

# Block 01 — Physical Quantities and SI System

## Student Group

First Name	Surname	Matrikel Nr.

## Table of Contents

<b>Block 01 — Physical quantities and SI system</b>	3
<b>Learning objectives</b>	3
<b>90-minute plan</b>	3
<b>Conceptual overview</b>	3
<b>Core content</b>	4
SI base quantities and units	4
Common derived quantities	4
Prefixes	5
Physical equations	5
Quantity Equations	5
normalized Quantity Equations	5
Example for a quantity equation	6
Letters for physical quantities	7
Notation & units	7
<b>Common pitfalls &amp; misconceptions</b>	9
<b>Exercises</b>	9
Quick checks	9
Exercise E1.1 Unit check (quantity equation)	9
Exercise E2.2 Work from lifting (quantity equation)	10
Exercise E3.1 Conversion	10
Exercise E4.2 Dimension	10
Exercise E5.3 Conversion	10
Exercise E6.4 Conversion	11
Longer exercises	11
Exercise E7 Conversions: Battery	11
Exercise E8 Conversions: Speed, Energy, and Power	12
Exercise E9 Conversion: Vacuum Cleaner	12
Exercise E10 Conversions: Video on Prefixes	12
Exercise E11 Conversion: Energy Consumption	13

Exercise E12 Conversions: Battery .....	13
Exercise E13 Conversions: Speed, Energy, and Power .....	14
Exercise E14 Conversion: Vacuum Cleaner .....	14
Exercise E15 Conversions: Video on Prefixes .....	15
Exercise E16 Conversion: Energy Consumption .....	15
Exercise E1 Machine-Vision Strobe Unit: Charging and Safe Discharge of a Flash Capacitor .....	15
Exercise E2 Sensor Input Buffer: Source, T-Network and Capacitor .....	19
Exercise E3 Hall-Sensor Calibration Coil: Short Air-Core Coil .....	22
Exercise E20 Conversion: Energy, Power and Area .....	26
Exercise E21 Conversion: Energy, Power and Area .....	26
<b>Learning Objectives</b> .....	27
<b>Preparation at Home</b> .....	27
<b>90-minute plan</b> .....	27
<b>Conceptual overview</b> .....	28
<b>Core content</b> .....	28
<i><b>Time Course of the Charging and Discharging Process</b></i> .....	31
<i><b>Charging a capacitor at time <math>t=0</math></b></i> .....	32
Notice: .....	34
<i><b>Discharging a capacitor at time <math>t=0</math></b></i> .....	35
<i><b>Periodic switching operations</b></i> .....	37
<i><b>Energy stored in a Capacitor</b></i> .....	38
Energy consideration of the capacitor .....	38
Energy Consideration on the Resistor .....	39
Consideration of total energy turnover .....	40
<b>Common pitfalls</b> .....	41
<b>Exercises</b> .....	41
<i><b>Worked examples</b></i> .....	41
Exercise 1.1 Capacitor charging/discharging practice Exercise .....	41
Exercise E 1.2 Capacitor charging/discharging .....	41
Exercise E1.3 Charge balance of two capacitors (educational exercise, not part of an exam) .....	43
Exercise E1.4 Machine-Vision Strobe Unit: Charging and Safe Discharge of a Flash Capacitor .....	44
Exercise E1.5 Sensor Input Buffer: Source, T-Network and Capacitor .....	48
Exercise E1.6 Hall-Sensor Calibration Coil: Short Air-Core Coil .....	51
<b>Embedded resources</b> .....	55
<i><b>Embedded resources</b></i> .....	56
<i><b>Mini-assignment / homework (optional)</b></i> .....	57

# Block 01 — Physical quantities and SI system

## Learning objectives

After this 90-minute block, you can

- Use the SI base quantities, units, and symbols correctly; convert between units with prefixes.
- Distinguish base vs. derived quantities; express key EE units (e.g.  $\text{V}$ ,  $\text{V}\cdot\text{s}$ ,  $\text{V}\cdot\text{A}$ ) in SI base units.
- Apply quantity equations and perform unit (dimensional) checks; contrast with normalized (dimensionless) equations.
- Read and use common Latin/Greek letter symbols; distinguish uppercase/lowercase and instantaneous vs. constant quantities.

## 90-minute plan

1. Warm-up (10 min):
  1. “What is the unit of conductivity? of energy?”
2. Quick prefix quiz; everyday magnitude estimates ( $\text{mA}$ ,  $\text{k}\Omega$ ,  $\mu\text{F}$ ).
3. Core concepts & derivations (60 min):
  1. SI base set  $\rightarrow$  derived units; prefix rules;
4. quantity vs. normalized equations;
5. dimensional checks.
  1. Prefix ladder ( $\text{E}\dots\text{a}$ ) and best-practice rounding/checks.
  2. Symbols & Greek letters in EE1; time-varying vs constant symbols.
6. Practice (15 min): Fast conversions and unit checks (individual  $\rightarrow$  pair).
7. Wrap-up (5 min): Summary table; common pitfalls checklist.

## Conceptual overview

1. Units are the grammar of engineering and physics.
2. The SI defines seven **base quantities** and units; all other (derived) units are built from these without extra numerical factors. The SI defines seven **base quantities** and units.
3. In EE1 we work strictly in the SI system, combining **numerical value  $\times$  unit** and tracking dimensions at every step (e.g.,  $2\text{ A}$  means “two times one ampere”).
4. Derived units (e.g.,  $\text{V}$ ,  $\Omega$ ,  $\text{S}$ ) must reduce to base units without hidden factors.
5. **Prefixes** scale units by powers of ten to keep numbers readable. Prefixes compress very large and very small numbers so we can compute and compare safely.
6. **Quantity equations** keep units; **normalized equations** cancel units to yield dimensionless ratios (e.g., efficiency).
7. In EE, symbol choices and letter case matter:  $\text{U}$  vs.  $u(t)$ ,  $\text{M}$  (mega) vs.  $\text{m}$  (milli). We adopt a consistent symbol set (Latin + Greek), and distinguish **constants** (capital letters) from **time functions** (lowercase, e.g.,  $u(t)$ ).

8. Finally, we preview the three anchor quantities for the next blocks: **charge** (what moves), **current** (how fast charge moves), and **voltage** (energy per charge). Physics describes **quantities** with a **numerical value × unit** (e.g.,  $I=2\text{~}\text{A}$ ).

## Core content

### SI base quantities and units

- For practical applications of physical laws of nature, **physical quantities** are put into mathematical relationships.
- There are basic quantities based on the SI system of units (French for *Système International d'Unités*), see below.
- In order to determine the basic quantities quantitatively (quantum = Latin for *how big*), **physical units** are defined, e.g.  $1\text{~}\text{m}$  for length.
- In electrical engineering, the first three basic quantities (cf. [table 1](#)) are particularly important.  
Mass is important for the representation of energy and power.
- Each physical quantity is indicated by a product of **numerical value** and **unit**:  
e.g.  $I = 2\text{~}\text{A}$ 
  - This is the short form of  $I = 2 \cdot 1\text{~}\text{A}$
  - $I$  is the physical quantity, here: electric current strength
  - $2$  is the numerical value
  - $1\text{~}\text{A}$  is the (measurement) unit, here:  $1\text{~}\text{A}$

Base quantity	Name	Unit	Definition
Time	Second	$\text{s}$	Oscillation of $^{133}\text{Cs}$ -Atom
Length	Meter	$\text{m}$	by $s$ and speed of light
el. Current	Ampere	$\text{A}$	by $s$ and elementary charge
Mass	Kilogram	$\text{kg}$	still by kg prototype
Temperature	Kelvin	$\text{K}$	by triple point of water
amount of substance	Mol	$\text{mol}$	via number of $^{12}\text{C}$ nuclides
luminous intensity	Candela	$\text{cd}$	via given radiant intensity

Tab. 1: SI base quantities (SI)

### Common derived quantities

- Besides the basic quantities, there are also quantities derived from them, e.g.  $[F] = [m] \cdot [a] \rightarrow 1\text{~}\text{N} = 1\text{~}\text{kg} \cdot \frac{1\text{~}\text{m}}{1\text{~}\text{s}^2}$ .
- SI units should be preferred for calculations. These can be derived from the basic quantities **without a numerical factor**.  
example:
  - The pressure unit bar ( $\text{bar}$ ) is an SI unit.
  - BUT: The obsolete pressure unit “Standard atmosphere” ( $=1.013\text{~}\text{bar}$ ) is **not** an SI unit.
- To prevent the numerical value from becoming too large or too small, it is possible to replace a decimal factor with a prefix.

We will see, that a lot of electrical quantities are derived quantities.

### Prefixes

- Use prefixes to keep magnitudes practical (see [table 2](#) and [table 3](#)).
- Instead of writing zeroes for like in  $0.000000004 \text{ ~}\mu\text{C}$  it is easier to write  $4 \text{ ~}\mu\text{C}$ .
- For calculation it is often easier to write  $4 \text{ ~nC} = 4 \cdot 10^{-9} \text{ ~C}$  or the notation  $4 \text{e-9 C}$

prefix	prefix symbol	meaning
Yotta	$\text{\rm Y}$	$10^{24}$
Zetta	$\text{\rm Z}$	$10^{21}$
Exa	$\text{\rm E}$	$10^{18}$
Peta	$\text{\rm P}$	$10^{15}$
Tera	$\text{\rm T}$	$10^{12}$
Giga	$\text{\rm G}$	$10^9$
Mega	$\text{\rm M}$	$10^6$
Kilo	$\text{\rm k}$	$10^3$
Hecto	$\text{\rm h}$	$10^2$
Deka	$\text{\rm de}$	$10^1$

Tab. 2: Prefixes I

### Physical equations

- Physical equations allow a connection of physical quantities.
- There are two types of physical equations to distinguish:
  - Quantity equations (in German: *Größengleichungen*)
  - Normalized quantity equations (also called related quantity equations, in German *normierte Größengleichungen*)

prefix	prefix symbol	meaning
Deci	$\text{\rm d}$	$10^{-1}$
Centi	$\text{\rm c}$	$10^{-2}$
Milli	$\text{\rm m}$	$10^{-3}$
Micro	$\text{\rm u}$ , $\mu$	$10^{-6}$
Nano	$\text{\rm n}$	$10^{-9}$
Piko	$\text{\rm p}$	$10^{-12}$
Femto	$\text{\rm f}$	$10^{-15}$
Atto	$\text{\rm a}$	$10^{-18}$
Zeppto	$\text{\rm z}$	$10^{-21}$
Yocto	$\text{\rm y}$	$10^{-24}$

Tab. 3: Prefixes II

Quantity Equations	normalized Quantity Equations
<p>The vast majority of physical equations result in a physical unit that does not equal \$1\$.</p> <p>Example: Force <math>F = m \cdot a</math> with <math>[F] = 1 \text{ ~kg} \cdot \frac{\text{m}}{\text{s}^2}</math></p> <ul style="list-style-type: none"> <li>• A unit check</li> </ul>	<p>In normalized quantity equations, the measured value or calculated value of a quantity equation is divided by a reference value. This results in a dimensionless quantity relative to the reference value.</p> <p>Example: The</p>

- should always be performed for quantity equations
- Quantity equations should generally be preferred

efficiency  $\eta = \frac{P_{\text{O}}}{P_{\text{I}}}$  is given as quotient between the outgoing power  $P_{\text{O}}$  and the incoming power  $P_{\text{I}}$ .

As a reference the following values are often used:

- Nominal values (maximum permissible value in continuous operation) or
- Maximum values (maximum value achievable in the short term)

For normalized quantity equations, the units should **always** cancel out.

### Example for a quantity equation

Let a body with the mass  $m = 100 \text{ kg}$  be given. The body is lifted by the height  $s = 2 \text{ m}$ .

What is the value of the needed work?

physical equation:

$W = F \cdot s$   
 $W = F \cdot s$  where  $F = m \cdot g$   
 $W = m \cdot g \cdot s$  where  $m = 100 \text{ kg}$ ,  $s = 2 \text{ m}$  and  $g = 9.81 \frac{\text{m}}{\text{s}^2}$   
 $W = 100 \text{ kg} \cdot 9.81 \frac{\text{m}}{\text{s}^2} \cdot 2 \text{ m}$   
 $W = 100 \cdot 9.81 \cdot 2 \text{ kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot \text{m}$   
 $W = 1962 \text{ kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot \text{m}$   
 $W = 1962 \text{ Nm} = 1962 \text{ J}$

## Letters for physical quantities

Latin/Greek letters are reused across physics.

In physics and electrical engineering, the letters for physical quantities are often close to the English term.

