

Exam Winter Semester 2022

Student Group

First Name	Surname	Matrikel Nr.

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Exercise E4 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a diameter of $d = 0.357 \text{ mm}$ and a length of $l = 3 \text{ m}$ is used for heating elements. The electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.

Determine the current I and the operating voltage U for heating elements.

The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m}$.

The heating element is $l = 3 \text{ m}$ long and has a diameter of $d = 0.357 \text{ mm}$.

∴ Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} \\ I &= \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{l}{\frac{1}{4} d^2 \cdot \pi} \quad | \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \text{ m}}{(0.357 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a diameter of $d = 0.357 \text{ mm}$ and a length of $l = 3 \text{ m}$ is used for heating elements. The electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.

Determine the current I and the operating voltage U for heating elements.

The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m}$.

The heating element is $l = 3 \text{ m}$ long and has a diameter of $d = 0.357 \text{ mm}$.

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Solution

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Exercise E1 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator exhibits a temperature coefficient of resistance in a refrigeration system. The refrigerator has a resistance of $10 \text{ k}\Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermal resistor at -40°C .

The power transfer resistor P is a part of the circuit and generates heat. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right)$$

**Exercise E2 Temperature-dependent Resistance
(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator exhibits a temperature coefficient of resistance in a refrigeration system. The refrigerator has a resistance of $10 \text{ k}\Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermal resistor at -40°C .

The power transfer resistor P is a part of the circuit and generates heat. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right)$$

Exercise E6 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at once, $R_2 = R_3 = 100 \Omega$ and the switch shall be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution

$$R_{eq} = 133.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

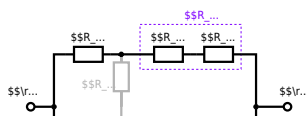
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel (R_Y + R_2) \parallel R_4$$

.. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel (500 \Omega) \parallel (200 \Omega) \parallel (500 \Omega \cdot 200 \Omega) / (500 \Omega + 200 \Omega)$$

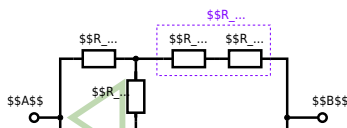
Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with $R_1 = 200 \Omega$, $R_2 = R_3 = 100 \Omega$ and the source $B = 10 \text{ V}$.
 Result given: $R_{\text{eq}} = 132.8 \Omega$.

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.

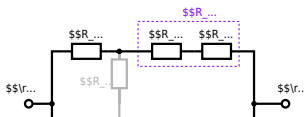


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \, \Omega + 200 \, \Omega + 200 \, \Omega) \parallel (100 \, \Omega + 100 \, \Omega) \parallel R_{\text{eq}} = (500 \, \Omega) \parallel (200 \, \Omega) \parallel R_{\text{eq}} = \frac{500 \, \Omega \cdot 200 \, \Omega}{500 \, \Omega + 200 \, \Omega} \parallel$$

Exercise E1 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

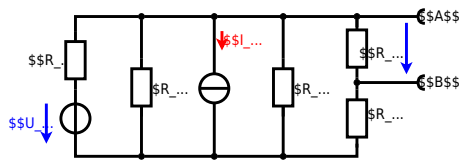
$$U_{\text{S}} = U_{\text{AB}} = 4.5 \, \text{V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \, \Omega$$



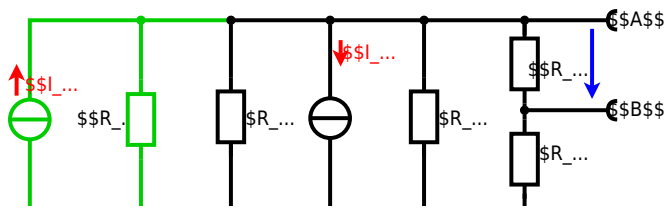
Calculate the internal resistance R_{in} and the source voltage U_{s} of an equivalent linear voltage source on the connectors A and B .
 $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \Omega$, $I_4=4.2 \text{ A}$,
 $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_2}{R_1} - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24}$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5\Omega || 10\Omega || 10\Omega = 5\Omega || 5\Omega = 2.5\Omega$:

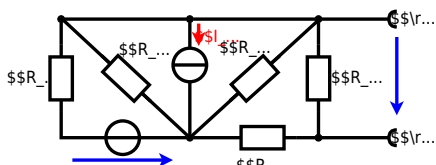
$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega}$$

$$R_{AB} = 15\Omega || (7.5\Omega + 2.5\Omega)$$

Exercise E3 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

$$U_s = U_{AB} = 4.5\text{V} \quad R_i = R_{AB} = 6\Omega$$



Calculated the internal resistance R_{int} and the source voltage U_s of an equivalent linear voltage source on the connectors A and B. $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot (R_6 || R_7)$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \left(\frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5} \right)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right) || R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E5 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a battery with an internal resistance of $R_1 = 1 \Omega$ and a charging capacitor $C = 2 \mu\text{F}$ connected in parallel with an open switch. The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

$$U_{eq} = \frac{U \cdot R_2}{R_1 + R_2} = \frac{12 \text{ V} \cdot 2 \Omega}{1 \Omega + 2 \Omega} = 8 \text{ V}$$

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U=12 \text{ V}$, a switch S_1 , a resistor of $R_1=20 \text{ }\Omega$ and a capacitor of $C=100 \text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0)=0 \text{ V}$.



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1)=0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1)=0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$. An equivalent linear voltage source can be given with $U_s = U \cdot \frac{R_B}{R_1 + R_B}$ and $R_i = R_1 \parallel R_B$ as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R_i = 0 \text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ ms} / (10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

Exercise E4 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (as shown in the figure) consists of a DC voltage source $U=6 \text{ V}$, a resistor $R_1=20 \text{ }\Omega$, a capacitor $C=20 \text{ }\mu\text{F}$ and a light bulb $R_B=20 \text{ }\Omega$. The switch S_1 is open. The voltage across the capacitor is again 0 V at the moment $t_0=0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1 \text{ ms}$ after closing the switch.

Solution To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 3 \text{ V}$$

$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

Solution

The ideal voltage source is $U = 12 \text{ V}$. The internal resistance is $R_1 = 20 \text{ }\Omega$. The voltage across the capacitor is $u_C(t)$. The voltage across the light bulb is $u_B(t)$. The voltage across the resistor R_2 is $u_{R_2}(t)$. The voltage across the capacitor is $u_C(t)$. The voltage across the light bulb is $u_B(t)$. The voltage across the resistor R_2 is $u_{R_2}(t)$.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0) = 0 \text{ V}$.

First do not consider the light bulb - it is not connected to the RC circuit.

Calculate the point of time t_1 when $u_C(t_1) = 0.5 \cdot U$.



Solution

An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \text{ }\Omega$, short-circuit). $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2 / (R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2 / (10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$: $u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$. It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

Exercise E2 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage \underline{U} and the phasor current \underline{I} through the components ($\$R\$ and $\$X_L\$) shall be given.$$

After analysis, the full width dimensioned complex impedance \underline{Z} can be extracted and the phase φ in phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right)$.

Solution
 .. Calculation of physical values of the two components.
 Solution
$$R = 0.24 \Omega \quad X_L = 2\pi \cdot 300 \text{ Hz} \cdot 4.68 \text{ mH} = 87.96 \Omega$$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \text{with } \underline{Z} = R + jX_L = 0.24 + j87.96 \Omega$$
 The current I and voltage U are in phase since Z is purely real (no imaginary part).
 The resulting impedance Z is $Z = 0.24 + j87.96 \Omega$.
 Therefore, the component 4.68 mH is in series with the 0.24Ω resistor.
 Impedance $Z = 0.24 + j87.96 \Omega$.

$$\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right) = \arctan\left(\frac{87.96}{0.24}\right) \approx 89.8^\circ$$
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right) = \arctan\left(\frac{87.96}{0.24}\right) \approx 89.8^\circ$.

Exercise E5 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage \underline{U} and the phasor current \underline{I} through the components ($\$R\$ and $\$X_L\$) shall be given.$$

After analysis, the full width dimensioned complex impedance \underline{Z} can be extracted and the phase φ in phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right)$.

Solution
 .. Calculation of physical values of the two components.
 Solution
$$R = 0.24 \Omega \quad X_L = 2\pi \cdot 300 \text{ Hz} \cdot 4.68 \text{ mH} = 87.96 \Omega$$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \text{with } \underline{Z} = R + jX_L = 0.24 + j87.96 \Omega$$
 The current I and voltage U are in phase since Z is purely real (no imaginary part).
 The resulting impedance Z is $Z = 0.24 + j87.96 \Omega$.
 Therefore, the component 4.68 mH is in series with the 0.24Ω resistor.
 Impedance $Z = 0.24 + j87.96 \Omega$.

$$\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right) = \arctan\left(\frac{87.96}{0.24}\right) \approx 89.8^\circ$$
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right) = \arctan\left(\frac{87.96}{0.24}\right) \approx 89.8^\circ$.

