

Exam Winter Semester 2022

Student Group

| First Name | Surname | Matrikel Nr. |
|------------|---------|--------------|
| | | |
| | | |
| | | |

Table of Contents

| | |
|---|----|
| Exercise E4 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022) | 3 |
| Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022) | 3 |
| Exercise E1 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022) | 4 |
| Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022) | 4 |
| Exercise E6 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022) | 5 |
| Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022) | 6 |
| Exercise E1 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022) | 8 |
| Exercise E3 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022) | 12 |
| Exercise E5 Charging Capacitors (written test, approx. 16 % of a 60-minute written test, WS2022) | 16 |
| Exercise E4 Charging Capacitors (written test, approx. 16 % of a 60-minute written test, WS2022) | 17 |
| Exercise E2 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022) | 19 |
| Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022) | 19 |
| Exercise E3 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022) | 20 |
| Exercise E6 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022) | 20 |
| Exercise E1 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022) | |

| | |
|--|----|
| test, WS2022) | 21 |
| Exercise E7 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022) | 24 |

G

Exercise E4 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of nichrome wire with a temperature coefficient of $1.80 \cdot 10^{-5} \text{ K}^{-1}$ is used for electric power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.

Calculate the current I needed to operate it for heating elements.

The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .

∴ Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \\ &= \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad | \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad | \quad R = \\ &= 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of nichrome wire with a temperature coefficient of $1.80 \cdot 10^{-5} \text{ K}^{-1}$ is used for electric power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.

Calculate the current I needed to operate it for heating elements.

The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .

∴ Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \\ &= \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad | \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad | \quad R = \\ &= 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

Exercise E1 Temperature-dependent Resistance
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator, explains a thermoelectric resistance in a refrigeration system. The thermistor has a resistance of $10 \text{ k}\Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result
 The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermistor at -40°C .

The resistance of the thermistor is $R = 6.5 \text{ k}\Omega$.

The resistance of the thermistor is $R = 6.5 \text{ k}\Omega$. Therefore, a solution is to use a heat sink to cool the thermistor.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

```
\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \\ \text{with } \Delta T &= T_{\text{end}} - T_{\text{start}} \\ R &= 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right) \\ &= 6.5 \text{ k}\Omega \end{align*}
```

Exercise E2 Temperature-dependent Resistance
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator, explains a thermoelectric resistance in a refrigeration system. The thermistor has a resistance of $10 \text{ k}\Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result
 The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermistor at -40°C .

The resistance of the thermistor is $R = 6.5 \text{ k}\Omega$.

The resistance of the thermistor is $R = 6.5 \text{ k}\Omega$. Therefore, a solution is to use a heat sink to cool the thermistor.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

```
\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \\ \text{with } \Delta T &= T_{\text{end}} - T_{\text{start}} \\ R &= 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right) \\ &= 6.5 \text{ k}\Omega \end{align*}
```

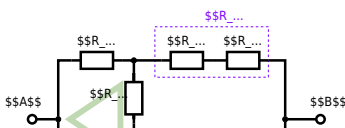
Exercise E6 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at once, $R_2 = R_3 = 100 \Omega$ and the switch is given. $R_1 = 400 \Omega$. Calculate the equivalent resistance R_{eq} between A and B .

Solution

$$R_{eq} = 133.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

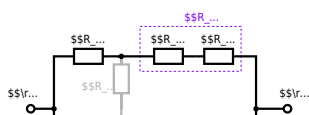
$$R_Y = \frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{100 \cdot 100}{100 + 100} = 50 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1) \parallel (R_Y + R_2) \parallel (R_Y + R_3 + R_4)$$

The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel (500 \Omega) \parallel (200 \Omega) \parallel (500 \Omega \cdot 200 \Omega) / (500 \Omega + 200 \Omega)$$

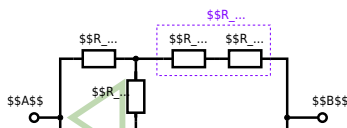
Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with $R_1 = 200 \Omega$, $R_2 = R_3 = 100 \Omega$ and the source $B = 10 \text{ V}$.
 Result given: $R_{\text{eq}} = 132.8 \Omega$.

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.