Thus explains  $C$  for **C**apacity,  $Q$  for **Q**uantity and  $\epsilon_0$  for the **E**lectrical Field Constant. But, maybe you already know that  $C$  is used for the thermal capacity as well as for the electrical capacity. The Latin alphabet does not have enough letters to avoid conflicts for the scope of physics. For this reason, Greek letters are used for various physical quantities (see [table 4](#)).

Especially in electrical engineering, **upper/lower case letters** are used to distinguish between

- a constant (time-independent) quantity, e.g. the period  $T$
- or a time-dependent quantity, e.g. the instantaneous voltage  $u(t)$
- EE relies on case and context (e.g.,  $U$  vs.  $u(t)$ ). Time-varying quantities often use lowercase, constants uppercase.

## Notation & units

The course consistently uses the following symbols, units, and typical values:

Symbol	Quantity	SI unit	name of the unit	Typical values
--------	----------	---------	------------------	----------------

Uppercase letters	Lowercase letters	Name	Application
$A$	$\alpha$	Alpha	angles, linear temperature coefficient
$B$	$\beta$	Beta	angles, quadratic temperature coefficient, current gain
$\Gamma$	$\gamma$	Gamma	angles
$\Delta$	$\delta$	Delta	small deviation, length of a air gap
$E$	$\epsilon$ , $\epsilon_0$	Epsilon	electrical field constant, permittivity
$Z$	$\zeta$	Zeta	- (math function)
$H$	$\eta$	Eta	efficiency
$\Theta$	$\theta$ , $\vartheta$	Theta	temperature in Kelvin
$I$	$\iota$	Iota	-
$K$	$\kappa$	Kappa	specific conductivity
$\Lambda$	$\lambda$	Lambda	- (wavelength)
$M$	$\mu$	Mu	magnetic field constant, permeability
$N$	$\nu$	Nu	-
$\Xi$	$\xi$	Xi	-
$O$	$\omicron$	Omicron	-

Symbol	Quantity	SI unit	name of the unit	Typical values	Uppercase letters	Lowercase letters	Name	Application
$\$q\$$	Electric charge	$\$\\rm C\$$	Coulomb	$\$10^{-19} \\sim \\rm C\$$ (electron) to $\$\\rm mC\$$	$\$\\Pi\$$	$\$\\pi\$$	Pi	math. product operator, math. constant
$\$I\$$	Electric current	$\$\\rm A\$$	Ampere	$\$\\rm \\mu A\$$ (sensors) to $\$\\rm kA\$$ (lightning)	$\$R\$$	$\$\\rho\$$ , $\$\\varrho\$$	Rho	specific resistivity
$\$U\$$	Voltage (potential difference)	$\$\\rm V\$$	Volt	$\$\\rm \\mu V\$$ (noise) to $\$\\rm MV\$$ (transmission lines)	$\$\\Sigma\$$	$\$\\sigma\$$	Sigma	math. sum operator, alternatively for specific conductivity
$\$\\varphi\$$	Electric potential	$\$\\rm V\$$	Volt	—	$\$T\$$	$\$\\tau\$$	Tau	time constant
$\$P\$$	Power	$\$\\rm W\$$	Watt	$\$\\rm mW\$$ (electronics) to $\$\\rm MW\$$ (machines)	$\$\\Upsilon\$$	$\$\\upsilon\$$	Upsilon	-
$\$W\$$	Energy	$\$\\rm J\$$	Joule	$\$\\rm \\mu J\$$ (capacitors) to $\$\\rm MJ\$$ (batteries)	$\$\\Phi\$$	$\$\\phi\$$ , $\$\\varphi\$$	Phi	magnetic flux, angle, potential
$\$R\$$	Resistance	$\$\\rm \\Omega\$$	Ohm	$\$\\rm m\\Omega\$$ to $\$\\rm M\\Omega\$$	$\$X\$$	$\$\\chi\$$	Chi	-
$\$G\$$	Conductance	$\$\\rm S\$$	Siemens	$\$\\rm \\mu S\$$ to $\$\\rm S\$$	$\$\\Psi\$$	$\$\\psi\$$	Psi	linked magnetic flux
$\$\\rho\$$	Resistivity	$\$\\rm \\Omega \\cdot m\$$	—	$\$1.7 \\cdot 10^{-8} \\sim \\rm \\Omega m\$$ (Cu)	$\$\\Omega\$$	$\$\\omega\$$	Omega	unit of resistance, angular frequency
$\$\\sigma\$$	Conductivity	$\$\\rm S/m\$$	—	$\$5.8 \\cdot 10^7 \\sim \\rm S/m\$$ (Cu)	Tab. 4: greek letters			
$\$C\$$	Capacitance	$\$\\rm F\$$	Farad	$\$\\rm pF\$$ (ceramic) to $\$\\rm F\$$ (supercaps)				
$\$L\$$	Inductance	$\$\\rm H\$$	Henry	$\$\\rm \\mu H\$$ to $\$\\rm H\$$				
$\$E\$$	Electric field strength	$\$\\rm V/m\$$	—	$\$\\rm 1 \\sim \\rm V/m\$$ to $\$\\rm MV/m\$$ (breakdown)				
$\$D\$$	Electric flux density	$\$\\rm C/m^2\$$	—	—				
$\$B\$$	Magnetic flux density	$\$\\rm T\$$	Tesla	$\$\\rm \\mu T\$$ (Earth) to several $\$\\rm T\$$ (MRI)				
$\$H\$$	Magnetic field strength	$\$\\rm A/m\$$	—	—				
$\$\\Phi\$$	Magnetic flux	$\$\\rm Wb\$$	Weber	$\$\\rm \\mu Wb\$$ to $\$\\rm mWb\$$				

Symbol	Quantity	SI unit	name of the unit	Typical values
$\theta$	magnetic voltage (Magnetomotive force)	$\text{A} \cdot \text{turn}$	—	—
$R$	Reluctance	$\text{A/Wb}$	—	—

Tab. 5: Course-wide notation and units

## Common pitfalls & misconceptions

- **Case matters:**  $\text{M}$  (mega,  $10^6$ ) vs.  $\text{m}$  (milli,  $10^{-3}$ );
- **Micro symbol:** use  $\mu$  (or  $u$  only when typing constraints exist);
- **usage of prefixes** never stack prefixes (no “ $\text{m}\mu\text{F}$ ”).
- **Mixed units:** keep SI consistently; avoid mixing  $\text{hours}$ / $\text{Wh}$  inside SI derivations.
- **Units vs. variables:** don't confuse  $W$  (work) with  $\text{W}$  (Watt = unit of power  $\text{P}$  = work per second).  
Don't confuse  $C$  (capacity = charge per voltage) with  $\text{C}$  (Coulomb = unit of charge  $\text{Q}$ ).
- **Units vs. prefixes:** don't confuse  $\text{mN}$  (Millinewton) with  $\text{Nm}$  (Newton meter).
- **Normalized vs. quantity equations:** dimensionless ratios should cancel units; if not, something's wrong.

## Exercises

### Quick checks

#### Exercise E1.1 Unit check (quantity equation)

Show that  $P=U \cdot I$  has unit watt. (Better to be calculated after reading Block02)

Result

1.  $[U]=\text{V}=\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3} \cdot \text{A}^{-1}$ ,  
 $[I]=\text{A}$ .
2.  $[P]=[U][I]=\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3}=\text{W}$ .

### Exercise E2.2 Work from lifting (quantity equation)

How much energy is needed to lift 100 kg for 2 meters?

Result

1.  $W = mgs$  with  $m = 100 \text{ kg}$ ,  $g = 9.81 \text{ m/s}^2$ ,  $s = 2 \text{ m}$
2.  $W = 100 \cdot 9.81 \cdot 2 = 1962 \text{ Nm}$

### Exercise E3.1 Conversion

Convert  $47 \text{ k}\Omega$  to  $\text{M}\Omega$  and  $\Omega$ .

Result

$$47 \text{ k}\Omega = 0.047 \text{ M}\Omega = 47,000 \Omega.$$

### Exercise E4.2 Dimension

Is  $\eta = \frac{P_{\text{O}}}{P_{\text{I}}}$  dimensionless?

Result

Yes. Units cancel ( $\text{W/W}$ ); normalized equation.

### Exercise E5.3 Conversion

Which is larger:  $5 \text{ mA}$  or  $4500 \text{ }\mu\text{A}$ ?

Result

$5 \text{ mA} = 5000 \text{ }\mu\text{A}$ , so  $5 \text{ mA}$  is larger.

### Exercise E6.4 Conversion

True/False:  $1 \text{ V} = 1 \text{ Nm/As}$ .

Result

True (from  $W = U \cdot Q$ ).

### Longer exercises

#### Exercise E7 Conversions: Battery

2. How many minutes could a battery with  $10 \text{ kWh}$  of stored energy provide a given power for the indicated time?

Reputation

$$\begin{aligned} t &= 200'000 \text{ min} \end{aligned}$$

There are additional losses:

$$\begin{aligned} W &= 10 \text{ kWh} &&= 10'000 \text{ Wh} \\ t &= \frac{W}{P} &&= \frac{10'000 \text{ Wh}}{5 \text{ W}} = 200'000 \text{ min} \end{aligned}$$

• The battery has an internal resistance. Depending on the current the battery provides, this leads to internal losses.

- The internal resistance of the battery depends on the state of charge (SoC) of the battery.
- The wires also add additional losses to the system.





### Exercise E11 Conversion: Energy Consumption

Convert the following values step by step:

Result

How much energy does an average household consume per day when consuming an average power of  $500\text{ W}$ ?

How many chocolate bars ( $2'000\text{ kJ}$  each) does this correspond to?

$22\text{ chocolate bars}$

Solution

$$\begin{aligned} W &= 500\text{ W} \cdot 24\text{ h} = 12000\text{ Wh} = \\ & 43'200'000\text{ Js} = 43'200\text{ kWs} \quad \&= 43'200\text{ kJ} \quad \text{\text{Or:}} \quad W \\ &= 0.5\text{ kW} \cdot 24\text{ h} = 12\text{ kWh} = 43'200\text{ kWs} \quad \&= \\ & 43'200\text{ kJ} \quad \text{\text{Or:}} \quad n_{\text{bars}} = \frac{43'200\text{ kJ}}{2'000\text{ kJ}} = \\ & 21.6\text{ chocolate bars} \end{aligned}$$

### Exercise E12 Conversions: Battery

2. How long can a battery with  $10\text{ kWh}$  supply a power of  $100\text{ W}$ ?

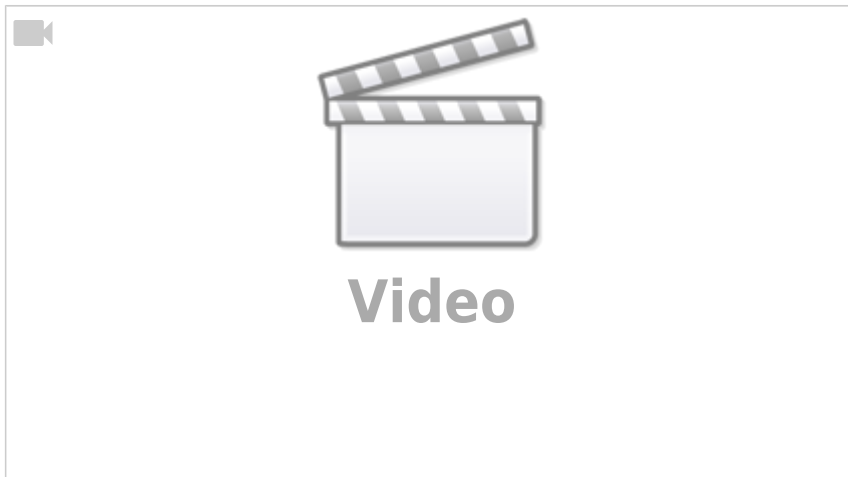
Result

$$t = 200'000\text{ min}$$

There are additional losses:



## Exercise E15 Conversions: Video on Prefixes



## Exercise E16 Conversion: Energy Consumption

Convert the following values step by step:

Result

How much energy does an average household consume per day when consuming an average power of  $500 \text{ W}$ ?

How many chocolate bars ( $2000 \text{ kJ}$  each) does this correspond to?

$22$  chocolate bars

Solution

$$\begin{aligned}
 W &= 500 \text{ W} \cdot 24 \text{ h} = 12000 \text{ Wh} = \\
 &= 43'200'000 \text{ Js} = 43'200 \text{ kWs} \quad \&= \quad 43'200 \text{ kJ} \quad \text{\text{Or:}} \quad W \\
 &= 0.5 \text{ kW} \cdot 24 \text{ h} = 12 \text{ kWh} = 43'200 \text{ kWs} \quad \&= \\
 &= 43'200 \text{ kJ} \quad \text{\text{Or:}} \quad n_{\text{bars}} = \frac{43'200 \text{ kJ}}{2'000 \text{ kJ}} = \\
 &= 21.6 \text{ chocolate bars}
 \end{aligned}$$



Start drawing by  
clicking here

## Exercise E1 Machine-Vision Strobe Unit: Charging and Safe Discharge of a Flash Capacitor

A machine-vision inspection system on a production line uses a short high-voltage flash pulse. For this purpose, an energy-storage capacitor is charged from a DC source and must be safely discharged before maintenance.