The absolute value of the impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$ with $R = 5 \Omega$, $X_L = \omega L = 2\pi \cdot 4 \text{ MHz} \cdot 100 \text{ nH} = 2.51 \text{ m}\Omega$ and $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 10 \text{ nF}} = 3.98 \text{ m}\Omega$.
 The phase ϕ is given by $\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{2.51 \text{ m}\Omega - 3.98 \text{ m}\Omega}{5 \Omega}\right) = -0.24 \text{ rad}$.
 With the complex part comes the physical value: $I = \frac{U}{Z} = \frac{10 \text{ V}}{\sqrt{5^2 + (2.51 - 3.98)^2}} = 1.97 \text{ A}$.
 The phase ϕ is $\phi = -0.24 \text{ rad} = -13.7^\circ$.

Exercise E3 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

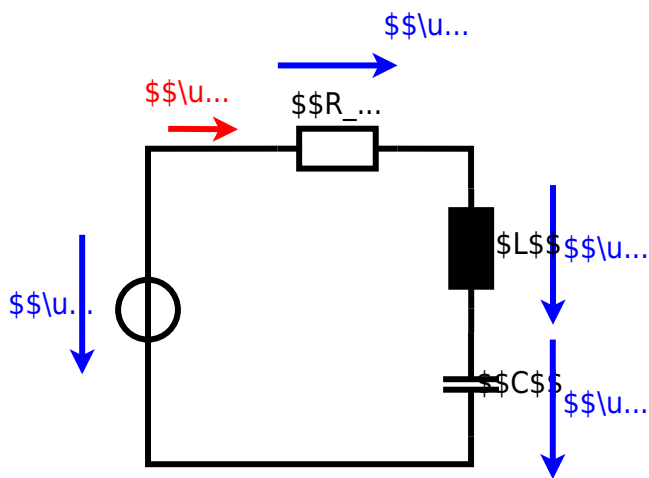
At a series circuit with a resistor $R_1 = 1 \text{ k}\Omega$, a capacitor $C_1 = 40 \text{ nF}$ and an inductor $L_1 = 4.7 \text{ }\mu\text{H}$ in AC with a voltage $U = 10 \text{ V}$ and a frequency $f = 4 \text{ MHz}$.
 Result: $Z = 1.00 \text{ }\Omega$, $I = 10 \text{ A}$, $\phi = 0^\circ$.
 A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution
 Solution $R_1 = 1.00 \Omega$
 Solution $R_2 = 10.0 \Omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z = \sqrt{R^2 + X_L^2}$.
 Parallel circuit means that the voltage is the same on R_2 and C_2 .
 The equivalent impedance for R_2 and C_2 combined is given by $Z = \frac{R_2 \cdot X_C}{\sqrt{R_2^2 + X_C^2}}$.
 Since Z is perpendicular to R_2 , this can be simplified to $Z = \frac{R_2 \cdot X_C}{X_C}$.
 Z is perpendicular to R_2 (It has to, since R_2 is perpendicular to X_C).
 Therefore, the resulting current of the parallel circuit is given as:
 $I = \frac{U}{Z} = \frac{U}{\frac{R_2 \cdot X_C}{\sqrt{R_2^2 + X_C^2}}} = \frac{U \cdot \sqrt{R_2^2 + X_C^2}}{R_2 \cdot X_C}$
 This can be rearranged to $I \cdot R_2 \cdot X_C = U \cdot \sqrt{R_2^2 + X_C^2}$
 $I^2 \cdot R_2^2 \cdot X_C^2 = U^2 \cdot (R_2^2 + X_C^2)$
 $I^2 \cdot R_2^2 \cdot X_C^2 - U^2 \cdot X_C^2 = U^2 \cdot R_2^2$
 $X_C^2 \cdot (I^2 \cdot R_2^2 - U^2) = U^2 \cdot R_2^2$
 $X_C^2 = \frac{U^2 \cdot R_2^2}{I^2 \cdot R_2^2 - U^2}$
 $X_C = \frac{U \cdot R_2}{\sqrt{I^2 \cdot R_2^2 - U^2}}$
 Back to the first formula: $R_3 \cdot I = X_C \cdot I$
 $R_3 = X_C = \frac{U \cdot R_2}{\sqrt{I^2 \cdot R_2^2 - U^2}}$

Exercise E6 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)





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Last update: **2023/04/02 00:28**

