

Exam Winter Semester 2022

Student Group

First Name	Surname	Matrikel Nr.

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Exercise E4 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a diameter of $d = 1.80 \text{ mm}$ is used for electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.

Determine the current I linked to the operating voltage for heating elements.

The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m}$.

The heating element is $l = 3 \text{ m}$ long and has a diameter of $d = 3.57 \text{ mm}$.

∴ Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \\ &= \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \text{and } R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \text{and } R = \\ &= 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a diameter of $d = 1.80 \text{ mm}$ is used for electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.

Determine the current I linked to the operating voltage for heating elements.

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Exercise E1 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator exhibits a temperature coefficient of resistance in the refrigeration system. The circuit has a resistance of $10 \text{ k}\Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermal circuit at -40°C .

The power transfer resistor P is a part of the circuit and generates heat. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right)$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator exhibits a temperature coefficient of resistance in the refrigeration system. The circuit has a resistance of $10 \text{ k}\Omega$ at 25°C .

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermal circuit at -40°C .

The power transfer resistor P is a part of the circuit and generates heat. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right)$$

Exercise E6 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at once, $R_2 = R_3 = 100 \Omega$ and the switch shall be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution

$$R_{eq} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

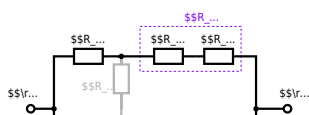
$$R_Y = \frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{100 \cdot 100}{100 + 100} = 50 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1) \parallel (R_Y + R_2) \parallel (R_Y + R_3)$$

.. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel (500 \Omega) \parallel (200 \Omega) \parallel (500 \Omega \cdot 200 \Omega) / (500 \Omega + 200 \Omega)$$

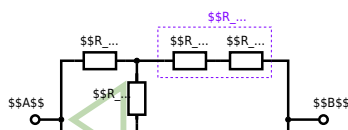
Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with $R_1 = 200 \Omega$, $R_2 = R_3 = 100 \Omega$ and the source $B = 10 \text{ V}$.
 Result given: $R_{\text{eq}} = 132.8 \Omega$.

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega}$$

**Exercise E1 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



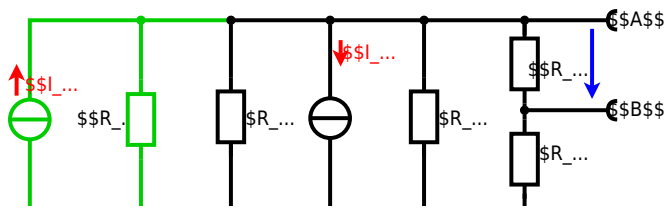
Calculate the internal resistance R_{in} and the source voltage U_{s} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3=10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$. Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135} \cdot \frac{R_6}{R_6 + R_7}$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - I_4 \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - (U_2 \cdot \frac{R_7}{R_1} - I_4) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

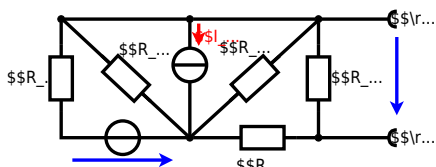
with $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \parallel R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

Exercise E3 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

$$U_s = U_{AB} = 4.5\text{V} \parallel R_i = R_{AB} = 6\Omega$$



Calculated the internal resistance R_{int} and the source voltage U_{oc} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \text{ } \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \text{ } \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \text{ } \Omega$, $R_6=7.5 \text{ } \Omega$, $R_7=15 \text{ } \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} = \left(\frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5}$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} = \left(\frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} || R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E5 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a battery with an internal resistance of $R_1 = 5 \Omega$ and a charging capacitor $C = 2 \mu\text{F}$ connected in parallel with an open switch. The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Result: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution: $U_{eq} = \frac{U \cdot R_2}{R_1 + R_2} = \frac{12 \text{ V} \cdot 2 \Omega}{5 \Omega + 2 \Omega} = 2 \text{ V}$

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U=12 \text{ V}$, a switch S_1 , a resistor of $R_1=20 \text{ }\Omega$ and a capacitor of $C=100 \text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0)=0 \text{ V}$.