Data:  $C = 1 \mu\text{F}$   $W_e = 0.1 \text{ J}$   $I_{\text{max}} = 100 \text{ mA}$   $R_i = 10 \text{ M}\Omega$

1. What voltage must the capacitor have so that it stores the required energy?

#### SolutionResult

$$\begin{aligned} W_e &= \frac{1}{2} C U^2 \\ U &= \sqrt{\frac{2W_e}{C}} \\ &= \sqrt{\frac{2 \cdot 0.1 \text{ J}}{1 \cdot 10^{-6} \text{ F}}} \\ &= \sqrt{200000} \text{ V} \approx 447.2 \text{ V} \end{aligned}$$

$$\begin{aligned} U &= 447.2 \text{ V} \\ \end{aligned}$$

2. The charging current must not exceed  $100 \text{ mA}$  at the start of charging. What charging resistor is required?

#### SolutionResult

At the beginning of charging, the capacitor behaves like a short circuit, so  $i_{C(\text{max})} = i_C(t=0) = \frac{U}{R}$   
Thus,  $R \geq \frac{U}{I_{\text{max}}} = \frac{447.2 \text{ V}}{0.1 \text{ A}} \approx 4472 \Omega = 4.47 \text{ k}\Omega$

$$\begin{aligned} R &\geq 4.47 \text{ k}\Omega \\ \end{aligned}$$

3. How long does the charging process take until the capacitor is practically fully charged?

## SolutionResult

The time constant is 
$$T = RC = 4.47 \text{ k}\Omega \cdot 1 \text{ }\mu\text{F} = 4.47 \text{ ms}$$
 In engineering practice, a capacitor is considered practically fully charged after about  $5T$ :

$$t \approx 5T = 5 \cdot 4.47 \text{ ms} = 22.35 \text{ ms}$$

$$t \approx 22.35 \text{ ms}$$

4. Give the time-dependent capacitor voltage and the voltage across the charging resistor.

## SolutionResult

For the charging process:

$$u_C(t) = U \left(1 - e^{-t/T}\right) \quad u_R(t) = U e^{-t/T}$$

with  $U = 447.2 \text{ V}$  and  $T = 4.47 \text{ ms}$ . So the capacitor voltage rises exponentially from  $0$  to  $447.2 \text{ V}$ , while the resistor voltage falls exponentially from  $447.2 \text{ V}$  to  $0$ .

$$u_C(t) = 447.2 \left(1 - e^{-t/4.47 \text{ ms}}\right) \text{ V}$$

$$u_R(t) = 447.2 e^{-t/4.47 \text{ ms}} \text{ V}$$

5. After charging, the capacitor is disconnected from the source. Its leakage can be modeled by an internal resistance of  $10 \text{ M}\Omega$ . After what time has the stored energy dropped to one half, and what is the capacitor voltage then?

## SolutionResult

Half the energy means  $W_e' = 0.5W_e$ . Since  $W_e = \frac{1}{2}CU^2$  the voltage at half energy is  $U' = \frac{U}{\sqrt{2}} = \frac{447.2 \text{ V}}{\sqrt{2}} = 316.2 \text{ V}$ . For the discharge through the internal resistance:  $u_C(t) = Ue^{-t/T_2}$  with  $T_2 = R_iC = 10 \text{ ms} \cdot 1 \text{ MHz} = 10 \text{ s}$ . Set  $u_C(t) = U'$ :  $e^{-t/T_2} = \frac{U'}{U} \Leftrightarrow t = T_2 \ln\left(\frac{U}{U'}\right) \approx 10 \text{ s} \cdot \ln\left(\frac{447.2}{316.2}\right) \approx 3.47 \text{ s}$ .

$$U' = 316.2 \text{ V} \quad t = 3.47 \text{ s}$$

6. The fully charged capacitor is discharged through the charging resistor before maintenance. How long does the discharge take, and how much energy is converted into heat in the resistor?

### SolutionResult

The discharge time constant through the same resistor is again  $T = RC = 4.47 \text{ ms}$ . Thus the practical discharge time is  $t \approx 5T = 22.35 \text{ ms}$ . The complete stored capacitor energy is converted into heat in the resistor:  $W_R = W_e = 0.1 \text{ J}$ .

$$t \approx 22.35 \text{ ms} \quad W_R = 0.1 \text{ J}$$

## Exercise E2 Sensor Input Buffer: Source, T-Network and Capacitor

A 12 V industrial sensor electronics unit feeds a buffered measurement node through a resistor T-network. A capacitor smooths the node voltage. At first, the load is disconnected. After the capacitor is fully charged, a measurement load is connected by a switch.

Data: 
$$\begin{aligned} U &= 12 \text{ V} \\ R_1 &= 2 \text{ k}\Omega \\ R_2 &= 10 \text{ k}\Omega \\ R_3 &= 3.33 \text{ k}\Omega \\ C &= 2 \text{ }\mu\text{F} \\ R_L &= 5 \text{ k}\Omega \end{aligned}$$

Initially, the capacitor is uncharged and the switch is open.

1. What is the capacitor voltage after it is fully charged?

### SolutionResult

Using the equivalent voltage source of the network on the left-hand side, the open-circuit voltage is

$$\begin{aligned} U_{0e} &= \frac{R_2}{R_1 + R_2} U \\ &= \frac{10 \text{ k}\Omega}{2 \text{ k}\Omega + 10 \text{ k}\Omega} \cdot 12 \text{ V} \\ &= 10 \text{ V} \end{aligned}$$

After full charging, the capacitor voltage equals this voltage.

$$\begin{aligned} U_C = U_{0e} &= 10 \text{ V} \end{aligned}$$

2. How long does the charging process take?

### SolutionResult

$$\begin{aligned} R_{ie} &= 5.00 \text{ }\mu\text{s} \end{aligned}$$

The internal resistance seen by the capacitor is  $R_{ie} = R_3 + (R_1 \parallel R_2) = 3.33 \text{ k}\Omega + \frac{2 \text{ k}\Omega}{2 \text{ k}\Omega + 10 \text{ k}\Omega} = 3.33 \text{ k}\Omega + 1.67 \text{ k}\Omega = 5.00 \text{ k}\Omega$ . So the time constant is  $T = R_{ie}C = 5.00 \text{ k}\Omega \cdot 2 \text{ }\mu\text{F} = 10 \text{ ms}$ . Practical charging time:  $t \approx 5T = 50 \text{ ms}$ .

$$t \approx 50 \text{ ms}$$

3. Give the time-dependent capacitor voltage.

#### SolutionResult

The charging law is  $u_C(t) = U_{0e} \left(1 - e^{-t/T}\right) = 10 \left(1 - e^{-t/10 \text{ ms}}\right) \text{ V}$ . So the capacitor voltage rises exponentially from  $0 \text{ V}$  to  $10 \text{ V}$ .

$$u_C(t) = 10 \left(1 - e^{-t/10 \text{ ms}}\right) \text{ V}$$

4. After the capacitor is fully charged, the switch is closed and the load resistor is connected. What is the stationary load voltage?

#### SolutionResult

Now use a second equivalent voltage-source step. The Thevenin source seen by the load has 
$$U_{0e} = 10 \text{ V} \parallel R_{ie} = 5.00 \text{ k}\Omega$$
 Thus, the stationary load voltage is 
$$U_{C'} = U_{0e}' = \frac{R_L}{R_{ie} + R_L} U_{0e} = \frac{5 \text{ k}\Omega}{5 \text{ k}\Omega + 5 \text{ k}\Omega} \cdot 10 \text{ V} = 5 \text{ V}$$

$$\begin{aligned} U_L &= 5 \text{ V} \\ \end{aligned}$$

5. How long does it take until this new stationary state is practically reached?

#### SolutionResult

The new internal resistance is 
$$R_{ie}' = R_{ie} \parallel R_L = 5.00 \text{ k}\Omega \parallel 5.00 \text{ k}\Omega = 2.50 \text{ k}\Omega$$
 Hence the new time constant is 
$$T' = R_{ie}' C = 2.50 \text{ k}\Omega \cdot 2 \text{ }\mu\text{F} = 5 \text{ ms}$$
 Practical settling time: 
$$t \approx 5T' = 25 \text{ ms}$$

$$\begin{aligned} R_{ie}' &= 2.50 \text{ k}\Omega \\ t &\approx 25 \text{ ms} \\ \end{aligned}$$

6. Give the time-dependent load voltage after the switch is closed.

#### SolutionResult

At the switching instant, the capacitor voltage cannot jump. Therefore:

$$\begin{aligned} u_L(0^+) &= 10 \text{ V} \\ u_L(\infty) &= 5 \text{ V} \end{aligned}$$
 The voltage therefore decays exponentially toward the new final value:
 
$$\begin{aligned} u_L(t) &= u_L(\infty) + (u_L(0^+) - u_L(\infty))e^{-t/T} \\ &= 5 + 5e^{-t/5 \text{ ms}} \text{ V} \end{aligned}$$

$$\begin{aligned} u_L(t) &= 5 + 5e^{-t/5 \text{ ms}} \text{ V} \end{aligned}$$

### Exercise E3 Hall-Sensor Calibration Coil: Short Air-Core Coil

A Hall-sensor calibration bench uses a short air-core coil to create a defined magnetic field. An air-core coil is chosen because it avoids hysteresis and remanence effects. The coil is wound as a short cylindrical coil.

Data:
 
$$\begin{aligned} l &= 22 \text{ mm} \\ d &= 20 \text{ mm} \\ d_{\text{Cu}} &= 0.8 \text{ mm} \\ N &= 25 \\ \rho_{\text{Cu}, 20^\circ \text{C}} &= 0.0178 \text{ } \Omega \cdot \text{mm}^2/\text{m} \end{aligned}$$

A DC current of  $1 \text{ A}$  shall flow through the coil.

1. Calculate the coil resistance  $R$  at room temperature.

#### SolutionResult

The wire cross section is
 
$$\begin{aligned} A_{\text{Cu}} &= \pi \left( \frac{d_{\text{Cu}}}{2} \right)^2 = \pi (0.4 \text{ mm})^2 \\ &= 0.503 \text{ mm}^2 \end{aligned}$$
 The total wire length is approximated by the number of turns times the circumference:
 
$$l_{\text{Cu}} = N \pi d$$

$$R = 55.6 \text{ m}\Omega$$

```

&= 25\pi \cdot 20~{\rm mm} \\ &=
1570.8~{\rm mm} = 1.571~{\rm m}
\end{align*} Thus, \begin{align*} R
&= \rho_{\rm Cu} \frac{l_{\rm
Cu}}{A_{\rm Cu}} \\ &=
0.0178~{\rm
\Omega, mm^2/m} \cdot \frac{1.571~
{\rm m}}{0.503~{\rm mm^2}} \\
&\approx 0.0556~{\rm \Omega}
\end{align*}

```

2. Calculate the coil inductance  $L$ .

### SolutionResult

For this short air-core coil, use

```

\begin{align*} L = N^2 \cdot
\frac{\mu_0
A}{l} \cdot \frac{1}{1 + \frac{d}{2l}}
\end{align*} with \begin{align*} A
&= \pi \left( \frac{d}{2} \right)^2 =
\pi (10~{\rm mm})^2 = 314.16~{\rm
mm^2} = 3.1416 \cdot
10^{-4}~{\rm m^2} \\ \mu_0 &=
4\pi \cdot 10^{-7}~{\rm Vs/(Am)}
\end{align*} Therefore,
\begin{align*} L &= 25^2 \cdot
\frac{4\pi \cdot 10^{-7}}{Vs/(Am)} \cdot
3.1416 \cdot
10^{-4}~{\rm m^2} \cdot \frac{1}{1 +
\frac{20}{2 \cdot 22}} \\
&\approx 7.71 \cdot 10^{-6}~{\rm H}
\end{align*}

```

```

\begin{align*} L = 7.71~{\rm \mu H}
\end{align*}

```

3. Which DC voltage must be applied so that the stationary current becomes  $I=1~\text{A}$ ? How large is the current density  $j$  in the copper wire?

