3 The magnetostatic Field

Student Group

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3 The magnetostatic Field

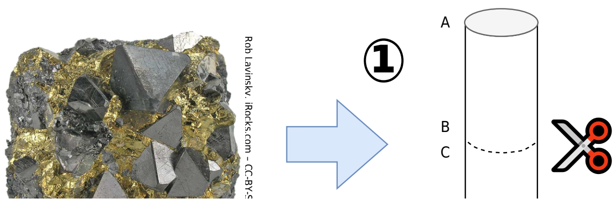
> The online book 'University Physics II' is strongly recommended as a reference for this and the following chapter - especially the following chapters: 11. Magnetic Forces and Fields (only 11.1 - 11.3 and 11.5)

3.1 Magnetic Phenomena

> **Learning Objectives**
>
> By the end of this section, you will be able to:
>
> 1. Know that forces act between magnetic poles and know the direction of the forces.
> 2. Know that a magnetic field is formed around a current-carrying conductor.
> 3. be able to sketch the field lines of the magnetic field. Know the direction of the field and where the field is densest.

Effects around Permanent Magnets

Fig. 1: First approximation to magnetism



First, permanent magnets made of magnetic magnetite ($\rm Fe_{3} O_{4}$) were found in Greece in the region around Magnesia. Besides the iron materials, other elements also show a similar “strong and permanent magnetic force effect”, which is also called ferromagnetism after iron: Cobalt and nickel, as well as many of their alloys, also exhibit such an effect. Chapter 3.5 Matter in the magnetic field describes the subdivision of magnetic materials in detail.

Here now the “magnetic force effect” is to be looked at more near. For this purpose, a few thought experiments are carried out with a magnetic iron stone figure 21 (This video gives a similar introduction).

1. From the iron ore should now first be separated a handy elongated part. If one is lucky, the given iron ore is already magnetic by itself. This case will be considered in the following. The elongated piece is now to be cut into two small pieces.
2. As soon as the two pieces are removed from each other, one notices that the two pieces attract each other again directly at the cut surface.
3. If one of the two parts is turned (the upper part in the picture below), a repulsive force acts on the two parts.

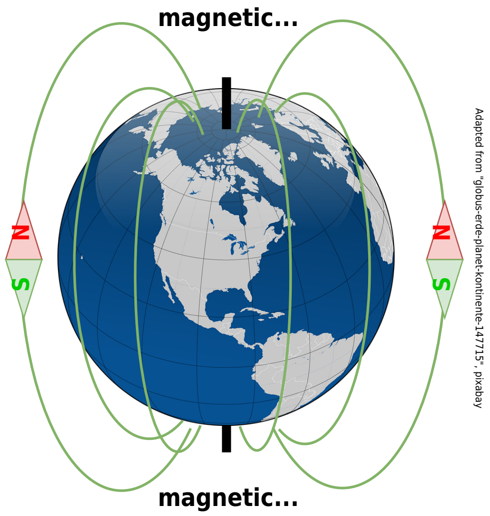
So it seems that there is a directed force around each of the two parts. If you dig a little deeper you will find that this force is focused on one part of the outer surface.

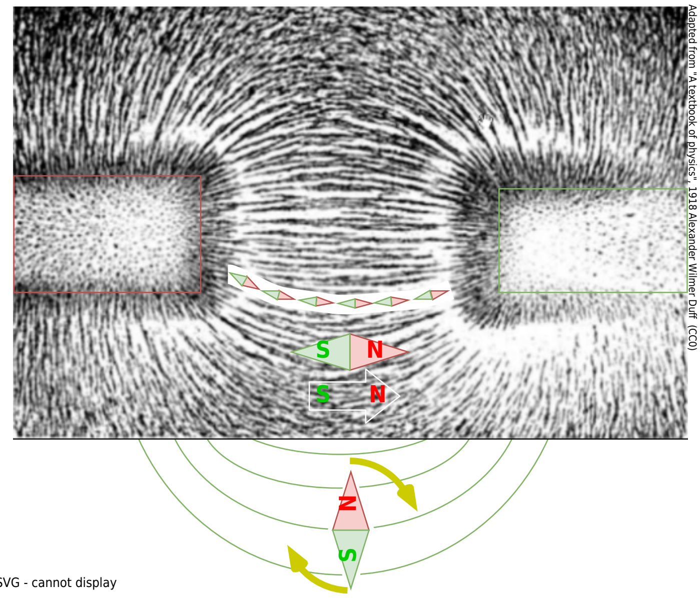
Of course, you already know magnets and also know that there are poles. The considered thought experiment shall clarify, how one could have proceeded at an unknown appearance. In further thought experiments, such magnet iron stones can also be cut into other directions and the forces analyzed.

The result here is:

1. There are two poles. These are called the north pole and the south pole. The north pole is colored red, and the south pole is green.
2. Poles with the same name repel each other. Unequal poles attract each other. This is similar to the electric field (opposite charges attract).
3. So magnets experience a force in the vicinity of other magnets.
4. A compass is a small rotating “sample” magnet and is also called a magnetic needle. This sample magnet can thus represent the effect of a magnet. This is also similar to the sample charge of the electric field.
5. The naming of the magnetic poles was done by the part of the compass which points to the geographic north pole. The reason for this is that the magnetic south pole is found at the geographic north pole.
6. Magnetic poles are not isolatable. Even the smallest fraction of a magnet shows either no magnetism or both north and south poles.

Fig. 2: Magnetic field becomes visible through iron filings





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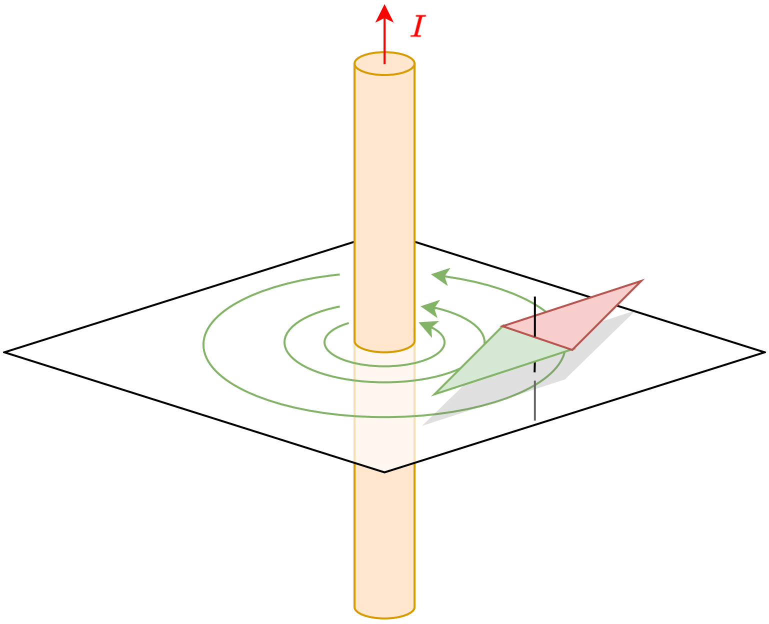
Interestingly, even non-magnetized, ferromagnetic materials experience a force effect in the magnetic field. A non-magnetic nail is attracted by a permanent magnet. This even happens independently of the magnetic pole. This also explains the visualization about iron filings (= small ferromagnetic parts), see figure 2. Also here there is a force effect and a torque, which aligns the iron filings. The visible field seems to form field lines here.

Notice:

- Field line images can be visualized by iron filings. Conceptually, these can be understood as a string of sample magnets.
- The **direction of the magnetic field** defined via the sample magnet: The north pole of the sample magnet points in the direction of the magnetic field.
- The **amount of magnetic field** is given by the torque experienced by a sample magnet oriented perpendicular to the field.
- Field lines seem to repel each other (“perpendicular push”). e.g. visible when the field exits the permanent magnet.
- Field lines attempt to travel as short a path as possible (“longitudinal pull”).