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1)=0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1)=0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$
 An equivalent linear voltage source can be given with U_s , R_1 and R_B as seen in yellow:

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$$
 The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 \text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

Therefore, the voltage of the equivalent linear voltage source is:

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ ms} / (10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

Exercise E4 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (as shown in the solution) consists of a DC voltage source $U=6 \text{ V}$, a resistor $R_1=20 \text{ }\Omega$, a capacitor $C=20 \text{ }\mu\text{F}$, a resistor $R_2=10 \text{ }\Omega$ and a light bulb $R_B=10 \text{ }\Omega$. The voltage across the capacitor is again 0 V at the moment $t_0=0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1 \text{ ms}$ after closing the switch.

Solution To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 3 \text{ V}$$

$$R_i = R_1 \parallel R_B = 13.3 \text{ }\Omega$$

Solution

The ideal voltage source is $U = 12 \text{ V}$. The internal resistance is $R_1 = 20 \text{ }\Omega$. The voltage across the capacitor is $u_C(t)$. The voltage across the light bulb is $u_B(t)$. The voltage across the resistor R_2 is $u_{R_2}(t)$. The voltage across the capacitor is $u_C(t)$. The voltage across the light bulb is $u_B(t)$. The voltage across the resistor R_2 is $u_{R_2}(t)$.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0) = 0 \text{ V}$.

First do not consider the light bulb - it is not connected to the RC circuit.

Calculate the point of time t_1 when $u_C(t_1) = 0.5 \cdot U$.



Solution

An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \text{ }\Omega$, short-circuit). $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2 / (R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2 / (10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$: $u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$. It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) \implies t = R_1 \cdot C \cdot \ln(0.5)$

Exercise E2 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage \underline{U} and the phasor current \underline{I} are both through the components ($\$R\$ and $\$X_L\$) shall be given.$$

After analysis, the full width dimensioned complex impedance \underline{Z} shall be extracted and the phase φ in phase (in $^\circ$) shall be given.

Solution
.. Calculation of physical values of the two components.
Solution
$$R = 0.24 \Omega \quad X_L = 2\pi \cdot 300 \text{ Hz} \cdot 4.68 \text{ mH} = 87.96 \Omega$$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \varphi = \arctan\left(\frac{\text{Im}(\underline{Z})}{\text{Re}(\underline{Z})}\right) = \arctan\left(\frac{87.96}{0.24}\right) = 87.96^\circ$$

The current I and voltage U are in phase since $\varphi = 0^\circ$ (pure real) resulting impedance $\underline{Z} = 0.24 + j87.96 \Omega$.
Therefore, the component 4.68 mH is in series with the same as the voltage 4.68 mH impedance $\underline{Z} = 0.24 + j87.96 \Omega$.
$$\underline{Z} = 0.24 + j87.96 \Omega$$

The phase φ shall be calculated as
$$\varphi = \arctan\left(\frac{\text{Im}(\underline{Z})}{\text{Re}(\underline{Z})}\right) = \arctan\left(\frac{87.96}{0.24}\right) = 87.96^\circ$$

With the complex part $\varphi = 87.96^\circ$
$$\varphi = \arctan\left(\frac{87.96}{0.24}\right) = 87.96^\circ$$

The phase φ shall be calculated as
$$\varphi = \arctan\left(\frac{\text{Im}(\underline{Z})}{\text{Re}(\underline{Z})}\right) = \arctan\left(\frac{87.96}{0.24}\right) = 87.96^\circ$$

Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage \underline{U} and the phasor current \underline{I} are both through the components ($\$R\$ and $\$X_L\$) shall be given.$$

After analysis, the full width dimensioned complex impedance \underline{Z} shall be extracted and the phase φ in phase (in $^\circ$) shall be given.

Solution
.. Calculation of physical values of the two components.
Solution
$$R = 0.24 \Omega \quad X_L = 2\pi \cdot 300 \text{ Hz} \cdot 4.68 \text{ mH} = 87.96 \Omega$$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \varphi = \arctan\left(\frac{\text{Im}(\underline{Z})}{\text{Re}(\underline{Z})}\right) = \arctan\left(\frac{87.96}{0.24}\right) = 87.96^\circ$$

The current I and voltage U are in phase since $\varphi = 0^\circ$ (pure real) resulting impedance $\underline{Z} = 0.24 + j87.96 \Omega$.
Therefore, the component 4.68 mH is in series with the same as the voltage 4.68 mH impedance $\underline{Z} = 0.24 + j87.96 \Omega$.
$$\underline{Z} = 0.24 + j87.96 \Omega$$

The phase φ shall be calculated as
$$\varphi = \arctan\left(\frac{\text{Im}(\underline{Z})}{\text{Re}(\underline{Z})}\right) = \arctan\left(\frac{87.96}{0.24}\right) = 87.96^\circ$$

The absolute value of the impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and the phase angle is $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$.
 With the complex part comes the physical value: $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$.
 The phase ϕ is given by $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$.