## SolutionResult

In the stationary DC state, the coil behaves like its ohmic resistance:

$$\begin{aligned} U &= RI \\ &= 55.6 \text{ m}\Omega \cdot 1 \text{ A} \\ &= 55.6 \text{ mV} \end{aligned}$$

The current density is

$$j = \frac{I}{A_{\text{Cu}}} = \frac{1 \text{ A}}{0.503 \text{ mm}^2} \approx 1.99 \text{ A/mm}^2$$

$$\begin{aligned} U &= 55.6 \text{ mV} \\ j &= 1.99 \text{ A/mm}^2 \end{aligned}$$

4. How much magnetic energy is stored in the coil in the stationary state?

## SolutionResult

$$\begin{aligned} W_m &= \frac{1}{2} LI^2 \\ &= \frac{1}{2} \cdot 7.71 \cdot 10^{-6} \text{ H} \cdot (1 \text{ A})^2 \\ &= 3.86 \cdot 10^{-6} \text{ Ws} \end{aligned}$$

$$W_m = 3.86 \cdot 10^{-6} \text{ Ws}$$

5. Give the time-dependent coil current  $i(t)$  when the coil is switched on.

## SolutionResult

A coil current cannot jump instantly. It starts at  $0$  and approaches the final

$$i(t) = I \left( 1 - e^{-t/T} \right)$$

value  $I=1\text{~}\text{A}$  exponentially:  

$$i(t) = I \left(1 - e^{-t/T}\right)$$
 So the sketch starts at  $0\text{~}\text{A}$ , rises quickly, and then slowly approaches  $1\text{~}\text{A}$ .

6. How long does it take until the current has practically reached its stationary value?

#### SolutionResult

The time constant is  $T = \frac{L}{R} = \frac{7.71\text{~}\text{mH}}{55.6\text{~}\text{m}\Omega} \approx 138.9\text{~}\mu\text{s}$   
 A practical final value is reached after about  $5T$ :  
 $t \approx 5T = 5 \cdot 138.9\text{~}\mu\text{s} \approx 695\text{~}\mu\text{s}$

$$t \approx 695\text{~}\mu\text{s}$$

7. How much energy is dissipated as heat in the coil resistance during the current build-up?

#### SolutionResult

Using the current from task 5,  

$$i(t) = I \left(1 - e^{-t/T}\right)$$
 the heat dissipated in the winding resistance up to the practical final time  $5T$  is  

$$W_R = \int_0^{5T} R i^2(t) dt = R I^2 \int_0^{5T} \left(1 - e^{-t/T}\right)^2 dt$$

$$W_R \approx 27.05 \cdot 10^{-6}\text{~}\text{Ws}$$

$$\int_0^T (1 - e^{-t/T})^2 dt \approx \frac{7}{2}T$$
 Thus,
 
$$W_R \approx RI^2 \cdot \frac{7}{2}T \approx 0.0556 \cdot \Omega \cdot (1 - A)^2 \cdot \frac{7}{2} \cdot 138.9 \mu s \approx 27.05 \cdot 10^{-6} \text{Ws}$$

**Exercise E20 Conversion: Energy, Power and Area**

1. The car has a usable battery capacity of 100 kWh. Solar panels produces per 1 m<sup>2</sup> in average in December 0.2 kWh/m<sup>2</sup>. The car is driven 50 km per day. The size of a distinct solar module with 460 Wp (Watt peak) is 1.9 m times 1.1 m.

Results:
 
$$A = \frac{100 \text{ kWh}}{0.2 \text{ kWh/m}^2} = 500 \text{ m}^2$$
 .. What is the average power consumption of the car per day?
 
$$P = \frac{100 \text{ kWh}}{24 \text{ h}} = 4.17 \text{ kW}$$
 Solution:
 
$$\frac{P}{A} = \frac{4.17 \text{ kW}}{500 \text{ m}^2} = 0.00834 \text{ kW/m}^2$$

$$\frac{W}{l} = \frac{16 \text{ kWh}}{100 \text{ km}} = 0.16 \text{ kWh/km}$$

$$W = 50 \text{ km} \cdot 0.16 \text{ kWh/km} = 8 \text{ kWh}$$

**Exercise E21 Conversion: Energy, Power and Area**

1. The car has a usable battery capacity of 100 kWh. Solar panels produces per 1 m<sup>2</sup> in average in December 0.2 kWh/m<sup>2</sup>. The car is driven 50 km per day. The size of a distinct solar module with 460 Wp (Watt peak) is 1.9 m times 1.1 m.

.. What is the average power consumption of the car per day?

$$\begin{aligned} \overline{P} &= \frac{1}{T} \int_0^T P(t) dt = \frac{1}{T} \int_0^T \frac{dW}{dt} dt = \frac{1}{T} \Delta W \\ \overline{P} &= \frac{1}{24 \text{ h}} \int_0^{24 \text{ h}} P(t) dt = \frac{1}{24 \text{ h}} \int_0^{24 \text{ h}} \frac{dW}{dt} dt = \frac{1}{24 \text{ h}} \Delta W \\ \overline{P} &= \frac{1}{24 \text{ h}} \int_0^{24 \text{ h}} P(t) dt = \frac{1}{24 \text{ h}} \int_0^{24 \text{ h}} \frac{dW}{dt} dt = \frac{1}{24 \text{ h}} \Delta W \\ \overline{P} &= \frac{1}{24 \text{ h}} \int_0^{24 \text{ h}} P(t) dt = \frac{1}{24 \text{ h}} \int_0^{24 \text{ h}} \frac{dW}{dt} dt = \frac{1}{24 \text{ h}} \Delta W \end{aligned}$$

$$\begin{aligned} \overline{P} &= \frac{1}{T} \int_0^T P(t) dt = \frac{1}{T} \int_0^T \frac{dW}{dt} dt = \frac{1}{T} \Delta W \\ \overline{P} &= \frac{1}{24 \text{ h}} \int_0^{24 \text{ h}} P(t) dt = \frac{1}{24 \text{ h}} \int_0^{24 \text{ h}} \frac{dW}{dt} dt = \frac{1}{24 \text{ h}} \Delta W \\ \overline{P} &= \frac{1}{24 \text{ h}} \int_0^{24 \text{ h}} P(t) dt = \frac{1}{24 \text{ h}} \int_0^{24 \text{ h}} \frac{dW}{dt} dt = \frac{1}{24 \text{ h}} \Delta W \end{aligned}$$

## Learning Objectives

After this 90-minute block, you

- know the time constant  $\tau$  and in particular calculate it.
- determine the time characteristic of the currents and voltages at the RC element for a given resistance and capacitance.
- know the continuity conditions of electrical quantities.
- know when (=according to which measure) the capacitor is considered to be fully charged/discharged, i.e. a steady state can be considered to have been reached.
- can calculate the energy content in a capacitor.
- can calculate the change in energy of a capacitor resulting from a change in voltage between the capacitor terminals.
- can calculate (initial) current, (final) voltage, and charge when balancing the charge of several capacitors (also via resistors).

## Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- ...

## 90-minute plan

1. Warm-up (x min):
  1. ....
2. Core concepts & derivations (x min):

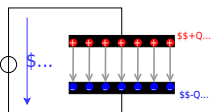
1. ...
3. Practice (x min): ...
4. Wrap-up (x min): Summary box; common pitfalls checklist.

## Conceptual overview

1. ...

## Core content

Fig. 1: Capacitor in electrical circuit



Here we will shortly introduce the basic idea behind a capacitor. A more detailed analysis will follow in electrical engineering II.

A capacitor consists of two insulated conductors (electrodes) separated by an insulator (cf. [figure 1](#)).

The electrodes serve as “charge carrier storage”. This is done in the following manner:

1. An external source draws charge carriers from one of the electrodes and carries them to the other electrode.
2. If the external source is a voltage source with the voltage  $U$ , a stationary state is reached after a certain time.  
In this state there is a fixed number of  $+Q$  on the positive electrode and  $-Q$  on the



\$...      \$...

In the following, the charging process of a capacitor is to be considered in more detail. For this purpose, one has to realize, that during the charging of the capacitor, besides the voltage source  $U_{\text{s}}$  and the capacitor  $C$ , there is always a resistance  $R$  in the circuit. This is composed of the internal resistance of the (non-ideal) voltage source, the internal resistance of the capacitor, and the parasitic (=interfering) resistance of the line. In practical applications, it is often desired that capacitors charge in a certain time range. For this purpose, another real resistor is inserted into the circuit. The resulting series of resistors and capacitors is called an **RC element**. It resembles a voltage divider in which a resistor has been replaced by a capacitor.

To start the charging, an (ideal) switch  $S$  is inserted. The circuit to be considered then looks like shown in [figure 2](#).

An ideal switch is characterized by:

- infinitely fast switching
- resistance of  $0 \sim \Omega$  in the closed state ("short circuit")
- resistance  $\rightarrow \infty$  in the open state ("open line")
- no capacitive effect

In this chapter also time-varying quantities are considered. These are generally marked with lowercase letters. Examples of time-varying quantities are:

- A **time-varying voltage  $u_C(t)$  across a capacitor** or the **voltage  $U_{\text{rms}}$  of an ac voltage source** as opposed to a constant voltage  $U_{\text{rms}}$  across a constant voltage source.
- A **time-varying current  $i_L(t)$  across a coil** or **time-varying current  $i_C(t)$  across a capacitor**.

Since the time dependence is already clear from the lowercase letter, these quantities are occasionally not indicated by the trailing  $(t)$ . So it is  $u = u(t)$ .

## Time Course of the Charging and Discharging Process

In the simulation below you can see the circuit mentioned above in a slightly modified form:

- The capacitance  $C$  can be charged via the resistor  $R$  if the toggle switch  $S$  connects the DC voltage source  $U_{\text{rms}}$  to the two.
- But it is also possible to short-circuit the series circuit of  $R$  and  $C$  via the switch  $S$ .
- Furthermore, the current  $i_C$  and the voltage  $u_C$  are displayed in the oscilloscope as data points over time and in the circuit as numerical values.
- Additionally it is possible to change the capacitance value  $C$  and resistance value  $R$  with the sliders Capacitance  $C$  and Resistance  $R$ .

Exercises:

1. Become familiar with how the capacitor current  $i_C$  and capacitor voltage  $u_C$  depend on the given capacitance  $C$  and resistance  $R$ .  
To do this, use for  $R = \{ 10 \sim \Omega, 100 \sim \Omega, 1 \sim k\Omega \}$  and  $C = \{ 1 \sim \mu\text{F}, 10 \sim \mu\text{F} \}$ . How fast does the capacitor voltage  $u_C$  increase in each case  $n$ ?
2. Which quantity ( $i_C$  or  $u_C$ ) is continuous here? Why must this one be continuous? Why must the other quantity be discontinuous?

In the following, this circuit is divided into two separate circuits, which consider only charging and only discharging.

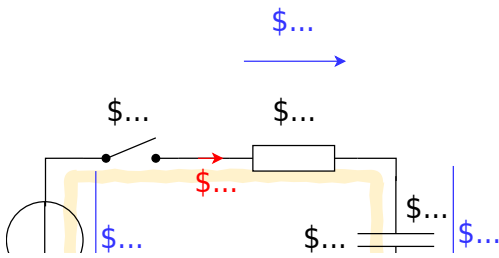
To understand the charging process of a capacitor, an initially uncharged capacitor with capacitance  $C$  is to be charged by a DC voltage source  $U_{\text{rms}}$  via a resistor  $R$ .

- In order that the voltage  $U_{\text{rms}}$  acts at a certain time  $t_0 = 0 \sim s$  the switch  $S$  is closed at this time.
- Directly after the time  $t_0$  the maximum current (“charging current”) flows in the circuit. This is only limited by the resistor  $R$ . The uncharged capacitor has a voltage  $u_C(t_0) = 0 \sim V$  at that time. The maximum voltage  $u_R(t_0) = U_{\text{rms}}$  is applied to the resistor. The current is  $i_C(t_0) = \frac{U_{\text{rms}}}{R}$ .
- The current causes charge carriers to flow from one electrode to the other. Thus the capacitor is

charged and its voltage increases  $u_C$ .

