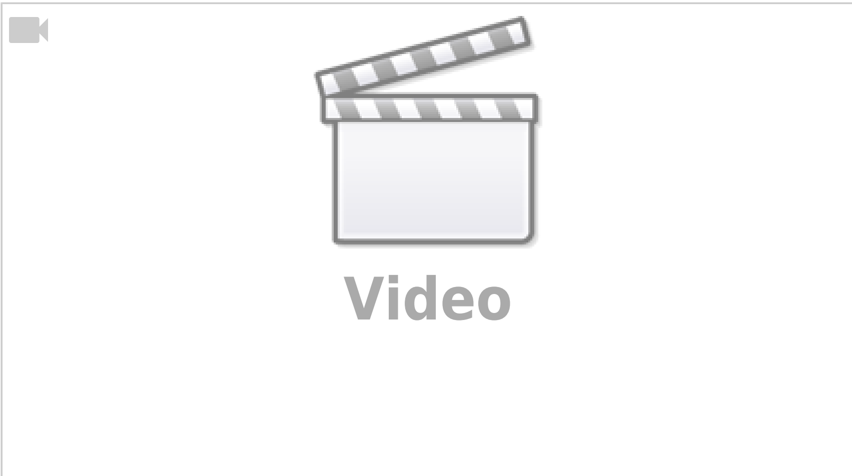
Exercise 3.2.1 Solving a circuit simplification I



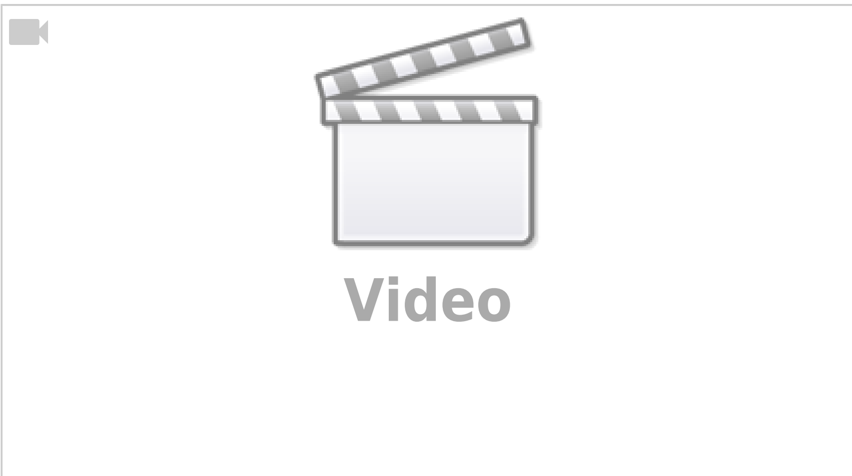
Exercise 3.2.2 Solving a circuit simplification II



Exercise 3.2.3 Solution sketch for a more difficult circuit simplification

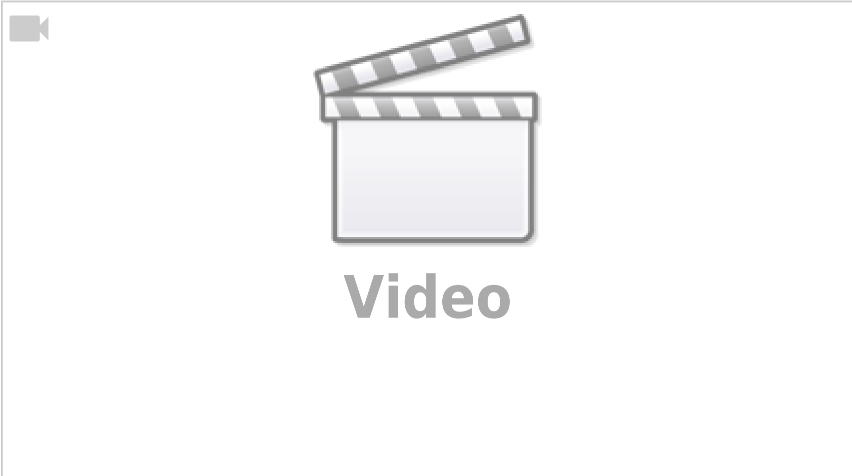


Exercise 3.2.4 Interesting circuit tasks

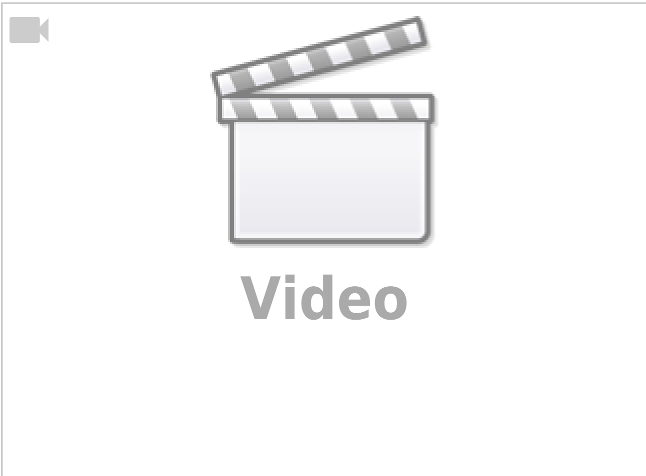


Embedded resources

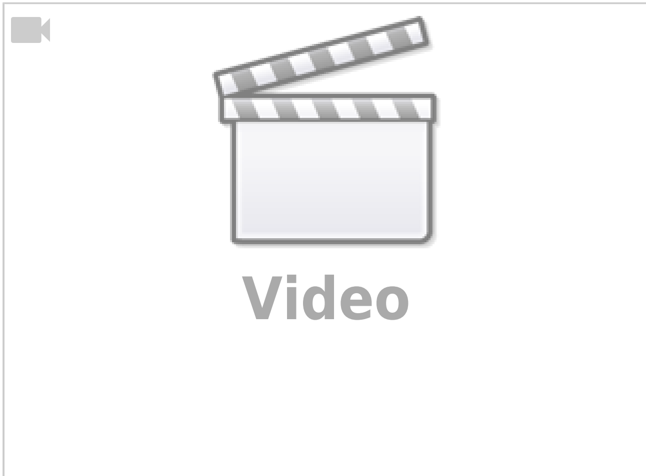
DC Voltage & Current Source Theory



Intro to superposition (method used when deactivating sources)



A more complex superposition example



Summary

- A **linear (real) source** is fully determined by $(U_{\rm OC},I_{\rm SC})$ or equivalently $(U_0,R_{\rm i})$ / $(I_0,G_{\rm i})$; both forms are **equivalent**.
- **Thevenin ↔ Norton**: $U_{\rm OC}=I_{\rm SC}R_{\rm i}$, $G_{\rm i}=1/R_{\rm i}$; deactivation rules let you find $R_{\rm i}$ quickly.
- **Efficiency vs. maximum power**: choose $R_{\rm L}\gg R_{\rm i}$ for high η, or $R_{\rm L}=R_{\rm i}$ for max $P_{\rm L}$.
- **Two-terminal reductions** (e.g., loaded divider) simplify analysis of larger networks.

1)
This concept will also be used in an electrical engineering lab experiment on resistors in the 3rd semester.