Effects around Current-carrying Wires

Fig. 3: Magnetic field around a current-carrying conductor

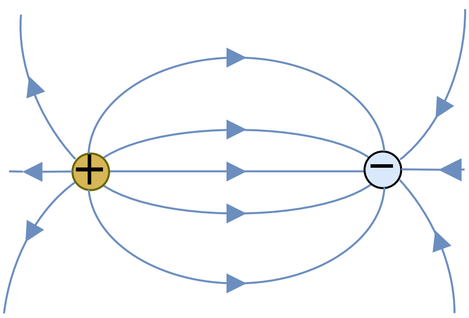
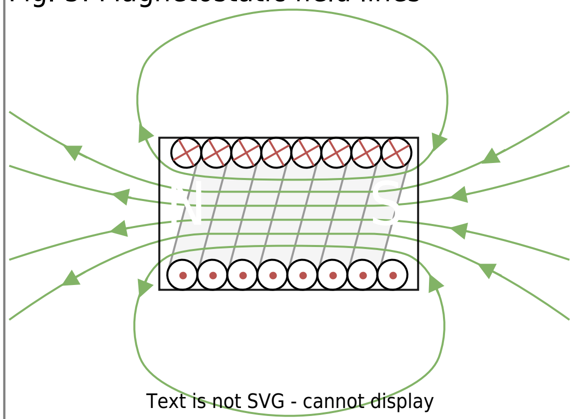


In 1820, Christian Ørsted discovered by chance during a lecture that current-carrying conductors also affect a compass. This experiment is illustrated in figure 3. A long, straight conductor with a circular cross-section has current I flowing through it. Due to symmetry considerations, the field line pattern must be radially symmetric concerning the conductor axis. In an experiment with a magnetic needle, it can be shown that the field lines form concentric circles.

Notice:

- If the technical direction of the current is considered, the magnetic field lines surround the current in the sense of a right-hand screw. ("right screw rule")
- This rule can also be remembered in another way: If the thumb of the **r**ight hand points in the (technical) cu**rr**ent direction, the fingers of the hand surround the conductor like the magnetic field lines. Likewise, if the thumb of the **l**eft hand points in the E**l**ectron flow direction, the fingers of the hand surround the conductor like the magnetic field lines.

Comparison of Electrostatics and Magnetostatics

Property	Electrostatic	Magnetostatic
Field line images	Fig. 4: Electrostatic field lines  Text is not SVG - cannot display	Fig. 5: Magnetostatic field lines  Text is not SVG - cannot display
sample for the field	positive sample charge	compass
field lines	- start on a positive charge - end on a negative charge	- have no start and no end - are closed
field line ends	there are sources and sinks	there are <u>no</u> sources and sinks
field type	vortex-free **<u>source field</u>**	source-free **<u>vortex field</u>**

3.2 Magnetic Field Strength

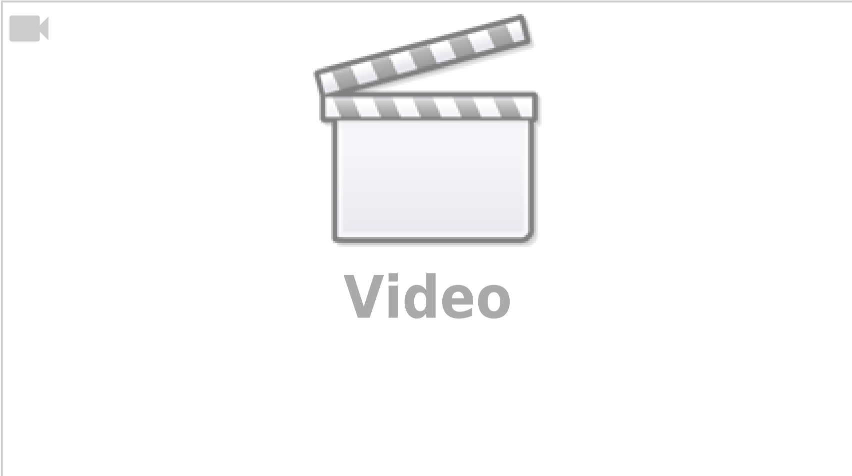
Learning Objectives

By the end of this section, you will be able to:

1. know the two field-describing quantities of the magnetic field.
2. be able to describe and apply the relationship between these two quantities.

Superposition of the magnetostatic Field

Superposition of magnetic fields



Before the magnetic field strength will be considered in more detail, the simulation and superposition of the magnetic field will be discussed in more detail here.

Magnetostatic fields can be superposed, just like electrostatic fields. This allows the fields of several current-carrying lines to be combined into a single one. This trick is used in the following chapter to examine the magnetic field in more detail.

On the right side, the magnetic field of a single current-carrying conductor is shown. This was already derived in the previous chapter by symmetry considerations. The representation in the simulation can be simplified a bit here to see the conditions more clearly: Currently, the field lines are displayed in 3D, which is done by selecting `Display: Field Lines` and `No Slicing`. If you change the selection to `Show Z Slice` instead of `No Slicing`, you can switch to a 2D display. In this display, small compass needles can also show the magnetic field. To do this, select `Display: Field Vectors` instead of `Display: Field Lines`. In addition, a “magnetic sample”, i.e. a moving compass, can be found at the mouse pointer in the 2D display.

If there is another current-carrying conductor near the first conductor, the fields overlap. In the simulation below, the current of both conductors is directed in the same direction. The field between the conductors overlaps just enough to weaken. This can also be deduced by previous knowledge if just the middle point between both conductors is considered: There, for the left conductor the right-hand rule results in a vector directed towards the observer. For the right conductor, it results in a vector that is directed away from the observer. These just cancel each other out. Further outward field lines go around both conductors. The North and south poles here are not fixed localized toward the outside.

If, on the other hand, the current in the second conductor is directed in the opposite direction to the current in the first conductor, the picture changes: Here there is a reinforcing superposition between the two conductors. Using the nomenclature from the previous chapter, it is also possible to assign north and south poles locally. Towards the outside, one pole appears to be located in front of the two conductors and another one behind.

in both simulations, the distances between the conductors can also be changed using the `Line Separation` slider. What do you notice in each case when the two lines are brought close together?

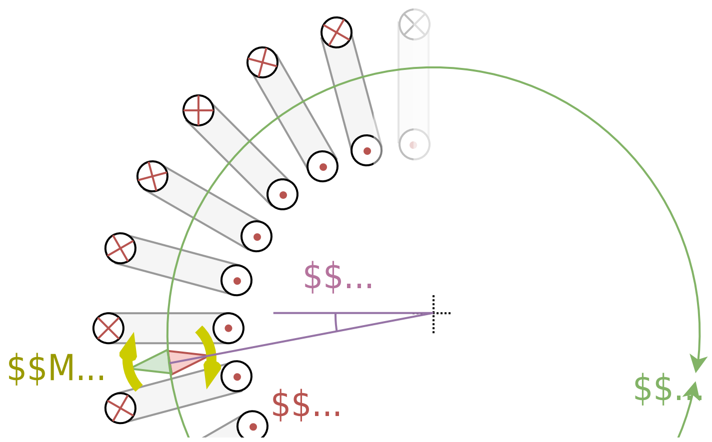
Magnetic Field Strength part 1: toroidal Coil

Fig. 6: Magnetic field in a toroidal coil

So far the magnetic field was defined quite pragmatically by the effect on a compass. For a deeper analysis of the magnetic field, the field is now to be considered again - as with the electric field - from two directions. The magnetic field will also be considered a “causer field” (a field caused by magnets) and an “acting field” (a field acting on a magnet). This chapter will first discuss the acting magnetic field. For this, it is convenient to consider the effects inside a toroidal coil (= donut-like setup). This can be seen in figure 6. For reasons of symmetry, it is also clear here that the field lines form concentric circles.

In an experiment, a magnetic needle inside the toroidal coil is now to be aligned perpendicular to the field lines. Then, the magnetic field will generate a torque M which tries to align the magnetic needle in the field direction.