- Thus the voltage  $u_R$  across the resistor is reduced and so is the current  $i_R$ .
- With the current thus reduced, less charge flows on the capacitor.
- Ideally, the capacitor is not fully charged to the specified voltage  $U_{\text{rms}}$  until  $t \rightarrow \infty$ . It then carries the charge:  $q(t \rightarrow \infty) = Q = C \cdot U_{\text{rms}}$

Fig. 2: circuit for viewing the charge curve



The process is now to be summarized in detail in formulas. Linear components are used in the circuit, i.e. the component values for the resistor  $R$  and the capacitance  $C$  are independent of the current or the voltage. Then definition equations for the resistor  $R$  and the capacitance  $C$  are also valid for time-varying or infinitesimal quantities:

$$\begin{aligned} R &= \frac{u_R(t)}{i_R(t)} = \frac{\mathrm{d}u_R}{\mathrm{d}i_R} = \text{const.} \\ C &= \frac{q(t)}{u_C(t)} = \frac{\mathrm{d}q}{\mathrm{d}u_C} = \text{const.} \end{aligned} \quad \text{\tag{5.1.1}}$$

## Charging a capacitor at time $t=0$

By considering the loop, the general result is: the voltage of the source is equal to the sum of the two voltages across the resistor and capacitor.

$$U_{\text{rms}} = u_R + u_C = R \cdot i_C + u_C \quad \text{\tag{5.1.2}}$$

At the first instant  $\mathrm{d}t$ , an infinitesimally small charge “chunk”  $\mathrm{d}q$  flows through the circuit driven by the current  $i_C$  from the voltage source. For this, (5.1.1) gives:

$$i_C = \frac{\mathrm{d}q}{\mathrm{d}t} \quad \text{\textit{and}} \quad \mathrm{d}q = C \cdot \mathrm{d}u_C$$

The charging current  $i_C$  can be determined from the two formulas:

$$i_C = C \cdot \frac{du_C}{dt} \quad \text{\tag{5.1.3}}$$

Thus (5.1.2) becomes:

$$U_{\text{rs}} = u_R + u_C \quad \&= R \cdot C \cdot \frac{du_C}{dt} + u_C$$

here follows some mathematics:

This result represents a 1st order differential equation. This should generally be rewritten so that the part that depends (on the variable) is on one side and the rest is on the other. This is already present here. The appropriate approach to such a problem is:

$$u_C(t) = \mathcal{A} \cdot e^{\mathcal{B} \cdot t} + \mathcal{C}$$

$$\begin{aligned} U_{\text{rs}} &= R \cdot C \cdot \frac{d}{dt}(\mathcal{A} \cdot e^{\mathcal{B} \cdot t} + \mathcal{C}) + \mathcal{A} \cdot e^{\mathcal{B} \cdot t} + \mathcal{C} \\ &= R \cdot C \cdot \mathcal{A} \mathcal{B} \cdot e^{\mathcal{B} \cdot t} + \mathcal{A} \cdot e^{\mathcal{B} \cdot t} + \mathcal{C} \\ &= (R \cdot C \cdot \mathcal{A} \mathcal{B} + \mathcal{A}) \cdot e^{\mathcal{B} \cdot t} + \mathcal{C} \end{aligned}$$

This equation must hold for every  $t$ . This is only possible if the left, as well as the right term, become equal to 0.

Thus:

$$\begin{aligned} \mathcal{C} &= U_{\text{rs}} - R \cdot C \cdot \mathcal{A} \mathcal{B} + \mathcal{A} = 0 \\ \mathcal{A} &= -\frac{U_{\text{rs}}}{R \cdot C \cdot \mathcal{B} + 1} \end{aligned}$$

So it follows:

$$u_C(t) = \mathcal{A} \cdot e^{\left(-\frac{t}{RC}\right)} + U_{\text{rs}}$$

For the solution it must still hold that at time  $t_0=0$   $u_C(t_0) = 0$  just holds:

$$0 = \mathcal{A} \cdot e^{\left(-\frac{0}{RC}\right)} + U_{\text{rs}} \quad \&= \mathcal{A} + U_{\text{rs}} \\ \mathcal{A} &= -U_{\text{rs}}$$

So the solution is:

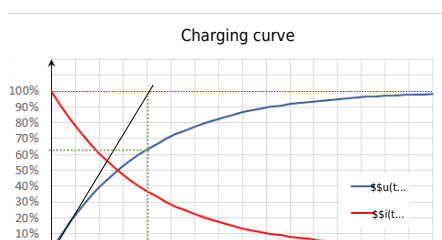
$$u_C(t) = -U_{\text{rs}} \cdot e^{\left(-\frac{t}{RC}\right)} + U_{\text{rs}}$$

And this results in: 
$$u_C(t) = U_{\text{rs}} \cdot (1 - e^{\left(-\frac{t}{RC}\right)})$$

And with (5.1.3),  $i_C$  becomes: 
$$i_C(t) = \frac{U_{\text{rs}}}{R} \cdot e^{\left(-\frac{t}{RC}\right)}$$

In [figure 4](#), the two time course diagrams for the charging voltage  $u_C(t)$  and the charging current  $i_C(t)$  of the capacitor are shown.

Fig. 4: charging curve

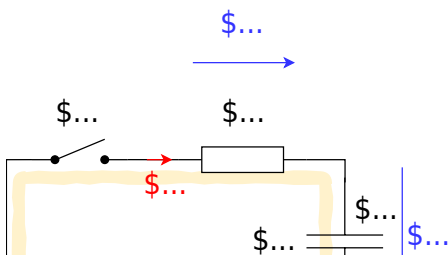


### Notice:

- There must be a unitless term in the exponent. So  $RC$  must also represent a time. This time is called **time constant**  $\tau = R \cdot C$ .
- At time  $t = \tau$ , we get:  $u_C(t) = U_{\text{rms}} \cdot (1 - \frac{1}{e}) = U_{\text{rms}} \cdot (1 - \frac{1}{2.718}) = U_{\text{rms}} \cdot \frac{1.718}{2.718} = 0.63 \cdot U_{\text{rms}} = 63\% \cdot U_{\text{rms}}$ .  
So, **the capacitor is charged to 63% after one  $\tau$ .**
- At time  $t = 2 \cdot \tau$  we get:  $u_C(t) = U_{\text{rms}} \cdot (1 - \frac{1}{e^2}) = 86\% \cdot U_{\text{rms}} = (63\% + (100\% - 63\%) \cdot 63\%) \cdot U_{\text{rms}}$ . So, **after each additional  $\tau$ , the uncharged remainder (1-63%) is recharged to 63%.**
- After about  $t = 5 \cdot \tau$ , the result is a capacitor charged to over 99%. In real circuits, **a charged capacitor can be assumed after  $5 \cdot \tau$ .**
- The time constant  $\tau$  can be determined graphically in several ways:
  - Plotting the voltage value corresponding to 63% on the y-axis. Finding the point of intersection with the graph. Reading the time (see green lines in [figure 4](#)).
  - Plotting the tangent to the (voltage) charge curve at the time of the discharged capacitor. This intersects a horizontal line at the level of the charging voltage at the point  $t = \tau$  (see black and light blue lines in [figure 4](#)).

## Discharging a capacitor at time $t=0$

Fig. 5: circuit for viewing discharge curve



The following situation is considered for the discharge:

- A capacitor charged to voltage  $U_{\text{rms}}$  with capacitance  $C$  is short-circuited across a resistor  $R$  at time  $t=t_0$ .
- As a result, the full voltage  $U_{\text{rms}}$  is initially applied to the resistor:  $u_R(t_0)=U_{\text{rms}}$
- The initial discharge current is thus defined by the resistance:  $i_C = \frac{u_R}{R}$
- The discharging charges lower the voltage of the capacitor  $u_C$ , since:  $u_C = \frac{q(t)}{C}$
- Ideally, the capacitor is not fully discharged before  $t \rightarrow \infty$ .

Also, this process now is to put into a formula in detail. By looking at the loop, the general result is: the sum of the two voltages across the resistor and capacitor adds up to zero.

$$\begin{aligned} 0 &= u_R + u_C = R \cdot i_C + u_C \end{aligned}$$

This gives (5.1.3):

$$\begin{aligned} 0 &= u_R + u_C = R \cdot C \cdot \frac{du_C}{dt} + u_C \end{aligned}$$

also here uses some mathematics:

This result again represents a 1st order differential equation. The appropriate approach to such a problem is:

$$\begin{aligned} u_C(t) &= \mathcal{A} \cdot \{\mathrm{e}^{\mathcal{B} \cdot t}\} + \mathcal{C} \\ \end{aligned}$$

$$\begin{aligned} 0 &= R \cdot C \cdot \frac{d}{dt}(\mathcal{A} \cdot \{\mathrm{e}^{\mathcal{B} \cdot t}\} + \mathcal{C}) + \mathcal{A} \cdot \{\mathrm{e}^{\mathcal{B} \cdot t}\} + \mathcal{C} \\ &= R \cdot C \cdot \mathcal{A} \mathcal{B} \cdot \{\mathrm{e}^{\mathcal{B} \cdot t}\} + \mathcal{A} \cdot \{\mathrm{e}^{\mathcal{B} \cdot t}\} + \mathcal{C} \\ &= (R \cdot C \cdot \mathcal{A} \mathcal{B} + \mathcal{A}) \cdot \{\mathrm{e}^{\mathcal{B} \cdot t}\} - \mathcal{C} \end{aligned}$$

This equation must hold for every  $t$ . This is only possible if the left, as well as the right term, become equal to 0. Thus:

$$\begin{aligned} \mathcal{C} &= 0 \\ R \cdot C \cdot \mathcal{A} \mathcal{B} + \mathcal{A} &= 0 \quad | : \mathcal{A} \\ -1 &= -\frac{1}{RC} \end{aligned}$$

So it follows:

$$\begin{aligned} u_C(t) &= \mathcal{A} \cdot \{\mathrm{e}^{-\frac{t}{RC}}\} + 0 \\ \end{aligned}$$

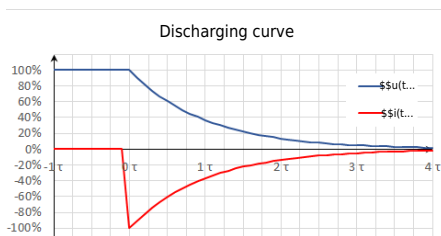
For the solution it must still hold that at time  $t_0=0$   $u_C(t_0) = U_{\mathrm{s}}$  just holds:

$$\begin{aligned} U_{\mathrm{s}} &= \mathcal{A} \cdot \{\mathrm{e}^{\{0\}}\} \\ U_{\mathrm{s}} &= \mathcal{A} \end{aligned}$$

Therefore, the result is:

$$u_C(t) = U_{\mathrm{s}} \cdot \{\mathrm{e}^{-\frac{t}{RC}}\}$$

Fig. 6: discharge curve



And this results in: 
$$u_C(t) = U_{\text{rms}} \cdot e^{-\frac{t}{\tau}}$$

$$\tau = R C$$

And with (5.1.3),  $i_C$  becomes: 
$$i_C(t) = -\frac{U_{\text{rms}}}{R} \cdot e^{-\frac{t}{RC}}$$

In figure 6 the two time course diagrams are again shown; now for the discharge voltage  $u_C(t)$  and the discharge current  $i_C(t)$  of the capacitor. Since the current now flows out of the capacitor, the sign of  $i_C$  is negative.

## Periodic switching operations

In the simulation on the right, a periodic switching operation can be seen. The capacitor is periodically charged and discharged via the switch. Three sliders are given in the simulation to change the resistance  $R$  (Resistance  $R$ ), the capacity  $C$  (Capacity  $C$ ), and the frequency  $f$  (Frequency  $f$ ). In the simulation below, the voltage  $u_C$  across the capacitor is shown in green and the current  $i_C$  is shown in yellow.

Exercises:

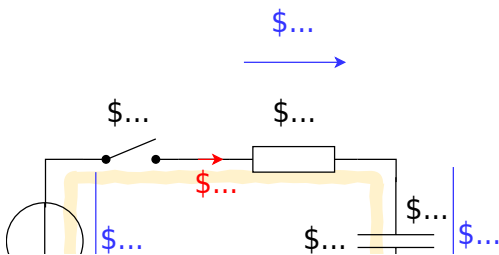
1. Increase the the frequency to  $f=10 \sim \{\text{rm kHz}\}$  using the appropriate slider. What is the change for  $u_C$  and  $i_C$ ?
2. Now increase the capacitance to  $C=10 \sim \{\text{rm }\mu\text{F}\}$  using the corresponding slider. What is the

change for  $u_C$  and  $i_C$ ?

- Now increase the resistance to  $R = 1 \text{ k}\Omega$  using the corresponding slider. What is the change for  $u_C$  and  $i_C$ ?