Exercise E3 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with $R_1 = 1.00 \text{ k}\Omega$, $R_2 = 4.70 \text{ k}\Omega$, $L = 4.70 \text{ mH}$ and $C = 100 \text{ nF}$ at $f = 4 \text{ MHz}$.
 Result: $Z = 10.0 \text{ k}\Omega$, $\phi = 10.0^\circ$.
 A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution

$$R_1 = 1.00 \text{ k}\Omega$$

$$R_2 = 4.70 \text{ k}\Omega$$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$.
 Parallel circuit means that the voltage is the same on R_2 and C_1 .
 The equivalent impedance for R_2 and C_1 combined is given by $Z_{parallel} = \frac{R_2 \cdot X_{C1}}{R_2 + X_{C1}}$.
 Since X_{C1} is perpendicular to R_2 , this can be simplified to $Z_{parallel} = \frac{R_2 \cdot X_{C1}}{\sqrt{R_2^2 + X_{C1}^2}}$.
 This is perpendicular to Z_{series} . (It has to, since R_2 is perpendicular to X_{C1} .)
 Therefore, the resulting current of the parallel circuit is given as:

$$I_{parallel} = \frac{U}{Z_{parallel}} = \frac{U \cdot \sqrt{R_2^2 + X_{C1}^2}}{R_2 \cdot X_{C1}}$$

 This current is the same as the current through R_1 .

$$I_{parallel} = I_{R1} = \frac{U}{Z_{series}} = \frac{U}{\sqrt{R_1^2 + (X_L - X_C)^2}}$$

 Back to the first formula: $R_1 \cdot I_{parallel} = X_{C1} \cdot I_{parallel}$

$$R_1 = X_{C1} = \frac{1}{\omega C_1} = \frac{1}{2\pi \cdot 4 \cdot 10^6 \cdot C_1}$$

Exercise E6 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)



Exercise E7 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the current $i(t)$ through the resistor R in the circuit shown in Figure 1. The voltage source is $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$ V. The circuit consists of a resistor $R = 10 \Omega$, an inductor $L = 330 \mu\text{H}$, and a capacitor $C = 0.22 \mu\text{F}$, all in series.

Result: $Z = 19.8 \Omega$

Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.

$$Z = \frac{U}{I} \quad I = \frac{U}{Z} \quad Z_C = \frac{1}{2\pi \cdot f \cdot C} = \frac{1}{2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} \approx 391.5 \Omega$$

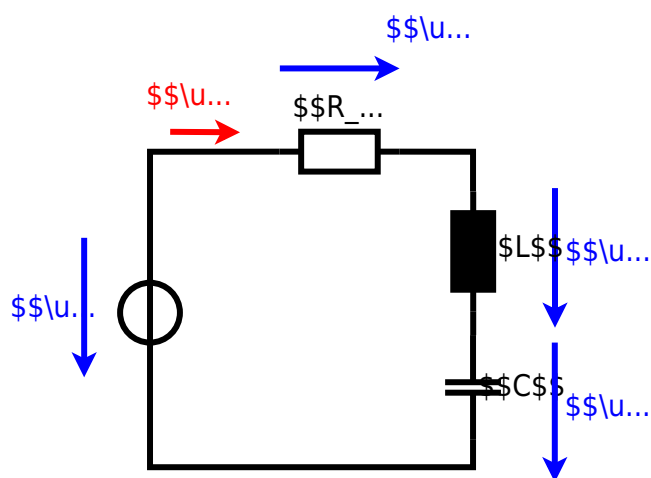
$$Z_L = 2\pi \cdot f \cdot L = 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} \approx 3.14 \Omega$$

$$Z = \sqrt{R^2 + (Z_L - Z_C)^2} = \sqrt{10^2 + (3.14 - 391.5)^2} \approx 391.5 \Omega$$

$$\underline{Z} = R + j(Z_L - Z_C) = 10 + j(3.14 - 391.5) \approx 10 - j388.4 \Omega$$

$$|\underline{Z}| = \sqrt{R^2 + (Z_L - Z_C)^2} = \sqrt{10^2 + 388.4^2} \approx 388.4 \Omega$$





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Last update: **2023/04/01 12:50**