Fig. 7: Toroidal Coil



It now follows:

1. $M = {\rm const.} \neq f(\varphi)$: For the same distance from the axis of symmetry, the torque M is independent of the angle ϕ.
2. $M \sim I$: The stronger the current flowing through a winding, the stronger the effect, i.e. the stronger the torque.
3. $M \sim N$: The greater the number N of windings, the stronger the torque M.
4. $M \sim {1 \over l}$: The smaller the average coil circumference l the greater the torque. The average coil circumference l is equal to the **mean magnetic path length** (=average field line length).

To summarize: \begin{align*} M \sim {{I \cdot N}\over{l}} \end{align*}

The **magnetic field strength** H inside the toroidal coil is given as: \begin{align*} \boxed{H ={{I \cdot N}\over{l}}} \quad \quad | \quad \text{applies to toroidal coil only} \end{align*}

For the unit of the magnetic field strength H we get $[H] = {{[I]}\over{[l]}}= \rm 1~{{A}\over{m}}$

Magnetic Field Strength part 2: straight conductor

The previous derivation from the toroidal coil is now used to derive the field strength around a long, straight conductor. For a single conductor the part $N \cdot I$ of the formula can be reduced to $ N \cdot I = 1 \cdot I = I$ since there is only one conductor. For the toroidal coil, the magnetic field strength was given by this current(s) divided by the (average) field line length. Because of the (same rotational) symmetry, this is also true for the single conductor. Also here the field line length has to be taken into account.

The length of a field line around the conductor is given by the distance r of the field line from the conductor: $l = l(r) = 2 \cdot \pi \cdot r$.
For the magnetic field strength of the single conductor we then get: \begin{align*} \boxed{H ={I\over{l}} = {{I}\over{2 \cdot \pi \cdot r}}} \quad \quad | \quad \text{applies only to the long, straight conductor} \end{align*}

Fig. 8: magnetic Field Lines around a Conductor

In the electric field, the field line density was a measure of the strength of the field. This is also used for the magnetic field. Looking at the simulations in Falstad (e.g. figure 8) with this understanding, one notices an inconsistency: contrary to the relationship just given, the field line density in the Falstad simulation **not** indicates the strength of the field. A realistic simulation is shown in figure 9 for comparison, which makes the difference clear: the field is stronger near the conductor. Thus the field line density must also be stronger there.

Fig. 9: correct Picture of Magnetic Field Lines around a Conductor

Attention:

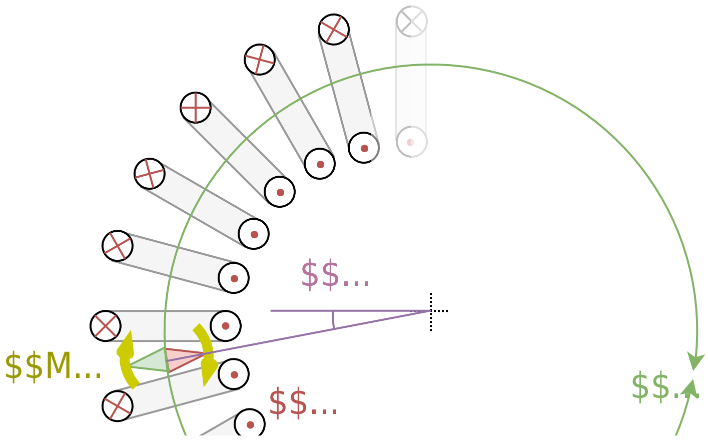
- The density of the field lines is a measure of the field strength.
- The simulation in Falstad cannot represent this in this way. Here the field strength is coded by the color intensity (dark green = low field strength, light green to white = high field strength).

Magnetic Voltage

The cause of a magnetic field is a current. As seen for the coil, sometimes the current has to be counted up ($N \cdot I$), e.g. by the number of windings of the coil. If this current I and/or the number N of windings is increased, the effect is amplified. To make this easier to handle, we introduce the **Magnetic Voltage**. The magnetic voltage θ is defined as

\begin{align*} \boxed{\theta = \sum I = N \cdot I} \end{align*} The unit of θ is: $[\theta]= 1~\rm A$ (obsoletely called ampere-turn). For the magnetic voltage the currents which flow through the surface enclosed by the closed path have to be considered. A detailed definition will be given below after more analysis.

Fig. 10: magnetic voltage on the edge of an enclosed surface



Thus, the magnetic field strength H of the toroidal coil is then given by: $H= {{\theta}\over{l}}$

Notice:

- For the sign of the magnetic voltage, one has to consider the orientation of the current and way on the enclosing path. The figure 10 shows the positive orientation: The positive orientation is given when the currents show out of the drawing plane and the path shows a counterclockwise orientation.
- This is again given as the right-hand rule (see figure 11): For the positive orientation the current shows along the thumb of the right hand, while the path is counted along the

direction of the fingers of the right hand.

Fig. 11: Right hand rule

In the English literature often the name **Magnetomotive Force** $\mathcal{F}$ is used instead of magnetic voltage θ. The naming refers to the Electromotive Force. The electromotive force describes the root cause of a (voltage) source to be able to drive a current and therefore generate a defined voltage. Both "forces" shall not be confused with the mechanical force $\vec{F}= m \cdot \vec{a}$. They only describe the driving cause behind the electric or magnetic fields. The German courses in higher semesters use the term *Magnetische Spannung* - therefore, the English equivalent is introduced here.

Magnetic Field Strength part 3: Generalization

So far, only rotational symmetric problems could be solved. Now, this shall be generalized. For this purpose, we will have a look back at the electric field. For the electric field strength E of a capacitor with two plates at a distance of s and the potential difference U holds:

\begin{align*} U = E \cdot s \quad \quad | \quad \text{applies to capacitor only} \end{align*}

In words: The potential difference is given by adding up the field strength along the path of a probe from one plate to the other. This was extended to $U = \int_s E {\rm d}s$. If we compare this idea to the magnetic field strength H of a toroidal coil with the mean magnetic path length l, we had

\begin{align*} \theta = H \cdot l \quad \quad | \quad \text{applies to toroidal coil only} \end{align*}

Can you see the similarities? Again, the magnitude of the field strength is summed up along a path to arrive at another field-describing quantity (here, the magnetic voltage θ). Because of the similarity - which continues below - the so-called **magnetic potential difference V_m** is introduced:

\begin{align*} V_m = H \cdot s \quad \quad | \quad \text{applies to toroidal coil only} \end{align*}

Now, what is the difference between the magnetic potential difference V_m and the magnetic voltage θ?

1. The first equation of the toroidal coil ($\theta = H \cdot l$) is valid for exactly <u>one turn</u> along a field line with the length l. In addition, the magnetic voltage was equal to the current times the number of windings: $\theta = N \cdot I$.
2. The second equation ($V_{\rm m} = H \cdot s$) is independent of the length of the field line l. Only if $s = l$ is chosen, the magnetic voltage equals the magnetic potential difference. The path length s can be a fraction or multiple of a single revolution l for the magnetic potential difference.