## Energy stored in a Capacitor

Fig. 2: circuit for viewing the charge curve



Now the capacitor as energy storage is to be looked at more closely. For this, we consider again the circuit in [figure 2](#) an. According to the chapter [Preparation, Properties, and Proportions](#), the power for constant values (DC) is defined as:

$$\begin{aligned} P &= \frac{\Delta W}{\Delta t} = U \cdot I \end{aligned}$$

For variable signals, the instantaneous power is given as:

$$\begin{aligned} p &= \frac{dw}{dt} = u \cdot i \end{aligned}$$

### Energy consideration of the capacitor

Charging the capacitor at time  $t_0 = 0$  results in  $\Delta W = \Delta W_C$  for the stored energy at a later time  $t_1 = t$ :

$$\begin{aligned} \Delta W_C &= \int_{t_0}^{t_1} dw = \int_0^t u \cdot i \cdot dt = \\ &= \int_0^t u_C \cdot i_C \cdot dt \end{aligned} \tag{5.2.1}$$

During the charging process 
$$u_C(t) = U_s \cdot (1 - e^{-\frac{t}{\tau}})$$
 
$$i_C(t) = \frac{U_s}{R} \cdot e^{-\frac{t}{\tau}}$$

$\tag{5.2.2}$

In particular:

$$\begin{aligned} C &= \frac{q(t)}{u_C(t)} \quad \& \rightarrow \quad \& q(t) \&= u_C(t) \cdot C \\ i_C(t) &= \frac{dq(t)}{dt} \quad \& \rightarrow \quad \& C = \text{const.} \quad \& i_C(t) \&= C \cdot \frac{du_C(t)}{dt} \end{aligned}$$

Thus, the stored energy from formula (5.2.1):

$$\begin{aligned} \Delta W_C &= \int_0^t u_C(t) \cdot C \cdot \frac{du_C(t)}{dt} dt \\ & \quad \& | \text{substitution of integration variable: } t \rightarrow u_C \quad \&= \\ & \int_{U_0}^{U_1} u_C(t) \cdot C \cdot du_C \quad \& | \text{Since the capacity is constant, it} \\ & \text{can be written ahead of the integral} \quad \&= C \cdot \int_{U_0}^{U_1} u_C \cdot du_C \\ & \left[ \frac{1}{2} u_C^2 \right]_{U_0}^{U_1} \quad \& \end{aligned} \quad \boxed{\Delta W_C = \frac{1}{2} C (U_1^2 - U_0^2)} \tag{5.2.3}$$

Thus, for a fully discharged capacitor ( $U_{\text{s}}=0 \sim V$ ), the energy stored when charging to voltage  $U_{\text{s}}$  is  $\Delta W_C = \frac{1}{2} C \cdot U_{\text{s}}^2$ .

## Energy Consideration on the Resistor

The converted energy can also be determined for the resistor:

$$\Delta W_R = \int_0^t u_R \cdot i_R dt = \int_0^t R \cdot i_R \cdot i_R dt = R \cdot \int_0^t i_R^2 dt$$

Since the current through the capacitor  $i_C$  is equal to that through the resistor  $i_R$ , it follows via (5.2.2):

$$\begin{aligned} \Delta W_R &= R \cdot \int_0^t \left( \frac{U_{\text{s}}}{R} \cdot e^{-\frac{t}{\tau}} \right)^2 dt \\ &= \frac{U_{\text{s}}^2}{R} \cdot \int_0^t e^{-\frac{2t}{\tau}} dt \\ &= \frac{U_{\text{s}}^2}{R} \cdot \left[ -\frac{\tau}{2} e^{-\frac{2t}{\tau}} \right]_0^t \\ &= -\frac{1}{2} \cdot U_{\text{s}}^2 \cdot C \cdot \left[ e^{-\frac{2t}{\tau}} - 1 \right] \end{aligned}$$

For  $t \rightarrow \infty$  we get:

$$\begin{aligned} \Delta W_R &= -\frac{1}{2} \cdot U_{\text{s}}^2 \cdot C \cdot \left[ e^{-\frac{2t}{\tau}} - 1 \right]_{t=0}^{t=\infty} \\ &= -\frac{1}{2} \cdot U_{\text{s}}^2 \cdot C \cdot \left[ 0 - 1 \right] \end{aligned} \quad \boxed{\Delta W_R = \frac{1}{2} C U_{\text{s}}^2} \tag{5.2.4}$$

This means that the energy converted at the resistor is independent of the resistance value (for an ideal constant voltage source  $U_{\text{s}}$  and given capacitor  $C$ )! At first, this doesn't really sound comprehensible. No matter if there is a very large resistor  $R_1$  or a tiny small resistor  $R_2$ : The same waste heat is always produced. Graphically, this apparent contradiction can be resolved like this: A higher resistor  $R_2$  slows down the small charge packets  $\Delta q_1$ ,  $\Delta q_2$ , ...  $\Delta q_n$  more strongly. But a considered single charge packet  $\Delta q_k$  will nevertheless pass the same voltage across the resistor  $R_1$  or  $R_2$  since this is given only by the accumulated packets in the capacitor:  $u_r = U_{\text{s}} - u_C = U_{\text{s}} - \frac{q}{C}$ .

In real applications, as mentioned in previous chapters, ideal voltage sources are not possible. Thus, without a real resistor, the waste heat is dissipated proportionally to the internal resistance of the source and the internal resistance of the capacitor. The internal resistance of the capacitor depends on the frequency but is usually smaller than the internal resistance of the source.

### Consideration of total energy turnover

In the previous considerations, the energy conversion during the complete charging process was also considered. It was found that the capacitor stores the energy  $W_C = \frac{1}{2} U_{\text{rms}}^2 \cdot C$  (see (5.2.3)) and at the resistor the energy  $W_R = \frac{1}{2} U_{\text{rms}}^2 \cdot C$  (see (5.2.4)) into heat. So, in total, the voltage source injects the following energy:

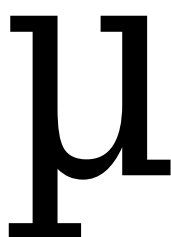
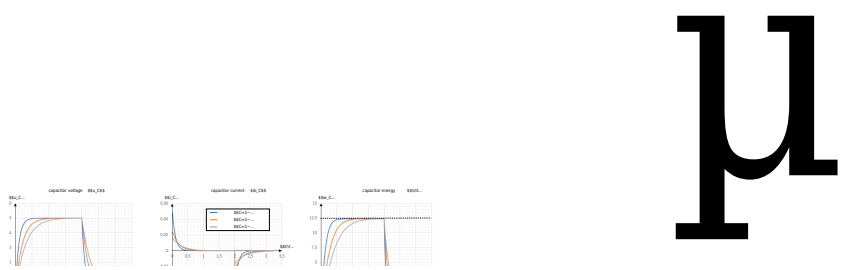
$$\Delta W_0 = \Delta W_R + \Delta W_C = U_{\text{rms}}^2 \cdot C$$

This also follows via (5.2.1):

$$\begin{aligned} \Delta W_0 &= \int_0^{\infty} u_0 \cdot i_0 \cdot dt \quad | \quad u_0 = U_{\text{rms}} \text{ \textit{is constant because constant voltage source!}} \\ &= U_{\text{rms}} \cdot \int_0^{\infty} i_C \cdot dt \\ &= U_{\text{rms}} \cdot \int_0^{\infty} \frac{dq}{dt} \cdot dt \\ &= U_{\text{rms}} \cdot Q \\ &= U_{\text{rms}}^2 \cdot C \end{aligned} \quad | \quad \textit{where } Q = C \cdot U_{\text{rms}}$$

This means that only half of the energy emitted by the source is stored in the capacitor! Again, This doesn't really sound comprehensible at first. And again, it helps to look at small packets of charge that have to be transferred from the ideal source to the capacitor. [figure 8](#) shows current and voltage waveforms across the capacitor and the stored energy for different resistance values. There, too, it can be seen that the maximum stored energy (dashed line in the figure at right) is given by  $\Delta W = \frac{1}{2} U_{\text{rms}}^2 \cdot C$  alone.  $U_{\text{rms}}^2 \cdot C = \frac{1}{2} \cdot (5 \text{ V})^2 \cdot 1 \text{ } \mu\text{F} = 12.5 \text{ } \mu\text{Ws}$  is given.

Fig. 8: Current, voltage, and energy during charging and discharging



This can also be tested in the following simulation. In addition to the RC element shown so far, a power meter and an integrator are also drawn here. It is possible to display the instantaneous power and the stored energy. Via the slider Resistance R the resistance value can be varied. The following values are shown in the oscilloscopes:

- left: Current  $i_C$  and voltage  $u_C$  at the capacitor.
- middle: Instantaneous power  $p_C = u_C \cdot i_C$  of the capacitor.

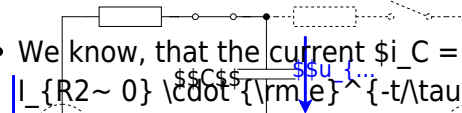


(1/5) This is also true for  $t_2$ , since between  $t_1$  and  $t_2$  the charge on  $C$  does not change:  $u_c(t_2) = 4 \text{ V}$ .

- In the first moment after closing  $S_2$  at  $t_2$ , the voltage drop on  $R_3 + R_2$  is:  $U_{R3+R2} = U_2 - u_c(t_2) = 6 \text{ V}$ .
- So the voltage divider of  $R_3 + R_2$  lead to  $u_{R2}(t_2 = 10 \text{ ms}) = \frac{R_2}{R_3 + R_2} \cdot U_{R3+R2} = \frac{2 \text{ k}\Omega}{3 \text{ k}\Omega + 2 \text{ k}\Omega} \cdot 6 \text{ V} = 2.4 \text{ V}$

We see that the voltage on  $R_2$  has to decrease from  $2.4 \text{ V}$  to  $1/10 \cdot U_2 = 1 \text{ V}$ .

To calculate this, there are multiple ways. In the following, one shall be retraced:



- We know, that the current  $i_C = i_{R2}$  subsides exponentially:  $i_{R2}(t) = I_{R2 \sim 0} \cdot e^{-t/\tau}$
- So we can rearrange the task to focus on the change in current instead of the voltage

We see that  $U_1 = u_C + u_{R2}$  and there is only one current in the loop:  $i = i_C = i_{R2}$ . The exponential decay is true regardless of where it starts.

The current is generally given with the exponential function:  $i_C = \frac{U_1}{R_1 + R_2} \cdot e^{-t/\tau}$ , with  $R$  given here as  $R = R_1 + R_2$ . Therefore  $u_{R2}$  can be written as:

$$u_{R2}(t_3) = \frac{R_2}{R_1 + R_2} \cdot U_1 \cdot e^{-\frac{t_3 - t_2}{\tau_2}} = \frac{R_2}{R_1 + R_2} \cdot U_1 \cdot e^{-\frac{t_3 - t_2}{\tau_2}}$$

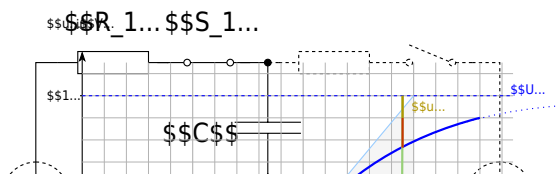
We see that for the time constant, we now need to use  $R = R_3 + R_2$ .

For the first task, the switch  $S_1$  gets closed at  $t_1 = 0 \text{ s}$ .

1.1 What is the value of the time constant  $\tau_1$ ?

Solution

The time constant  $\tau$  is generally given as:  $\tau = R \cdot C$ .  
 Now, we try to determine which  $R$  and  $C$  must be used here.  
 To find this out, we have to look at the circuit when  $S_1$  gets closed.



We see that for the time constant, we need to use  $R=R_1 + R_2$ .

### Exercise E1.3 Charge balance of two capacitors (educational exercise, not part of an exam)

In the simulation, you see the two capacitors  $C_1$  and  $C_2$  (The two small resistors with  $1 \sim \mu\Omega$  have to be there for the simulation to run). At the beginning,  $C_1$  is charged to  $10 \sim \text{V}$  and  $C_2$  to  $0 \sim \text{V}$ . With the switches  $S_1$  and  $S_2$  you can choose whether

1. the capacitances  $C_1$  and  $C_2$  are shorted, or
2. the capacitors  $C_1$  and  $C_2$  are connected via resistor  $R$ .

On the right side of the simulation, there are some additional “measuring devices” to calculate the stored potential energy from the voltages across the capacitors.

In the following, the charging and discharging of a capacitor are to be explained with this construction.

Under the electrical structure, the following quantities are shown over time:

Voltage $u_1(C_1)$ of the first capacitor	Voltage $u_2(C_2)$ of the second capacitor	Stored energy $w_1(C_1)$	Stored energy $w_2(C_2)$	Total energy $\sum w$
Initially charged to $10 \text{ V}$	Initially neutrally charged ( $0 \text{ V}$ )	Initially holds: $w_1(C_1) = \frac{1}{2} \cdot C \cdot U^2 = \frac{1}{2} \cdot 10 \text{ } \mu\text{F} \cdot (10 \text{ V})^2 = 500 \text{ } \mu\text{W}$ In the oscilloscope, equals $1 \text{ V}$ $\sim 1 \text{ W}$	Initially, $w_2(C_2) = 0$ , since the capacitor is not charged.	The total energy is $w_1 + w_2 = w_1$

The capacitor  $C_1$  has thus initially stored the full energy and via closing of the switch,  $S_2$  one would expect a balancing of the voltages and an equal distribution of the energy  $w_1 + w_2 = 500 \text{ } \mu\text{W}$ .