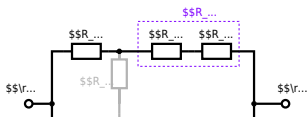


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E1 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

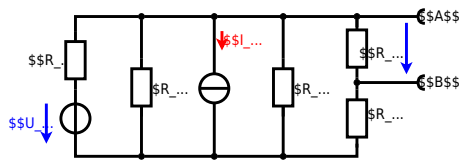
$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



Calculate the internal resistance R_{in} and the source voltage U_{s} of an equivalent linear voltage source on the connectors A and B .
 $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \Omega$, $I_4=4.2 \text{ A}$,
 $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot (R_6 || R_7)$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5\Omega || 10\Omega || 10\Omega = 5\Omega || 5\Omega = 2.5\Omega$:

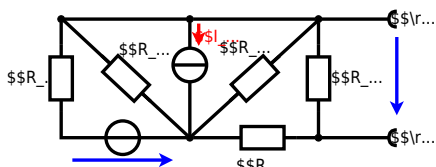
$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega}$$

$$R_{AB} = 15\Omega || (7.5\Omega + 2.5\Omega)$$

Exercise E3 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

$$U_s = U_{AB} = 4.5\text{V} \quad R_i = R_{AB} = 6\Omega$$



Calculated the internal resistance R_{int} and the source voltage U_{oc} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \text{ } \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \text{ } \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \text{ } \Omega$, $R_6=7.5 \text{ } \Omega$, $R_7=15 \text{ } \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} = \left(\frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5}$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} = \left(\frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} || R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E5 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a battery with an internal resistance of $R_1 = 5 \Omega$ and a charging capacitor $C = 2 \mu\text{F}$ connected in parallel with an open switch. The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

$$U_{eq} = \frac{U}{1 + \frac{R_1}{R_2}} = \frac{12 \text{ V}}{1 + \frac{5 \Omega}{10 \Omega}} = 8 \text{ V}$$

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U=12\text{ V}$, a switch S_1 , a resistor of $R_1=20\text{ }\Omega$ and a capacitor of $C=100\text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0\text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0)=0\text{ V}$.



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1)=0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1)=0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$
 An equivalent linear voltage source can be given with U_s , R_1 and R_B as seen in yellow:

$$e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) \implies t = R_1 \cdot C \cdot \ln(0.5)$$

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$
 The internal resistance is given by substituting the ideal voltage source with its resistance ($=0\text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1\text{ ms}/(10\text{ }\Omega \cdot 100\text{ }\mu\text{F})})$$

Exercise E4 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (as shown in the solution) consists of a DC voltage source $U=6\text{ V}$, a resistor $R_1=20\text{ }\Omega$, a capacitor $C=20\text{ }\mu\text{F}$ and a light bulb $R_B=20\text{ }\Omega$. The voltage across the capacitor is again 0 V at the moment $t_0=0\text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1\text{ ms}$ after closing the switch.

Solution To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

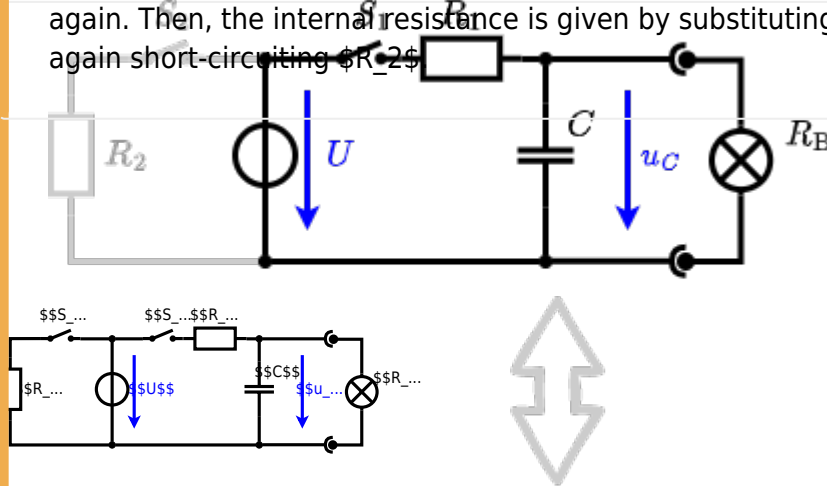
$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 3\text{ V}$$

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

Solution

The ideal voltage source is $U = 12 \text{ V}$. The internal resistance is $R_1 = 20 \text{ }\Omega$. The voltage across the capacitor is u_C . The voltage across the light bulb is u_B . The voltage across the resistor R_2 is u_{R_2} . The voltage across the capacitor is u_C . The voltage across the light bulb is u_B . The voltage across the resistor R_2 is u_{R_2} .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0) = 0$.