Thus, for each infinitesimally small path ${\rm d}s$ along a field line, the resulting infinitesimally small magnetic potential difference ${\rm d}V_{\rm m} = H \cdot {\rm d}s$ can be determined. If now along the field line the magnetic field strength $H = H(\vec{s})$ changes, then the magnetic potential difference from point $\vec{s_1}$ to point $\vec{s_2}$ results to:

\begin{align*} V_{\rm m12} = V_{\rm m}(\vec{s_1}, \vec{s_2}) = \int_\vec{s_1}^\vec{s_2} H(\vec{s}) {\rm d}s \end{align*}

Up to now, only the situation was considered that one always walks along one single field line. $\vec{s}$ therefore always arrived at the same spot of the field line. If one wants to extend this to arbitrary directions (also perpendicular to field lines), then only that part of the magnetic field strength $\vec{H}$ may be used in the formula, which is parallel to the path ${\rm d} \vec{s}$. This is made possible by scalar multiplication. Thus, it is generally valid:

\begin{align*} \boxed{V_{\rm m12} = \int_\vec{s_1}^\vec{s_2} \vec{H} \cdot {\rm d} \vec{s}} \end{align*}

The magnetic voltage θ (and therefore the current) is the cause of the magnetic field strength. From the chapter The stationary Electric Flow the general representation of the current through a surface is known. This leads to the **Ampere's Circuital Law**

$$\begin{align*} \boxed{\oint_{s} \vec{H} \cdot {\rm d} \vec{s} = \iint_A \; \vec{S} {\rm d}\vec{A} = \theta} \end{align*}$$

- The path integral of the magnetic field strength along an arbitrary closed path is equal to the free currents (= current density) through the surface enclosed by the path.
- The magnetic voltage θ can be given as
  - for a single conductor: $\theta = I$
  - for a coil: $\theta = N \cdot I$
  - for multiple conductors: $\theta = \sum_n \cdot I_n$
  - for spatial distribution: $\theta = \iint_A \; \vec{S} {\rm d}\vec{A}$
- ${\rm d}\vec{s}$ and ${\rm d}\vec{A}$ build a right-hand system: once the thumb of the right hand is pointing along ${\rm d}\vec{A}$, the fingers of the right hand show the correct direction for ${\rm d}\vec{s}$ for positive $\vec{H}$ and $\vec{S}$

Fig. 12: Right hand rule

Recap: Application of magnetic Field Strength

Ampere's Circuital Law shall be applied to find the magnetic field strength H inside the toroidal coil (figure 13).

Fig. 13: Toroidal Coil recap

- The closed path ${\rm s}$ is on a revolution of a field line in the center of the coil
- The surface A is the enclosed surface

This leads to:

$$\begin{align*} \oint_s \vec{H} \cdot {\rm d} \vec{s} &= \iint_A \vec{S} {\rm d}\vec{A} = \theta \end{align*}$$

Since $\vec{H} \uparrow \uparrow {\rm d} \vec{s}$ the term $\vec{H} \cdot {\rm d} \vec{s}$ can be substituted by $H {\rm d}s$:

$$\begin{align*} \oint_s H \cdot {\rm d}s &= \iint_A \vec{S} {\rm d}\vec{A} \end{align*}$$

The magnetic voltage is the current through the surface and is given as $N\cdot I$:

$$\begin{align*} \oint_{\rm s} H \cdot {\rm d}s &= N\cdot I \end{align*}$$

The magnetic field strength H can be considered constant:

$$\begin{align*} H \cdot \int_C {\rm d}s &= N\cdot I \end{align*}$$

The average coil circumference is $2\pi \cdot {{d}\over{2}}$:

$$\begin{align*} H \cdot 2\pi \cdot {{d}\over{2}} &= N\cdot I \end{align*}$$

Therefore, the magnetic field strength in the toroidal coil is $$\begin{align*} H &= {{N\cdot I}\over{ \pi \cdot d }} \end{align*}$$

Similarly, the magnetic field strength H in the distance r to a single conductor with the current I can be derived. In this situation, the result is:

$$\begin{align*} H &= {{I}\over{ 2\pi \cdot r }} \end{align*}$$

3.3 Magnetic Flux Density and Lorentz Law

Learning Objectives

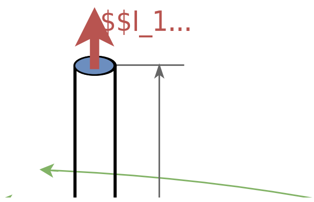
By the end of this section, you will be able to:

1. know the force law for current-carrying conductors.
2. Be able to determine the direction of the forces using given current directions and, if applicable, flux density.
3. be able to represent the vectors of the magnetic flux density in a sketch when several current-carrying conductors are specified.
4. be able to determine a force vector by superimposing several force vectors using vector calculus.
5. be able to state the following quantities for a force vector on a current-carrying conductor in a magnetostatic field:
    1. the force vector in coordinate representation
    2. the magnitude of the force vector
    3. the angle of the force vector

Definition of the Magnetic Flux Density

In the last sub-chapter, the field effect on a magnet caused by currents was analyzed. Now, the field acting onto currents will get deeper investigation. To do so, the effect between two parallel conductors has to be examined closer. The experiment consists of a part l of two very long[1)] conductors with the different currents I_1, I_2 in the distance r (see figure 14).

Fig. 14: Forces between two Conductors



When no current is flowing through the conductors the forces er equal to zero. Once the currents flow in the same direction (e.g. $I_1>0, I_2>0$) attracting forces $\vec{F}_{12} = - \vec{F}_{21}$ appear. The force $\vec{F}_{xy}$ shall be the force on the conductor x caused by conductor y. In the following the force $\vec{F}_{12}$ on the conductor 1 will be examined.

The following is detectable:

1. $|\vec{F}_{12}| \sim I_1$, $|\vec{F}_{12}| \sim I_2$: The stronger each current, the stronger the force F_{12}.
2. $|\vec{F}_{12}| \sim l$: As longer the conductor length l, as stronger the force F_{12} gets.
3. $|\vec{F}_{12}| \sim {{1}\over{r}}$: A smaller distance r leads to stronger force F_{12}.

To summarize: \begin{align*} {F}_{12} \sim {I_1 \cdot I_2 \cdot {{l}\over{r}}} \end{align*}

The proportionality factor is arbitrarily chosen as: \begin{align*} {{{F}_{12} \cdot r} \over {I_1 \cdot I_2 \cdot {l}}} = {{\mu}\over{2\pi}} \end{align*}

Here μ is the magnetic permeability and for vacuum (vacuum permeability): \begin{align*} \mu = \mu_0 = 4\pi \cdot 10^{-7} {{\rm Vs}\over{\rm Am}} = 1.257 \cdot 10^{-7} {{\rm Vs}\over{\rm Am}} \end{align*}

This leads to the **Ampere's Force Law**: \begin{align*} |\vec{F}_{12}| = {{\mu}\over{2 \pi}} \cdot {{I_1 \cdot I_2 }\over{r}} \cdot l \end{align*}

Since we want to investigate the effect on the current I_1, the following rearrangement can be done: \begin{align*} |\vec{F}_{12}| &= {{\mu}\over{2 \pi}} \cdot {{I_2 }\over{r}} &\cdot I_1 \cdot l \\ &= B &\cdot I_1 \cdot l \\ \end{align*}

The properties of the field from I_2 acting on I_1 are summarized to B - the **magnetic flux density**.
B has the unit: \begin{align*} [B] &= {{[F]}\over{[I]\cdot[l]}} = 1 \rm {{N}\over{Am}} = 1 {{{VAs}\over{m}}\over{Am}} = 1 {{Vs}\over{m^2}} \\ &= 1 {\rm T} \quad\quad({\rm Tesla}) \end{align*}

This formula can be generalized with the knowledge of the directions of the conducting wire $\vec{l}$, the magnetic field strength $\vec{B}$ and the force $\vec{F}$ using vector multiplication too:

\begin{align*} \boxed{\vec{F_L} = I \cdot \vec{l} \times \vec{B}} \end{align*}

The absolute value can be calculated by

\begin{align*} \boxed{|\vec{F_L}| = I \cdot |l| \cdot |B| \cdot \sin(\angle \vec{l},\vec{B})} \end{align*}

The force is often called **Lorentz Force** F_L. For the orientation, another right-hand rule can be applied.

Notice:
Right-hand rule for the Lorentz Force:

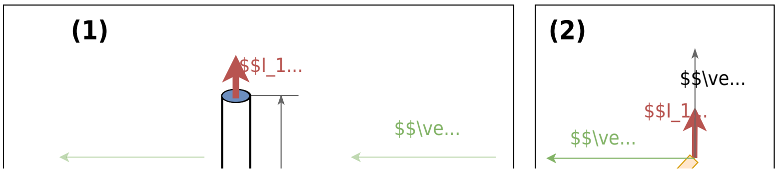
- The causing current I is on the thumb. Since the current is not a vector, the direction is given by the direction of the conductor $\vec{l}$
- The mediating external magnetic field $\vec{B}$ is on the index finger
- The resulting force $\vec{F}$ on the conductor is on the middle finger

This is shown in figure 14.