- Close the switch  $S_2$  (the toggle switch  $S_1$  should point to the switch  $S_2$ ).  
What do you find?
  - What do the voltages  $u_1$  and  $u_2$  do?
  - What are the energies and the total energy?  
How is this understandable with the previous total energy?
- Open  $S_2$  - the changeover switch  $S_1$  should not be changed. What do you find?
  - What do the voltages  $u_1$  and  $u_2$  do?
  - What are the energies and the total energy?  
How is this understandable with the previous total energy?
- Repeat 1. and 2. several times. Can anything be deduced regarding the distribution of energy?
- Change the switch  $S_2$  to the resistor. What do you observe?
  - What do the voltages  $u_1$  and  $u_2$  do?
  - What are the energies and the total energy?  
How is this understandable with the previous total energy?

### Exercise E1.4 Machine-Vision Strobe Unit: Charging and Safe Discharge of a Flash Capacitor

A machine-vision inspection system on a production line uses a short high-voltage flash pulse. For this purpose, an energy-storage capacitor is charged from a DC source and must be safely discharged before maintenance.

Data:  $C = 1 \text{ } \mu\text{F}$   $W_e = 0.1 \text{ J}$   $I_{\text{max}} = 100 \text{ mA}$   $R_i = 10 \text{ M}\Omega$

- What voltage must the capacitor have so that it stores the required energy?

## SolutionResult

```
\begin{align*} W_e &= \frac{1}{2} C \\ U^2 \\ U &= \sqrt{\frac{2W_e}{C}} \\ &= \sqrt{\frac{2 \cdot 0.1 \text{ J}}{1 \\ \cdot 10^{-6} \text{ F}}} \\ &= \sqrt{200000} \text{ V} \approx \\ 447.2 \text{ V} \end{align*}
```

```
\begin{align*} U &= 447.2 \text{ V} \\ \end{align*}
```

2. The charging current must not exceed  $100 \text{ mA}$  at the start of charging. What charging resistor is required?

## SolutionResult

At the beginning of charging, the capacitor behaves like a short circuit, so  $i_{C \text{ max}} = i_C(t=0) = \frac{U}{R}$ . Thus,  $R \geq \frac{U}{i_{\text{max}}} = \frac{447.2 \text{ V}}{0.1 \text{ A}} \approx 4472 \text{ } \Omega = 4.47 \text{ k}\Omega$ .

```
\begin{align*} R &\geq 4.47 \text{ k}\Omega \\ \end{align*}
```

3. How long does the charging process take until the capacitor is practically fully charged?

## SolutionResult

The time constant is  $T = RC = 4.47 \text{ k}\Omega \cdot 1 \text{ } \mu\text{F} = 4.47 \text{ ms}$

```
\begin{align*} t &\approx 22.35 \text{ ms} \\ \end{align*}
```

\end{align\*} In engineering practice, a capacitor is considered practically fully charged after about  $5T$ :

$$\begin{align*} t \approx 5T = 5 \cdot 4.47 \text{ ms} = 22.35 \text{ ms} \end{align*}$$

4. Give the time-dependent capacitor voltage and the voltage across the charging resistor.

#### SolutionResult

For the charging process:  

$$\begin{align*} u_C(t) &= U \left(1 - e^{-t/T}\right) \\ u_R(t) &= U e^{-t/T} \end{align*}$$
 with 
$$\begin{align*} U &= 447.2 \text{ V} \\ T &= 4.47 \text{ ms} \end{align*}$$
 So the capacitor voltage rises exponentially from  $0$  to  $447.2 \text{ V}$ , while the resistor voltage falls exponentially from  $447.2 \text{ V}$  to  $0$ .

$$\begin{align*} u_C(t) &= 447.2 \left(1 - e^{-t/4.47 \text{ ms}}\right) \text{ V} \\ u_R(t) &= 447.2 e^{-t/4.47 \text{ ms}} \text{ V} \end{align*}$$

5. After charging, the capacitor is disconnected from the source. Its leakage can be modeled by an internal resistance of  $10 \text{ M}\Omega$ . After what time has the stored energy dropped to one half, and what is the capacitor voltage then?

#### SolutionResult

Half the energy means 
$$\begin{align*} W_e' = 0.5 W_e \end{align*}$$
 Since 
$$\begin{align*} W_e = \frac{1}{2} C U^2 \end{align*}$$
 the voltage at half energy is

$$\begin{align*} U' &= 316.2 \text{ V} \\ t &= 3.47 \text{ s} \end{align*}$$

```

\begin{align*} U' = \\
\frac{U}{\sqrt{2}} = \\
\frac{447.2 \sim \{\rm V\}}{\sqrt{2}} = \\
316.2 \sim \{\rm V\} \end{align*} For the
discharge through the internal
resistance: \begin{align*} u_C(t) = \\
Ue^{-t/T_2} \end{align*} with
\begin{align*} T_2 = R_i C = 10 \sim \{\rm \\
M\Omega\} \cdot 1 \sim \{\rm \mu F\} = \\
10 \sim \{\rm s\} \end{align*} Set
u_C(t)=U': \begin{align*} Ue^{- \\
t/T_2} \&= U' \ \&t \&= T_2 \\
\ln\left(\frac{U}{U'}\right) \ \&= \\
10 \sim \{\rm \\
s\} \cdot \ln\left(\frac{447.2}{316.2}\right) \\
\&\approx 3.47 \sim \{\rm s\} \\
\end{align*}

```

6. The fully charged capacitor is discharged through the charging resistor before maintenance. How long does the discharge take, and how much energy is converted into heat in the resistor?

### SolutionResult

The discharge time constant through the same resistor is again

```

\begin{align*} T = RC = 4.47 \sim \{\rm \\
ms\} \end{align*} Thus the practical
discharge time is \begin{align*} t \\
\approx 5T = 22.35 \sim \{\rm ms\} \\
\end{align*} The complete stored
capacitor energy is converted into
heat in the resistor: \begin{align*} \\
W_R = W_e = 0.1 \sim \{\rm Js\} \\
\end{align*}

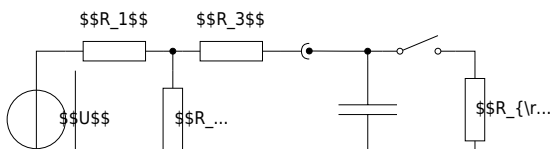
```

```

\begin{align*} t \approx 22.35 \sim \{\rm \\
ms\} \ \&W_R = 0.1 \sim \{\rm Js\} \\
\end{align*}

```

## Exercise E1.5 Sensor Input Buffer: Source, T-Network and Capacitor



A 12 V industrial sensor electronics unit feeds a buffered measurement node through a resistor T-network. A capacitor smooths the node voltage. At first, the load is disconnected. After the capacitor is fully charged, a measurement load is connected by a switch.

Data: 
$$\begin{aligned} U &= 12 \text{ V} \\ R_1 &= 2 \text{ k}\Omega \\ R_2 &= 10 \text{ k}\Omega \\ R_3 &= 3.33 \text{ k}\Omega \\ C &= 2 \text{ }\mu\text{F} \\ R_L &= 5 \text{ k}\Omega \end{aligned}$$

Initially, the capacitor is uncharged and the switch is open.

1. What is the capacitor voltage after it is fully charged?

### SolutionResult

Using the equivalent voltage source of the network on the left-hand side, the open-circuit voltage is

$$\begin{aligned} U_{0e} &= \frac{R_2}{R_1 + R_2} U \\ &= \frac{10 \text{ k}\Omega}{2 \text{ k}\Omega + 10 \text{ k}\Omega} \cdot 12 \text{ V} \\ &= 10 \text{ V} \end{aligned}$$

After full charging, the capacitor voltage equals this voltage.

$$\begin{aligned} U_C = U_{0e} &= 10 \text{ V} \end{aligned}$$

2. How long does the charging process take?

### SolutionResult

The internal resistance seen by the capacitor is  $R_{ie} = R_3 + (R_1 \parallel R_2) = 3.33 \text{ k}\Omega + \frac{2 \text{ k}\Omega}{2} = 3.33 \text{ k}\Omega + 1 \text{ k}\Omega = 4.33 \text{ k}\Omega$ . So the time constant is  $T = R_{ie}C = 4.33 \text{ k}\Omega \cdot 2 \text{ }\mu\text{F} = 8.66 \text{ ms}$ . Practical charging time:  $t \approx 5T = 43.3 \text{ ms}$ .

$$R_{ie} = 5.00 \text{ k}\Omega \quad t \approx 50 \text{ ms}$$

3. Give the time-dependent capacitor voltage.

### SolutionResult

The charging law is  $u_C(t) = U_{0e} \left(1 - e^{-t/T}\right) = 10 \left(1 - e^{-t/10 \text{ ms}}\right) \text{ V}$ . So the capacitor voltage rises exponentially from  $0 \text{ V}$  to  $10 \text{ V}$ .

$$u_C(t) = 10 \left(1 - e^{-t/10 \text{ ms}}\right) \text{ V}$$

4. After the capacitor is fully charged, the switch is closed and the load resistor is connected. What is the stationary load voltage?

### SolutionResult

Now use a second equivalent voltage-source step. The Thevenin source seen by the load has 
$$U_{0e} = 10 \text{ V} \quad R_{ie} = 5.00 \text{ k}\Omega$$
 Thus, the stationary load voltage is 
$$U_{C'} = U_{0e}' = \frac{R_L}{R_{ie} + R_L} U_{0e} = \frac{5 \text{ k}\Omega}{5 \text{ k}\Omega + 5 \text{ k}\Omega} \cdot 10 \text{ V} = 5 \text{ V}$$

$$\begin{aligned} U_L &= 5 \text{ V} \\ \end{aligned}$$

5. How long does it take until this new stationary state is practically reached?

### SolutionResult

The new internal resistance is 
$$R_{ie}' = R_{ie} \parallel R_L = 5.00 \text{ k}\Omega \parallel 5.00 \text{ k}\Omega = 2.50 \text{ k}\Omega$$
 Hence the new time constant is 
$$T' = R_{ie}' C = 2.50 \text{ k}\Omega \cdot 2 \text{ }\mu\text{F} = 5 \text{ ms}$$
 Practical settling time: 
$$t \approx 5T' = 25 \text{ ms}$$

$$\begin{aligned} R_{ie}' &= 2.50 \text{ k}\Omega \\ t &\approx 25 \text{ ms} \\ \end{aligned}$$

6. Give the time-dependent load voltage after the switch is closed.

### SolutionResult

At the switching instant, the capacitor voltage cannot jump. Therefore:

$$\begin{aligned} u_L(0^+) &= 10 \text{ V} \\ u_L(\infty) &= 5 \text{ V} \end{aligned}$$

The voltage therefore decays exponentially toward the new final value:

$$\begin{aligned} u_L(t) &= u_L(\infty) + \left( u_L(0^+) - u_L(\infty) \right) e^{-t/T'} \\ &= 5 + 5e^{-t/5 \text{ ms}} \text{ V} \end{aligned}$$

$$\begin{aligned} u_L(t) &= 5 + 5e^{-t/5 \text{ ms}} \text{ V} \end{aligned}$$

### Exercise E1.6 Hall-Sensor Calibration Coil: Short Air-Core Coil

A Hall-sensor calibration bench uses a short air-core coil to create a defined magnetic field. An air-core coil is chosen because it avoids hysteresis and remanence effects. The coil is wound as a short cylindrical coil.

Data: 
$$\begin{aligned} l &= 22 \text{ mm} \\ d &= 20 \text{ mm} \\ d_{\text{Cu}} &= 0.8 \text{ mm} \\ N &= 25 \\ \rho_{\text{Cu}, 20^\circ \text{C}} &= 0.0178 \text{ } \Omega \cdot \text{mm}^2/\text{m} \end{aligned}$$

A DC current of  $1 \text{ A}$  shall flow through the coil.