First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_C(t_1) = 0.5 \cdot U$.



Solution

An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \text{ }\Omega$, short-circuit). $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$.

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2/(10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$: $u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$. It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

Exercise E2 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source $\underline{U}_s = 50 \angle 0^\circ \text{ V}$ and the admittance $\underline{Y} = 0.24 \text{ S}$ are both in the components. ($\$R\$$ and $\$X_L\$$) shall be given.

After analysis, the full width dimensioned complex impedance \underline{Z} shall be extracted and the phase φ in phase (rad) shall be given. $\underline{Z} = (2 + j4) \Omega$

Solution
 .. Calculation of physical values of the two components.
 Solution $\underline{R} = \frac{1}{\text{Re}\{\underline{Y}\}} = \frac{1}{0.24} = 4.167 \Omega$ and $\underline{X}_L = \frac{1}{\text{Im}\{\underline{Y}\}} = \frac{1}{-0.06} = -16.667 \Omega$

Solution

```
\begin{align*} \underline{I} &= \frac{\underline{U}}{\underline{Z}} \quad \&= \{50 \angle 0^\circ \text{ V}\} \over \{ (2 + j4) \Omega \} \\ \text{The current } I & \text{ is } 12.5 \angle -63.4^\circ \text{ A} \\ \text{The voltage } U & \text{ is } 50 \angle 0^\circ \text{ V} \\ \text{The phase } \varphi & \text{ is } -63.4^\circ \end{align*}
```

Exercise E5 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source $\underline{U}_s = 50 \angle 0^\circ \text{ V}$ and the admittance $\underline{Y} = 0.24 \text{ S}$ are both in the components. ($\$R\$$ and $\$X_L\$$) shall be given.

After analysis, the full width dimensioned complex impedance \underline{Z} shall be extracted and the phase φ in phase (rad) shall be given. $\underline{Z} = (2 + j4) \Omega$

Solution
 .. Calculation of physical values of the two components.
 Solution $\underline{R} = \frac{1}{\text{Re}\{\underline{Y}\}} = \frac{1}{0.24} = 4.167 \Omega$ and $\underline{X}_L = \frac{1}{\text{Im}\{\underline{Y}\}} = \frac{1}{-0.06} = -16.667 \Omega$

Solution

```
\begin{align*} \underline{I} &= \frac{\underline{U}}{\underline{Z}} \quad \&= \{50 \angle 0^\circ \text{ V}\} \over \{ (2 + j4) \Omega \} \\ \text{The current } I & \text{ is } 12.5 \angle -63.4^\circ \text{ A} \\ \text{The voltage } U & \text{ is } 50 \angle 0^\circ \text{ V} \\ \text{The phase } \varphi & \text{ is } -63.4^\circ \end{align*}
```

The absolute value of the impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and the phase angle is $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$.
 With the complex part comes the physical value: $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$.
 The phase ϕ is given by $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$.

Exercise E3 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with $R_1 = 1.00 \text{ k}\Omega$, $R_2 = 4.70 \text{ k}\Omega$, $C_1 = 40 \text{ nF}$ and $L_1 = 4.70 \text{ }\mu\text{H}$ is connected to a voltage source $U = 10 \text{ V}$ at $f = 4 \text{ MHz}$.
 Result: $Z = 10.0 \text{ }\Omega$, $\phi = 90^\circ$.
 A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution

$$R_1 = 1.00 \text{ }\Omega$$

$$R_2 = 10.0 \text{ }\Omega$$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z_{RL} = \sqrt{R^2 + X_L^2}$.
 Parallel circuit means that the voltage is the same on R_2 and C_1 .
 $Z_{RC} = \frac{R_2 \cdot X_C}{\sqrt{R_2^2 + X_C^2}}$. Since X_C is perpendicular to R_2 , this can be simplified to $Z_{RC} = \frac{R_2 \cdot X_C}{X_C} = R_2$.
 X_C is perpendicular to Z_{RL} . (It has to, since R_2 is perpendicular to X_C .)
 Therefore, the resulting current of the parallel circuit is given as:

$$I = \frac{U}{\sqrt{R_2^2 + Z_{RL}^2}} = \frac{U}{\sqrt{R_2^2 + R^2 + X_L^2}}$$

 This can be rearranged to get $R = \sqrt{Z_{RC}^2 - X_L^2}$.
 Back to the first formula: $R_3 \cdot I_{3R} = X_{3C} \cdot I_{3C}$

$$R_3 = \frac{X_{3C} \cdot I_{3C}}{I_{3R}} = \frac{X_{3C} \cdot \frac{U}{\sqrt{R_3^2 + X_{3C}^2}}}{\frac{U}{\sqrt{R_3^2 + X_L^2}}}$$

Exercise E6 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

Resistor values $20 = 450 \text{ kHz}$ $4.7 \text{ } \mu\text{H}$ $30 \text{ } \mu\text{F}$ $330 \text{ } \mu\text{H}$ $10 \text{ } \mu\text{F}$ 10 mA
 A resistor R_1 shall have the same absolute value of the impedance as a capacitor

$C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution
 $R_1 = 1.00 \text{ } \Omega$
 $R_2 = 10.0 \text{ } \Omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by
 $Z = R + j\omega L$
 Parallel circuit means that the voltage is the same on R_1 and C_2
 $\frac{1}{Z} = \frac{1}{R_2} + \frac{1}{j\omega C_1}$
 $Z = \frac{R_2 \cdot j\omega C_1}{1 + j\omega C_1 R_2}$
 $|Z| = \frac{R_2 \cdot \omega C_1}{\sqrt{1 + (\omega C_1 R_2)^2}}$
 $I = \frac{U}{|Z|} = \frac{3.0 \text{ V}}{\frac{10 \text{ } \Omega \cdot 2\pi \cdot 15 \text{ kHz} \cdot 30 \text{ } \mu\text{F}}{\sqrt{1 + (2\pi \cdot 15 \text{ kHz} \cdot 30 \text{ } \mu\text{F} \cdot 10 \text{ } \Omega)^2}}}$
 $I = 60 \text{ mA}$
 Back to the first formula:
 $R_3 \cdot I = X_C \cdot I$
 $R_3 = \frac{1}{2\pi f C_3} \cdot \sqrt{I^2 - I_C^2}$

Exercise E1 Complex Impedance Circuit
 (written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the current $i(t)$ through the resistor R in the circuit shown below.
 The voltage source is $u(t) = 3.0 \text{ V} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$
 The circuit consists of a resistor $R = 10 \text{ } \Omega$, an inductor $L = 330 \text{ } \mu\text{H}$, and a capacitor $C = 10 \text{ } \mu\text{F}$ in series.

Result
 $Z = 48.2 \text{ } \Omega$ $Z_C = 19.8 \text{ } \Omega$
 $Z_L = j\omega L = j2\pi \cdot 15 \text{ kHz} \cdot 330 \text{ } \mu\text{H} = j31.1 \text{ } \Omega$
 $Z = R + jZ_L - Z_C = 10 \text{ } \Omega + j31.1 \text{ } \Omega - 19.8 \text{ } \Omega = 10.2 \text{ } \Omega + j11.3 \text{ } \Omega$
 $|Z| = \sqrt{10.2^2 + 11.3^2} = 15.4 \text{ } \Omega$
 $I = \frac{U}{|Z|} = \frac{3.0 \text{ V}}{15.4 \text{ } \Omega} = 195 \text{ mA}$
 $i(t) = 195 \text{ mA} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t - \phi)$
 $\phi = \arctan\left(\frac{Z_L - Z_C}{R}\right) = \arctan\left(\frac{11.3}{10.2}\right) = 48^\circ$





From:
<https://mexle.te.hs-heilbronn.de/> - **MEXLE Wiki**

Permanent link:
https://mexle.te.hs-heilbronn.de/electrical_engineering_1/ws2022_exam?rev=1680385439

Last update: **2023/04/01 23:43**