A way to remember the orientation is the mnemonic **FBI** (from middle finger to thumb):

- $\vec{F}$orce on middle finger
- $\vec{B}$-Field on index finger
- Current I on thumb (direction with length $\vec{l}$)

Fig. 14: Force onto a single Conductor in a B-Field



Lorentz Law and Lorentz Force

The true Lorentz force is not the force on the whole conductor but the single force onto an (elementary) charge.
To find this force the previous force onto a conductor can be used as a start. However, the formula will be investigated infinitesimally for small parts ${\rm d} \vec{l}$ of the conductor:

\begin{align*} \vec{{\rm d}F}_{\rm L} = I \cdot {\rm d}\vec{l} \times \vec{B} \end{align*}

The current is now substituted by $I = {\rm d}Q/{\rm d}t$, where ${\rm d}Q$ is the small charge packet in the length $\vec{{\rm d}l}$ of the conductor.

\begin{align*} \vec{{\rm d}F}_{\rm L} = {{{\rm d}Q}\over{{\rm d}t}} \cdot {\rm d}\vec{l} \times

\vec{B} \end{align*}

Mathematically not quite correct, but in a physical way true the following rearrangement can be done:

$$\begin{align*} \vec{{\rm d}F}_{\rm L} &= {{{\rm d}Q \cdot {\rm d}\vec{l}}\over{{\rm d}t}} \times \vec{B} \\ &= {\rm d}Q \cdot {{{\rm d}\vec{l}}\over{{\rm d}t}} \times \vec{B} \\ &= {\rm d}Q \cdot {{{\rm d}\vec{l}}\over{{\rm d}t}} \times \vec{B} \\ \end{align*}$$

Here, the part ${{{\rm d}\vec{l}}\over{{\rm d}t}}$ represents the speed $\vec{v}$ of the small charge packet ${\rm d}Q$.

$$\begin{align*} \vec{{\rm d}F}_{\rm L} &= {\rm d}Q \cdot \vec{v} \times \vec{B} \end{align*}$$

The **Lorenz Force** on a finite charge packet is the integration:

$$\begin{align*} \boxed{\vec{F}_{\rm L} = Q \cdot \vec{v} \times \vec{B}} \end{align*}$$

Notice:

- A charge Q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ experiences a force of $\vec{F_{\rm L}}$.
- The direction of the force is given by the right-hand rule.

Please have a look at the German contents (text, videos, exercises) on the page of the KIT-Brückenkurs >> 5.2.3 Lorentz-Kraft. Make sure that `Gesamt` is selected in the selection bar at the top. The last part “Magnetic field within matter” can be skipped.

3.4 Matter in the Magnetic Field

Learning Objectives

By the end of this section, you will be able to:

1. know the two field-describing quantities of the magnetostatic field.
2. be able to describe and apply the relationship between these two quantities via the material law.
3. know the classification of magnetic materials.
4. be able to read the relevant data from a magnetization characteristic curve.

The magnetic interaction is also represented via two fields in the previous subchapters similar to the electric field. Since the magnetic field strength $\vec{H}$ is only dependent on the field causing current I, this field is independent of surrounding matter.
The magnetic flux density $\vec{B}$ on the other hand, showed with the magnetic permeability a factor that can contain the material effects.

Both fields are connected. The Ampere's Force Law gave the formula

\begin{align*} |\vec{F}_{12}| &= {{\mu}\over{2 \pi}} \cdot {{I_2 }\over{r}} &\cdot I_1 \cdot l \\ &= B &\cdot I_1 \cdot l \\ \end{align*}

Therefore, the magnetic flux density $\vec{B}$ is equal to:

\begin{align*} B &= {{\mu}\over{2 \pi}} \cdot {{I_2 }\over{r}} \end{align*}

I_2 was here the field-causing current.
For the field strength of the straight conductor, we had:

\begin{align*} H &= {{I}\over{2 \pi \cdot r }} \end{align*}

The connection between the two fields is also $B = \mu \cdot H$. Since the field lines of both fields are always parallel to each other it results to

\begin{align*} \boxed{ \vec{B} = \mu \cdot \vec{H} } \end{align*}

This is similar to the $\vec{D} = \varepsilon \cdot \vec{E}$. Similarly, the permeability is separated into two parts:

\begin{align*} \mu &= \mu_0 \cdot \mu_{\rm r} \end{align*}

where

- $\mu_0 $ is the permeability of the vacuum
- $\mu_{\rm r}$ is the relative permeability, for the case that there is material in the magnetic field

The material can be divided into different types by looking at its relative permeability. figure 16 shows the relative permeability in the **magnetization curve** (also called B-H-curve). In this diagram, the different effect (B-field on y-axis) based on the causing external H-field (on x-axis) for different materials is shown. The three most important material types shall be discussed shortly.

Fig. 16: Magnetization Curve of different materials

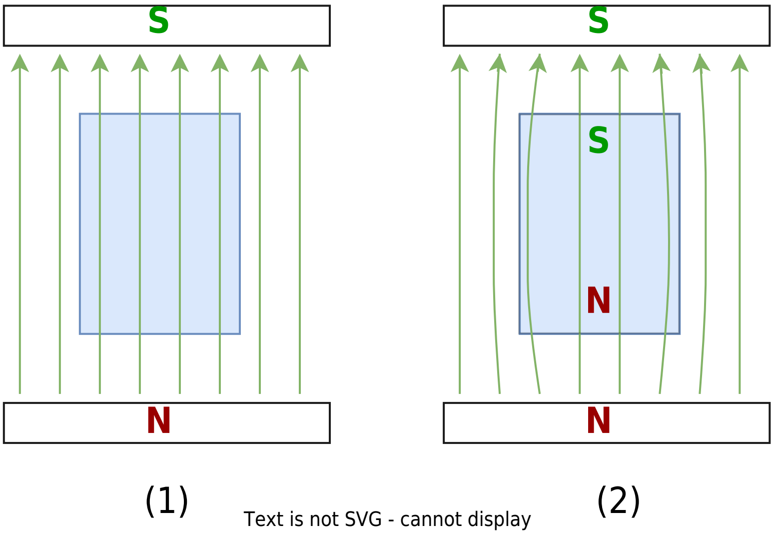
Diamagnetic Materials

- Diamagnetic materials weaken the magnetic field, compared to the vacuum.
- The weakening is very low (see table 1).
- For diamagnetic materials applies $0<\mu_{\rm r}<1$
- The principle behind the effect is based on quantum mechanics (see figure 17):
  - Without the external field no counteracting field is generated by the matter.
  - With an external magnetic field an antiparallel-orientated magnet is induced.
  - The reaction weakens the external field. This is similar to the weakening of the electric field by the dipoles of materials.
- Due to the repulsion of the outer magnetic field the material tends to move out of a magnetic field.
  For very strong magnetic fields small objects can be levitated (see clip).