1. Calculate the coil resistance  $R$  at room temperature.

### SolutionResult

The wire cross section is

$$\begin{aligned} A_{\text{Cu}} &= \pi \left( \frac{d_{\text{Cu}}}{2} \right)^2 = \pi (0.4 \text{ mm})^2 \\ &= 0.503 \text{ mm}^2 \end{aligned}$$

The total wire length is approximated by the number of turns times the circumference:

$$\begin{aligned} l_{\text{Cu}} &= N \pi d \\ &= 25 \pi \cdot 20 \text{ mm} = 1570.8 \text{ mm} = 1.571 \text{ m} \end{aligned}$$

Thus,

$$\begin{aligned} R &= \rho_{\text{Cu}} \frac{l_{\text{Cu}}}{A_{\text{Cu}}} \\ &= 0.0178 \text{ m} \Omega \cdot \frac{1.571 \text{ m}}{0.503 \text{ mm}^2} \\ &\approx 0.0556 \text{ m} \Omega \end{aligned}$$

$$\begin{aligned} R &= 55.6 \text{ m} \Omega \end{aligned}$$

2. Calculate the coil inductance \$L\$.

### SolutionResult

For this short air-core coil, use

$$\begin{aligned} L &= N^2 \cdot \frac{\mu_0}{4\pi} \cdot \frac{1}{1 + \frac{d}{2l}} \\ &\text{with } A = \pi \left( \frac{d}{2} \right)^2 = \pi (10 \text{ mm})^2 = 314.16 \text{ mm}^2 \\ &= 3.1416 \cdot 10^{-4} \text{ m}^2 \\ \mu_0 &= 4\pi \cdot 10^{-7} \text{ Vs/(Am)} \end{aligned}$$

Therefore,

$$\begin{aligned} L &= 25^2 \cdot \frac{4\pi \cdot 10^{-7}}{4\pi} \cdot \frac{1}{1 + \frac{0.01}{0.22}} \cdot 3.1416 \cdot 10^{-4} \cdot 10^{-3} \end{aligned}$$

$$\begin{aligned} L &= 7.71 \text{ } \mu\text{H} \end{aligned}$$

```
\frac{1}{1+\frac{20}{2\cdot 22}} \\
&\approx 7.71\cdot 10^{-6}\sim{\rm H} \\
\end{align*}
```

3. Which DC voltage must be applied so that the stationary current becomes  $I=1\sim{\rm A}$ ?  
How large is the current density  $j$  in the copper wire?

#### SolutionResult

In the stationary DC state, the coil behaves like its ohmic resistance:

```
\begin{align*} U &= RI \\ &= 55.6\sim{\rm m\Omega}\cdot 1\sim{\rm A} \\ &= 55.6\sim{\rm mV} \\ \end{align*}
```

The current density is

```
\begin{align*} j &= \frac{I}{A_{\rm Cu}} \\ &= \frac{1\sim{\rm A}}{0.503\sim{\rm mm}^2} \\ &\approx 1.99\sim{\rm A/mm}^2 \\ \end{align*}
```

```
\begin{align*} U &= 55.6\sim{\rm mV} \\ j &= 1.99\sim{\rm A/mm}^2 \\ \end{align*}
```

4. How much magnetic energy is stored in the coil in the stationary state?

#### SolutionResult

```
\begin{align*} W_m &= \frac{1}{2}LI^2 \\ &= \frac{1}{2}\cdot 7.71\cdot 10^{-6}\sim{\rm H}\cdot (1\sim{\rm A})^2 \\ &= 3.86\cdot 10^{-6}\sim{\rm Ws} \\ \end{align*}
```

```
\begin{align*} W_m &= 3.86\cdot 10^{-6}\sim{\rm Ws} \\ \end{align*}
```

5. Give the time-dependent coil current  $i(t)$  when the coil is switched on.

### SolutionResult

A coil current cannot jump instantly. It starts at  $0$  and approaches the final value  $I=1\text{~}\text{A}$  exponentially:  

$$i(t) = I \left(1 - e^{-t/T}\right)$$
 So the sketch starts at  $0\text{~}\text{A}$ , rises quickly, and then slowly approaches  $1\text{~}\text{A}$ .

$$i(t) = I \left(1 - e^{-t/T}\right)$$

6. How long does it take until the current has practically reached its stationary value?

### SolutionResult

The time constant is  $T = \frac{L}{R} = \frac{7.71\text{~}\text{mH}}{55.6\text{~}\text{m}\Omega} \approx 138.9\text{~}\mu\text{s}$   
 A practical final value is reached after about  $5T$ :  

$$t \approx 5T = 5 \cdot 138.9\text{~}\mu\text{s} \approx 695\text{~}\mu\text{s}$$

$$t \approx 695\text{~}\mu\text{s}$$

7. How much energy is dissipated as heat in the coil resistance during the current build-up?

## SolutionResult

Using the current from task 5,  

$$i(t) = I \left(1 - e^{-t/T}\right)$$
the heat dissipated in the winding resistance up to the practical final time  $5T$  is  

$$W_R = \int_0^{5T} R i^2(t) dt = R I^2 \int_0^{5T} \left(1 - e^{-t/T}\right)^2 dt$$
For this interval, the integral is approximately  

$$\int_0^{5T} \left(1 - e^{-t/T}\right)^2 dt \approx \frac{7}{2} T$$
Thus,  

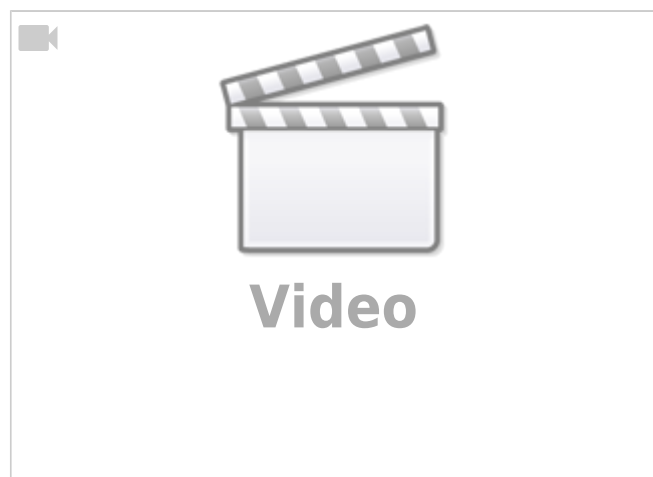
$$W_R \approx R I^2 \cdot \frac{7}{2} T = 0.0556 \cdot \Omega \cdot (1 \text{ A})^2 \cdot \frac{7}{2} \cdot \mu\text{s} \approx 27.05 \cdot 10^{-6} \text{ Ws}$$

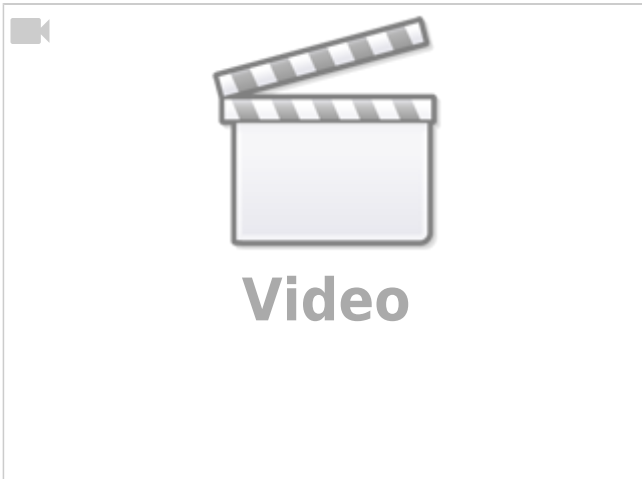
$$W_R \approx 27.05 \cdot 10^{-6} \text{ Ws}$$

## Embedded resources

Here is a short introduction about the transient behavior of an RC element (starting at 15:07 until 24:55)

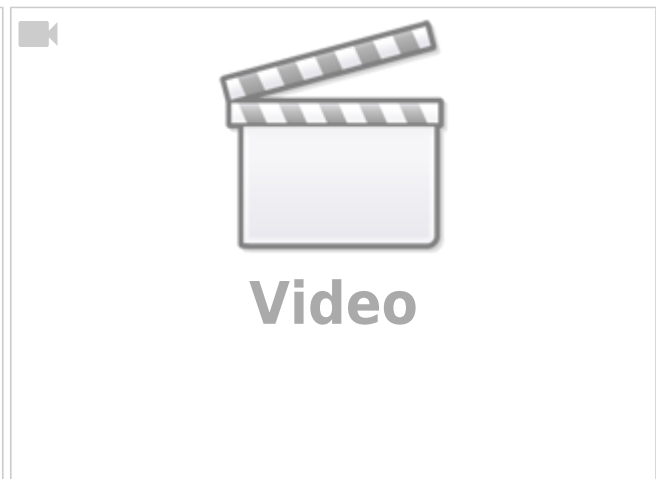
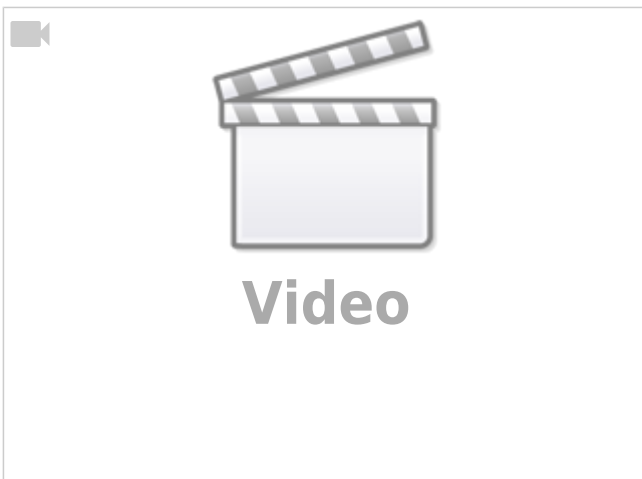
Mathematical explanation of charging a capacitor





Mathematical explanation of discharging a capacitor

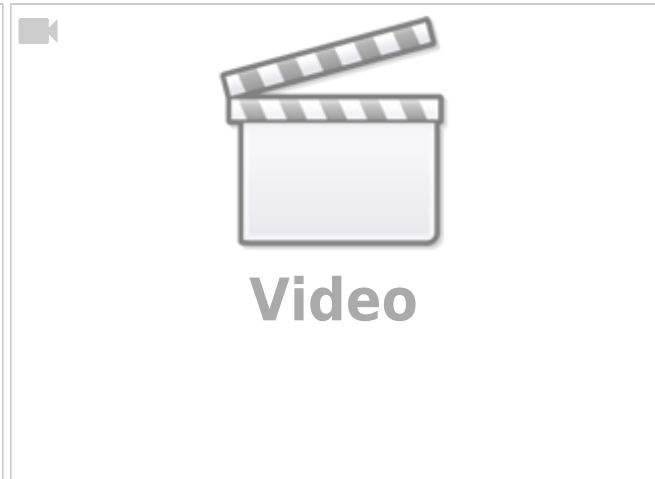
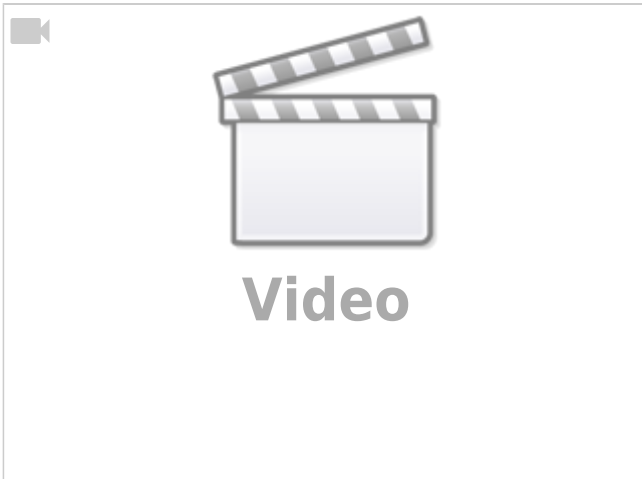
Mathematical explanation of the energy stored in the capacitor



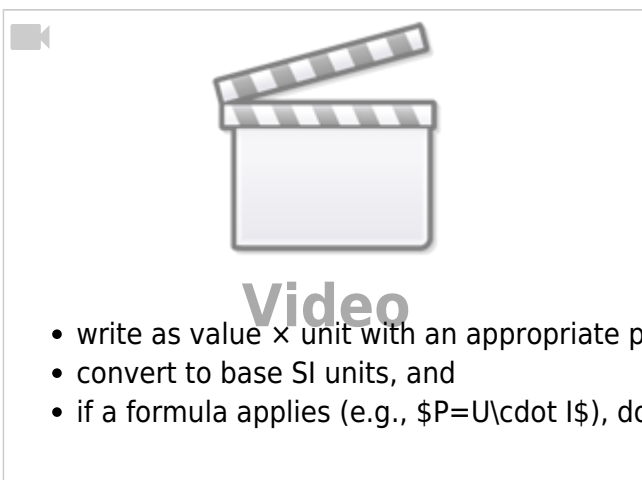
## Embedded resources

A nice 10-minute intro into some of the main topics of this chapter

Short presentation of the SI units



Orders of magnitude and why prefixes matter.



### Mini-assignment / homework (optional)

List 10 everyday EE-relevant quantities (e.g., USB current, phone battery energy, LED forward voltage). For each:

From:

<https://mexle.te.hs-heilbronn.de/> - MEXLE Wiki

Permanent link:

[https://mexle.te.hs-heilbronn.de/electrical\\_engineering\\_and\\_electronics\\_1/block01?rev=1759175537](https://mexle.te.hs-heilbronn.de/electrical_engineering_and_electronics_1/block01?rev=1759175537)

Last update: 2025/09/29 21:52

