

# Exam Winter Semester 2022

## Student Group

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### Exercise E4 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a diameter of  $1.80 \text{ mm}$  is used for electric power dissipation (= heat flow) of  $P=40 \text{ W}$  is necessary.

Determine the current  $I$  linked to the operating voltage for heating elements.

The Nichrome wire has a resistivity of  $1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$ .

The heating element is  $3 \text{ m}$  long and has a diameter of  $3.57 \text{ mm}$ .

∴ Calculate the resistance  $R$  of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \\ &= \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad | \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad | \quad R = \\ &= 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

### Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a diameter of  $1.80 \text{ mm}$  is used for electric power dissipation (= heat flow) of  $P=40 \text{ W}$  is necessary.

Determine the current  $I$  linked to the operating voltage for heating elements.

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The heating element is  $3 \text{ m}$  long and has a diameter of  $3.57 \text{ mm}$ .

∴ Calculate the resistance  $R$  of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \\ &= \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad | \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad | \quad R = \\ &= 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

### Exercise E1 Temperature-dependent Resistance

**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator exhibits a temperature coefficient of resistance in its refrigeration system. The refrigerator has a resistance of  $10 \text{ k}\Omega$  at  $25^\circ\text{C}$ . Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

Result: The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

Calculate the resistance of the thermal system at  $-40^\circ\text{C}$ .

The power transfer resistor  $P$  is a part of the circuit and generates heat. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

```
\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad | \\ \text{with } \Delta T &= T_{\text{end}} - T_{\text{start}} \quad | \\ R &= 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right) \quad | \\ &\end{align*}
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**Exercise E2 Temperature-dependent Resistance  
(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator exhibits a temperature coefficient of resistance in its refrigeration system. The refrigerator has a resistance of  $10 \text{ k}\Omega$  at  $25^\circ\text{C}$ . Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

Result: The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

Calculate the resistance of the thermal system at  $-40^\circ\text{C}$ .

The power transfer resistor  $P$  is a part of the circuit and generates heat. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

```
\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad | \\ \text{with } \Delta T &= T_{\text{end}} - T_{\text{start}} \quad | \\ R &= 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right) \quad | \\ &\end{align*}
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**Exercise E6 Pure Resistor Network Simplification**  
**(written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved at once,  $R_2 = R_3 = 100 \Omega$  and the switch shall be open. Calculate the equivalent resistance  $R_{eq}$  between  $A$  and  $B$ .

Solution

$$R_{eq} = 133.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2 = R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

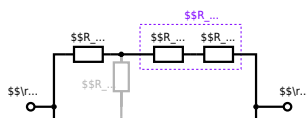
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel (R_Y + R_2) \parallel R_4$$

$$= 33.33 \Omega + (33.33 \Omega + 100 \Omega + 100 \Omega) \parallel (33.33 \Omega + 100 \Omega) \parallel (33.33 \Omega + 100 \Omega) \parallel 100 \Omega$$

The switch shall now be open. Calculate the equivalent resistance  $R_{eq}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim\Omega + 200 \sim\Omega + 200 \sim\Omega) \parallel (100 \sim\Omega + 100 \sim\Omega) \parallel R_{\text{eq}} = (500 \sim\Omega) \parallel (200 \sim\Omega) \parallel R_{\text{eq}} = \frac{500 \sim\Omega \cdot 200 \sim\Omega}{500 \sim\Omega + 200 \sim\Omega}$$

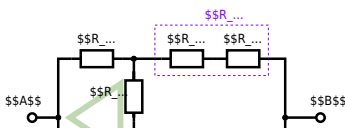
**Exercise E3 Pure Resistor Network Simplification**  
**(written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved with  $R_1 = 200 \Omega$ ,  $R_2 = R_3 = 100 \Omega$  and the switch  $S$  is given.  $R_{\text{eq}}$  is given.

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.

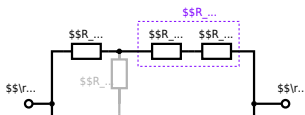


Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as: 
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series: 
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{eq}$  between A and B.

Solution



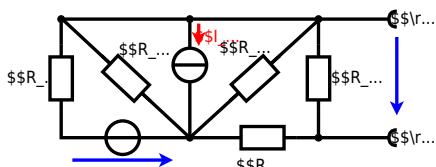
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \, \Omega + 200 \, \Omega + 200 \, \Omega) \parallel (100 \, \Omega + 100 \, \Omega) \parallel R_{\text{eq}} = (500 \, \Omega) \parallel (200 \, \Omega) \parallel R_{\text{eq}} = \frac{500 \, \Omega \cdot 200 \, \Omega}{500 \, \Omega + 200 \, \Omega} \parallel$$

### Exercise E1 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.  
Result

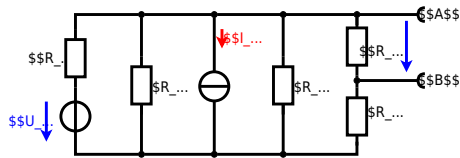
$$U_{\text{s}} = U_{\text{AB}} = 4.5 \, \text{V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \, \Omega$$



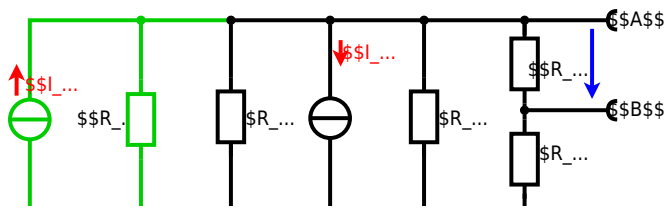
Calculate the internal resistance  $R_{in}$  and the source voltage  $U_{s}$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3=10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$ . Use equivalent sources in order to simplify the circuit!

### Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ : 
$$R_{135} = R_1 || R_3 || R_5$$
 
$$I_{24} = I_2 - I_4 = \frac{U_2}{R_1} - I_4$$
 The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following: 
$$U_{24}$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0\Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5\Omega || 10\Omega || 10\Omega = 5\Omega || 5\Omega = 2.5\Omega$ :

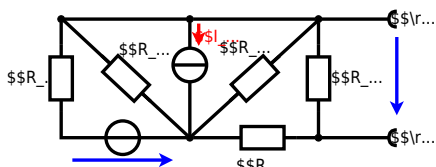
$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega}$$

$$R_{AB} = 15\Omega || (7.5\Omega + 2.5\Omega)$$

### Exercise E3 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.  
Result

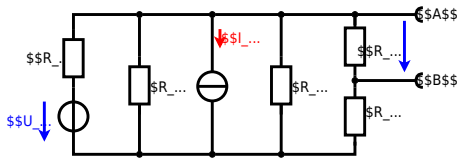
$$U_s = U_{AB} = 4.5\text{V} \quad R_i = R_{AB} = 6\Omega$$



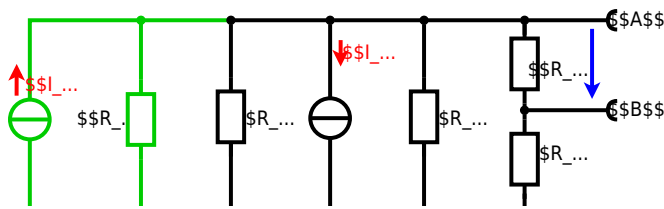
Calculate the internal resistance  $R_{in}$  and the source voltage  $U_{s}$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3=10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$ . Use equivalent sources in order to simplify the circuit!

### Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ : 
$$R_{135} = R_1 || R_3 || R_5$$
 
$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following: 
$$U_{24} = I_{24} \cdot R_{135} + I_1 \cdot R_2$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} = \left( \frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5}$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} = \left( \frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} || R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

**Exercise E5 Charging Capacitors**  
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a battery with an internal resistance of  $R_1 = 5 \Omega$  and a charging capacitor  $C = 2 \mu\text{F}$  connected in parallel with a switch  $S_1$ . The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution:** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

$$U_{eq} = \frac{U \cdot R_2}{R_1 + R_2} = \frac{12 \text{ V} \cdot 2 \Omega}{5 \Omega + 2 \Omega} = 2 \text{ V}$$

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .



The circuit contains a voltage source  $U=12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1=20 \text{ }\Omega$  and a capacitor of  $C=100 \text{ }\mu\text{F}$ . The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first task. At the moment  $t_0=0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0)=0 \text{ V}$ .



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1)=0.5 \cdot U$ .

Solution



So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1)=0.5 \cdot U$ :  

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \Rightarrow e^{-t/\tau} = 0.5 \Rightarrow t/\tau = \ln(0.5) \Rightarrow t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

An equivalent linear voltage source can be given with  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$  and  $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$ .  
 Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$ . The internal resistance is given by substituting the ideal voltage source with its resistance ( $=0 \text{ }\Omega$ , short-circuit).

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ ms} / (10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

**Exercise E4 Charging Capacitors**  
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (see the solution) consists of a  $12 \text{ V}$  DC voltage source, a  $20 \text{ }\Omega$  resistor, a  $100 \text{ }\mu\text{F}$  capacitor, a  $10 \text{ }\Omega$  resistor, and a light bulb. The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0=0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2=1 \text{ ms}$  after closing the switch.

**Solution** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_B$ .  

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U = 6 \text{ V}$$

$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

Solution

The ideal voltage source is  $U = 12 \text{ V}$ . The internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .



The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ }\Omega$  and a capacitor of  $C = 100 \text{ }\mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first asks. At the moment  $t_0 = 0$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0$ .

First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .



Solution

An equivalent linear voltage source can be given with  $U$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ . The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \text{ }\Omega$ , short-circuit).  $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2 / (R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2 / (10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ :  $u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$ . It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

### Exercise E2 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source  $\underline{U} = 50 \angle 0^\circ \text{ V}$  and the admittance  $\underline{Y} = 0.24 \text{ S}$  are both in the components. ( $\$R\$$  and  $\$X_L\$$ ) shall be given.