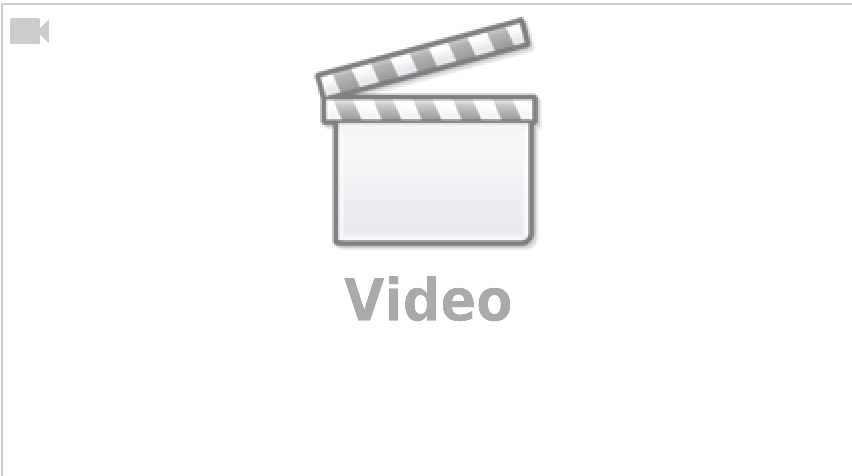
Material	Symbol	$\mu_{\rm r}$
Antimon	$\rm Sb$	$0.999 946$
Copper	$\rm Cu$	$0.999 990$
Mercury	$\rm Hg$	$0.999 975$
Silver	$\rm Ag$	$0.999 981$
Water	$\rm H_2O$	$0.999 946$
Bismut	$\rm Bi$	$0.999 830$

Tab. 1: Diamagnetic Materials

Fig. 17: Magnetic field in diamagnetic materials



A living insect (“diamagnet”) floats in a very strong magnetic field



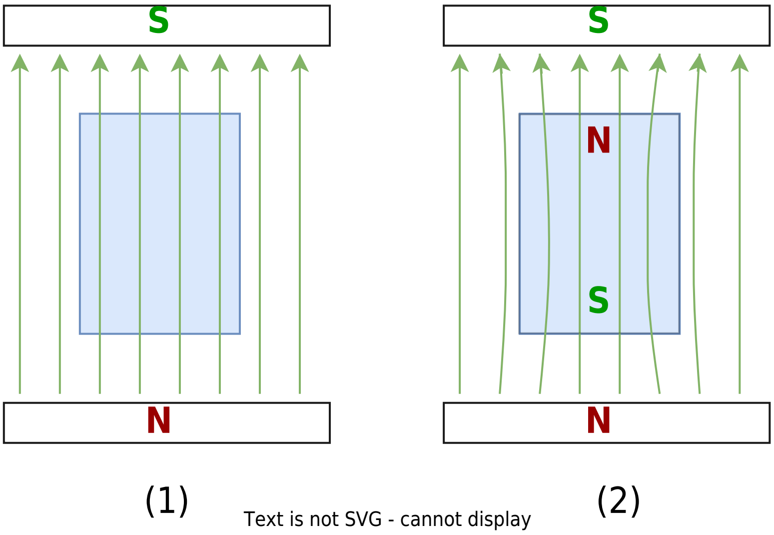
Paramagnetic Materials

- Paramagnetic materials strengthen the magnetic field, compared to the vacuum.
- The strengthening is very low (see table 2).
- For paramagnetic materials applies $\mu_{\rm r}>1$
- The principle behind the effect is again based on quantum mechanics (see figure 18):
  - Without the external field no counteracting field is generated by the matter.
  - With an external magnetic field internal “tiny magnets” based on the electrons in their orbitals are orientated similarly.
  - This reaction strengthens the external field.

Material	Symbol	$\mu_{\rm r}$
Aluminum	$\rm Al$	$1.000 022$
Air		$1.000 000 4$
Oxygen	$\rm O_2$	$1.000 001 3$
Platinum	$\rm Pt$	$1.000 36$
Tin	$\rm Sn$	$1.000 003 8$

Tab. 2: Paramagnetic Materials

Fig. 18: Magnetic field in paramagnetic materials



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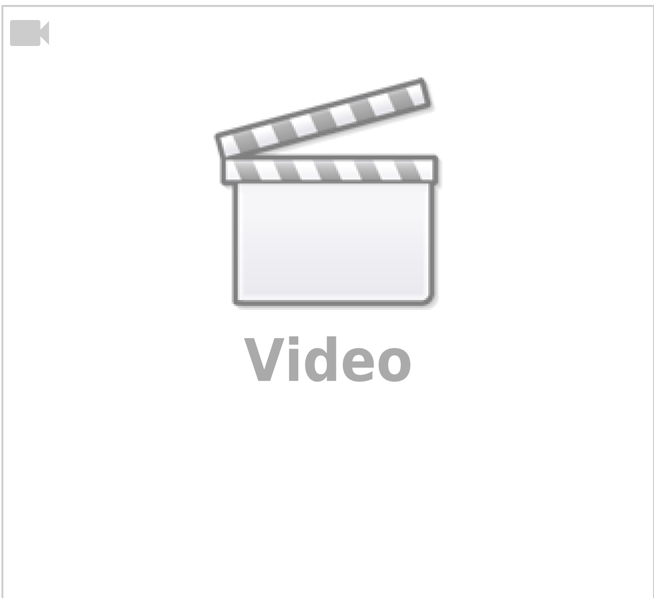
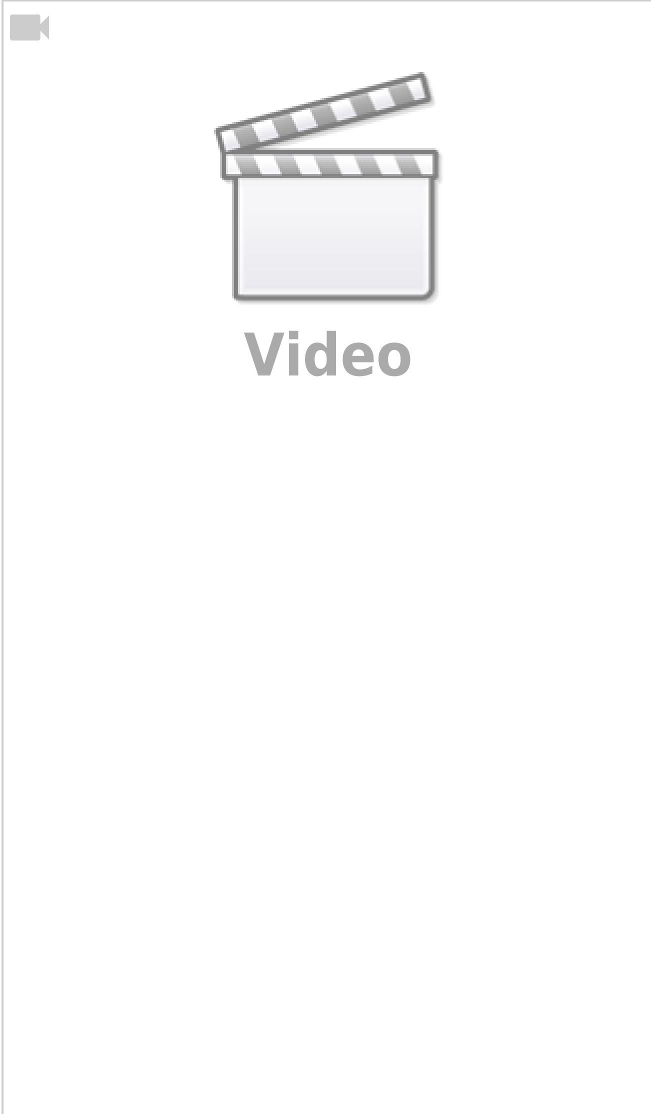
Explanation of diamagnetism and paramagnetism



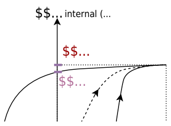


Ferromagnetic Materials

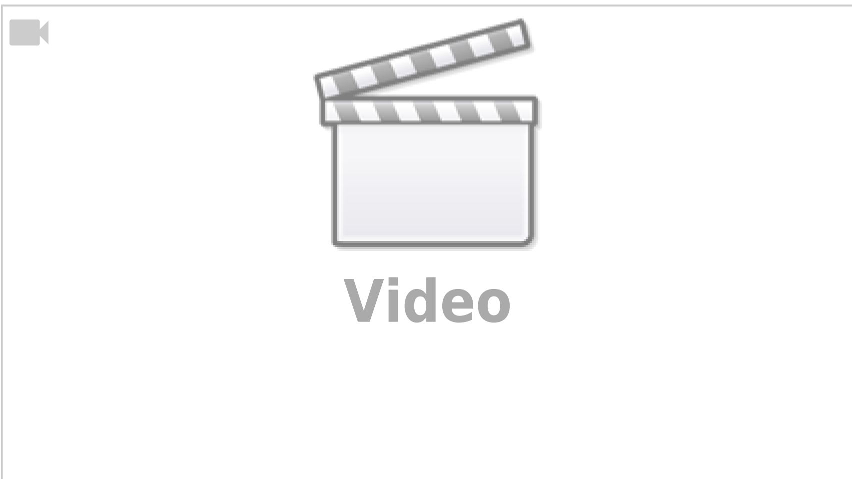
- Ferromagnetic materials strengthen the magnetic field strongly, compared to the vacuum.
- The strengthening can create a field multiple times stronger than in a vacuum.
- For ferromagnetic materials applies $\mu_{\rm r} \gg 1$
- The principle behind the effect is based on quantum mechanics:
    - Also here internal “tiny magnets” based on the electrons in their orbitals are oriented based on the external field.
    - However, these “tiny magnets” interact with each other in such a way that the field becomes stable even without the external field.
    - Ferromagnetic materials can be permanently magnetized. They build up **permanent magnets**.
    - The magnetic flux density of the ferromagnetic matter is depending on the external field strength and the history of the material.
    - The dependency between $\mu_{\rm r}$ and the external field strength H is nonlinear.
- Ferromagnetic materials are characterized by the magnetization curve (see figure 19)
    - Non-magnetized ferromagnets are located in the origin.
    - With an external field H the initial magnetization curve (in German: *Neukurve*, dashed in figure 19) is passed.
    - Even without an external field ($H=0$) and the internal field is stable.
      The stored field without external field is called **remanence** $B(H=0) = B_{\rm R}$ (or remanent magnetization).

  - In order to eliminate the stored field the counteracting **coercive field strength** $H_{\rm C}$ (also called coercivity) has to be applied.
  - The **saturation flux density** $B_{\rm sat}$ is the maximum possible magnetic flux density (at the maximum possible field strength $H_{\rm sat}$)

Fig. 19: Magnetization Curve



Explanation of the hysteresis curve



The ferromagnetic materials can again be subdivided into two groups: magnetically soft and magnetically hard materials.

magnetically soft ferromagnetic Materials

- high permeability
- high saturation flux density $B_{\rm sat}$ (= good magnetic conductor)
- small coercive field strength $H_{\rm C} < 1 ~\rm kA/m$ (= easy to reverse the magnetization)
- low magnetic losses for reversing the magnetization
- Important materials: ferrites

Applications:

- mainly for coil core material
- rotor and stator material for electric motors
- relays, contactors, transformers, compact antennas

Magnetically hard ferromagnetic Materials

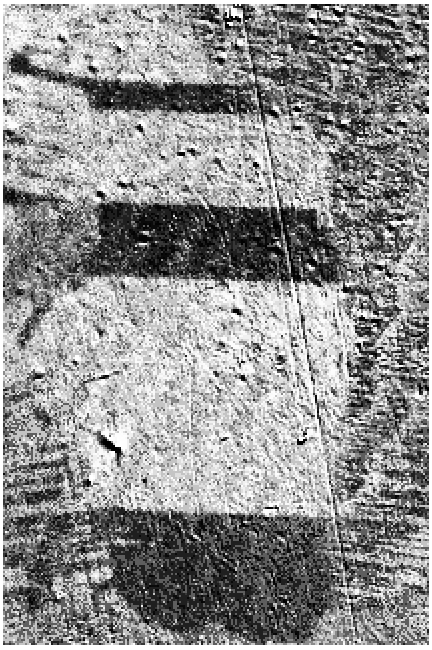
- low permeability
- low saturation flux density $B_{\rm sat}$ (= good magnetic conductor)
- high coercive field strength $H_{\rm C}$ up to $2'700~\rm kA/m$ (= easy to reverse the magnetization)
- high magnetic losses for reversing the magnetization
- Important materials: ferrites, alloys of iron and cobalt: $\rm AlNiCo$[2], $\rm SmCo$[3], $\rm NdFeB$[4]

Applications:

- mainly for permanent magnets
- permanent electric motors
- loudspeakers and microphones
- mechanical actuators

Fig. 13: Magnetization Curve of soft and hard magnetic materials

Wandering magnetic domains in a ferromagnetic material when the external field is reversed (from Zureks@en.wikipedia.org).



3.5 Poynting Vector (not part of the curriculum)

- Clear picture of the Poynting vector along an electric circuit: https://de.cleanpng.com/png-jyy1vj/
- Good explanation of the Energy flow via a current model: http://amasci.com/elect/poynt/poynt.html
- Very detailed view of the energy flow in an electric circuit: http://sharif.edu/~aborji/25733/files/Energy%20transfer%20in%20electrical%20circuits.pdf

Tasks

Task 3.2.1 Magnetic Field Strength around a horizontal straight Conductor

The current $I_0 = 100~\rm A$ flows in a long straight conductor with a round cross-section. The conductor shall have constant electric properties everywhere. The radius of the conductor is $r_{\rm L}= 4~\rm mm$.

1. What is the magnetic field strength H_1 at a point P_1, which is <u>outside</u> the conductor at a distance of $r_1 = 10~\rm mm$ from the conductor axis?

Path

- The H-field is given as: the current I through an area divided by the "specific" length l of the closed path around the area. This shall give you the formula (when not in already known)
- The relevant current is the given I_0.

Solution

The H-field is given as:

$$\begin{align*} H(r) &= {{I_0}\over{2\pi \cdot r}} \\ H(10~\rm mm) &= {{100~\rm A}\over{2\pi \cdot 0.1 ~\rm m}} \\ \end{align*}$$

$$\begin{align*} H(10~\rm mm) &= 159.15...~\rm{{A}\over{m}} \\ \rightarrow H(3~\rm mm) \end{align*}$$

2. What is the magnetic field strength H_2 at a point P_2, which is <u>inside</u> the conductor at the distance of $r_2 = 3~\rm mm$ from the conductor axis?

Path

- Again, the H-field is given as: the current I through an area divided by the "specific" length l of the closed path around the area.
- Here, the relevant current is **not** the given one. There is only a fraction of the current flowing through the part of the conductor inside the r_2.

Solution

But now I is not I_0 anymore, but only a fraction, so ΔI. I_0 is evenly distributed over the cross-section A of the conductor. The cross-sectional area is given as $A= r^2 \cdot \pi$

So the current ΔI is given as: current divided by the full area and then times the fractional area:

$$\begin{align*} \Delta I &= I_0 \cdot {{r_2^2 \cdot \pi}\over{r_{\rm L}^2 \cdot \pi}} \\ &= I_0 \cdot {{r_2^2 }\over{r_{\rm L}^2 }} \end{align*}$$

Therefore, the H-field is:

$$\begin{align*} H_2(r) &= {{\Delta I}\over{2\pi \cdot r_2}} &&= {{I_0 \cdot {{ r_2^2}\over{r_{\rm L}^2}} }\over{2\pi \cdot r_2}} \\ &= {{I_0 \cdot {{ r_2}\over{r_{\rm L}^2}} }\over{2\pi}} &&= {{1}\over{2\pi}} I_0 \cdot {{ \end{align*}$$

r_2}\over{r_{\rm L}^2}} \end{align*}

Task 3.2.2 Superposition

Fig. 21: Conductor Arrangement

2. Now, the current in one of the conductors is reversed. To which value does the magnetic field strength $H({\rm P})$ change?

Result

Solution

Path

\begin{align*} H &= 0 ~\rm{A}\over{m}} \end{align*} \rightarrow H &= 31.8 ~\rm{A}\over{m}} \end{align*}

In detail, the H-field of the single conductor is given as: \begin{align*} H &= {{I}\over{2\pi\cdot d}} \\ &= {{2 ~\rm A}\over{2\pi\cdot 0.02 ~\rm m}} \end{align*}

The H-field of the single reversed conductor I_3 is given as: \begin{align*} H &= H(I_3) \\ &= {{I}\over{2\pi \cdot r}} \\ &= {{2 ~\rm A}\over{2\pi \cdot 0.02 ~\rm m}} \\ \end{align*}

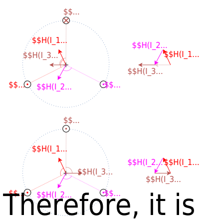
- Now, the formula for a single wire has to be used to calculate the field of a single conductor
- For the resulting field, the single wire fields again have to be superimposed.
- The symmetry and the result of question 1 give a strong hint about how much stronger the resulting field has to be compared to the field of the reversed one.

- However, even without calculation, the constant distance between point $\rm P$ and the three conductors dictates, that the H-field has a similar magnitude.
- By the symmetry of the conductor, the angles of the H-field vectors are defined and evenly distributed on the revolution.

Fig. 2: Conductor Arrangement

Once again, one can try to sketch the situation of the field vectors:

Fig. 2: Conductor Arrangement



Therefore, it is visible, that the resulting field is twice the value of $H(I_3)$:
The vectors of $H(I_1)$ plus $H(I_2)$ had in the task 1 just the length of $H(I_3)$.

Three long straight conductors are arranged in a vacuum to lie at the vertices of an equilateral triangle (see figure 21). The radius of the circumcircle is $r = 2 ~\rm cm$; the current is given by $I = 2 ~\rm A$.

1. What is the magnetic field strength $H({\rm P})$ at the center of the equilateral triangle?

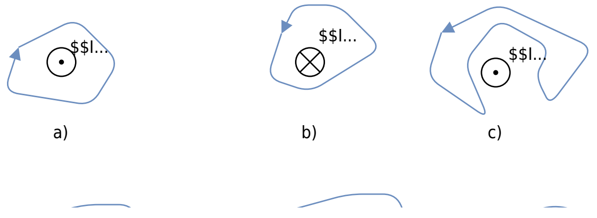
Path

- The formula for a single wire can calculate the field of a single conductor.
- For the resulting field, the single wire fields have to be superimposed.
- Since it is symmetric the resulting field has to be neutral.

Task 3.2.3 Magnetic Voltage

Fig. 10: different trajectories around current-carrying conductors
Result a)

a) $\theta_\rm d = 0 ~\rm A$ … = - 2.5~\rm A$



Given are the adjacent closed trajectories in the magnetic field of current-carrying conductors (see figure 10). Let $I_1 = 2~\rm A$ and $I_2 = 4.5~\rm A$ be valid.

In each case, the magnetic voltage $V_{\rm m}$ along the drawn path is sought.

Path

- The magnetic voltage is given as the **sum of the current through the area within a closed path**.
- The direction of the current and the path have to be considered with the righthand rule.

Task 3.3.1 magnetic Flux Density

[illegible]

1. For comparison, the same flux density shall be created inside a toroidal coil with $10'000$ windings and a toroidal diameter for the average field line of $d = 1~\rm m$.
How much current I is necessary for one of the windings of the toroidal coil?

Solution

[illegible]

The B-field is given as: \begin{align*} B &= \mu \cdot H \\ &= \mu \cdot {{I \cdot N}\over{l}} \\ \end{align*}

Path

[illegible]

This can be rearranged to the current I: \begin{align*} I &= {{B \cdot l}\over{\mu \cdot N}} \\ &= {{1.2 ~\rm T \cdot 1 ~\rm m}\over{10'000 \cdot 4\pi\cdot 10^-7 {\rm{Vs}\over{Am}} \cdot 10'000}} \end{align*}

- The B-field can be calculated by the H-field.
- The H-field is given as: the current I through an area divided by the "specific" length l of the closed path around the area. This shall give you the formula (when not already known)
- The current is number of windings times I.

Task 3.3.2 Electron in Plate Capacitor with magnetic Field

An electron shall move with the velocity $\vec{v}$ in a plate capacitor parallel to the plates, which have a potential difference U and a distance d. In the vacuum in between the plates acts additionally a magnetic field $\vec{B}$.

Fig. 25: Electron in B- and E-Field

Calculate the velocity depending on the other parameters $\vec{v} = f(U, |\vec{B}|, d)$!

Solution

Within the electric field, the Coulomb force acts on the electron:

$$\begin{align*} \vec{F}_C = q_e \cdot \vec{E} \end{align*}$$

Within the magnetic field, also the Lorentz force acts on the electron:

$$\begin{align*} \vec{F}_L = q_e \cdot \vec{v} \times \vec{B} \end{align*}$$

The absolute value of both forces must be equal to compensate each other:

$$\begin{align*} |\vec{F}_C| &= |\vec{F}_L|\\ |q_e \cdot \vec{E}| &= |q_e \cdot \vec{v} \times \vec{B}| \\ q_e \cdot |\vec{E}| &= q_e \cdot |\vec{v} \times \vec{B}| \\ |\vec{E}| &= |\vec{v} \times \vec{B}| \\ \end{align*}$$

Since $\vec{v}$ is perpendicular to $\vec{B}$ the right side is equal to $|\vec{v}| \cdot |\vec{B}| = v \cdot B$.
Additionally, for the plate capacitor $|\vec{E}|= U/d$.

Therefore, it leads to the following:

$$\begin{align*} {{U}\over{d}} &= v \cdot B \\ v &= {{U}\over{B \cdot d}} \end{align*}$$

Result

$$\begin{align*} v = {{U}\over{B\cdot d}} \end{align*}$$

Task 1

1. Which hand can be used to infer magnetic field direction from currents?

- ○ The right hand
- ○ The left hand

Tip for 1

For the current, you use which hand?

2. In the derivation from 1. how are the fingers to be assigned?

- ○ Thumb for current direction, remaining fingers for magnetic field direction
- ○ Thumb for magnetic field direction, remaining fingers for current direction
- ○ both possibilities are correct

Tip for 2

- Imagine a coil with a winding pictorially, or draw it on.
- Now think of a generated field through this to it. What direction must the current flow, that causes the field? Does this fit the rule of thumb?
- Then try it the other way round: If a current is given, where do the field lines go in and where out? What poles are created there?

3. Two conductors carrying current are parallel and close to each other. The current in both is flowing in the same direction. What force effect can be seen?

- ○ none
- ○ The conductors attract
- ○ The conductors repel

Tip for 3

See 3rd video.

- Picture the two wires, or draw them on.
- In which direction would the outer field run in each case?
- The field is a linear vector field. So the total field can be created from several individual fields by adding them together. Does adding the field in between make it larger, or smaller?

4. Two conductors carrying current are at right angles to each other. Current flows through both of them. What force effect can be seen?

- ○ none
- ○ The conductors attract
- ○ The conductors repel

5. What is the magnetic field inside the earth or a permanent magnet?

- ○ from the magnetic north pole to the

south pole
- ○ from the magnetic south pole to the north pole
- ○ the inside is free of field

6. At which location of a current-carrying coil are the field lines densest?

- ○ at the magnetic north pole
- ○ at the magnetic south pole
- ○ inside the coil
- ○ at both poles

Check Answers
You Scored % - /

Tip for 4

- First imagine the parallel wires again. What happens when the current flows in the same direction and what happens when the current flows in opposite directions? Are the resulting forces equal in magnitude?
- The reversal of the direction of the current can now also be produced by turning the wire instead of changing the current - just so that the wires are perpendicular to each other in the meantime when turning.
- With parallel wires and different current directions, the amount-wise same force arises. So, this is also true for every angle in between (in detail given by integration of the force over single wire pieces).
- But then there must be a point at which the force becomes 0.

Tip for 5

- The magnetic field lines must be closed.
- Compare the field curve between the coil and permanent magnet.

Tip for 6

- In video 1 you can see the course outside and inside the coil.

References to the media used

Element	License	Link
figure 21	CC-BY-SA 3.0	https://en.wikipedia.org/wiki/Magnetite?oldformat=true
figure 2	Public Domain	https://commons.wikimedia.org/wiki/File:Magnetic_field_of_bar_magnets_attracting.png
figure 10	CC-BY-SA 3.0	https://commons.wikimedia.org/wiki/File:VFPt_Solenoid_correct.svg

[1] ideally: infinite long; in reality much longer, than the distance between them

[2] Aluminum-Nickel-Cobalt

[3] Samarium-Cobalt

[4] Neodymium-Iron-Bor