After analysis, the full width dimensioned complex impedance  $\underline{Z}$  shall be extracted and the magnitude  $|\underline{Z}|$  in phase (rad)  $\angle \varphi$  shall be given.  $\varphi = \varphi(\underline{Z}) = \varphi(\underline{U}) - \varphi(\underline{I})$

Solution  
.. Calculation of physical values of the two components.  
Solution  $\varphi = \varphi(\underline{Z}) = \varphi(\underline{U}) - \varphi(\underline{I}) = 0^\circ - \arctan\left(\frac{-4.68}{0.24}\right) = 87.06^\circ$

Solution  
 $\underline{I} = \frac{\underline{U}}{\underline{Z}} \Leftrightarrow \underline{Z} = \frac{\underline{U}}{\underline{I}} \Leftrightarrow \underline{Z} = \frac{50 \angle 0^\circ}{0.24 \angle -87.06^\circ} = 208.33 \angle 87.06^\circ \Omega$   
The current  $\underline{I}$  and voltage  $\underline{U}$  are in phase since  $\varphi(\underline{I}) = \varphi(\underline{U}) = 0^\circ$  (pure real) resulting in  $\underline{Z} = 208.33 \angle 87.06^\circ \Omega$ .  
Therefore, the component  $4.68 \text{ } \Omega$  is in series with the same admittance  $0.24 \text{ S}$  in parallel.  
Impedance  $\underline{Z} = \frac{1}{\frac{1}{4.68 \text{ } \Omega} + \frac{1}{\frac{1}{0.24 \text{ S}}}} = \frac{1}{\frac{1}{4.68} + 0.24} = 4.68 \text{ } \Omega$   
The phase  $\varphi$  shall be calculated as  $\varphi = \arctan\left(\frac{\text{Im}(\underline{Z})}{\text{Re}(\underline{Z})}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -87.06^\circ$   
With the complex part  $\varphi = \varphi(\underline{Z}) = \varphi(\underline{U}) - \varphi(\underline{I}) = 0^\circ - \varphi(\underline{I}) = \varphi(\underline{I})$   
 $\varphi = \frac{X_L}{R} = \frac{4.68 \text{ } \Omega}{0.24 \text{ S}} = 19.5 \text{ } \Omega$   
The phase  $\varphi$  shall be calculated as  $\varphi = \arctan\left(\frac{\text{Im}(\underline{Z})}{\text{Re}(\underline{Z})}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -87.06^\circ$

### Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source  $\underline{U} = 50 \angle 0^\circ \text{ V}$  and the admittance  $\underline{Y} = 0.24 \text{ S}$  are both in the components. ( $\$R\$$  and  $\$X_L\$$ ) shall be given.

After analysis, the full width dimensioned complex impedance  $\underline{Z}$  shall be extracted and the magnitude  $|\underline{Z}|$  in phase (rad)  $\angle \varphi$  shall be given.  $\varphi = \varphi(\underline{Z}) = \varphi(\underline{U}) - \varphi(\underline{I})$

Solution  
.. Calculation of physical values of the two components.  
Solution  $\varphi = \varphi(\underline{Z}) = \varphi(\underline{U}) - \varphi(\underline{I}) = 0^\circ - \arctan\left(\frac{-4.68}{0.24}\right) = 87.06^\circ$

Solution  
 $\underline{I} = \frac{\underline{U}}{\underline{Z}} \Leftrightarrow \underline{Z} = \frac{\underline{U}}{\underline{I}} \Leftrightarrow \underline{Z} = \frac{50 \angle 0^\circ}{0.24 \angle -87.06^\circ} = 208.33 \angle 87.06^\circ \Omega$   
The current  $\underline{I}$  and voltage  $\underline{U}$  are in phase since  $\varphi(\underline{I}) = \varphi(\underline{U}) = 0^\circ$  (pure real) resulting in  $\underline{Z} = 208.33 \angle 87.06^\circ \Omega$ .  
Therefore, the component  $4.68 \text{ } \Omega$  is in series with the same admittance  $0.24 \text{ S}$  in parallel.  
Impedance  $\underline{Z} = \frac{1}{\frac{1}{4.68 \text{ } \Omega} + \frac{1}{\frac{1}{0.24 \text{ S}}}} = \frac{1}{\frac{1}{4.68} + 0.24} = 4.68 \text{ } \Omega$   
The phase  $\varphi$  shall be calculated as  $\varphi = \arctan\left(\frac{\text{Im}(\underline{Z})}{\text{Re}(\underline{Z})}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -87.06^\circ$   
With the complex part  $\varphi = \varphi(\underline{Z}) = \varphi(\underline{U}) - \varphi(\underline{I}) = 0^\circ - \varphi(\underline{I}) = \varphi(\underline{I})$   
 $\varphi = \frac{X_L}{R} = \frac{4.68 \text{ } \Omega}{0.24 \text{ S}} = 19.5 \text{ } \Omega$   
The phase  $\varphi$  shall be calculated as  $\varphi = \arctan\left(\frac{\text{Im}(\underline{Z})}{\text{Re}(\underline{Z})}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -87.06^\circ$

The absolute value of the impedance is  $Z = \sqrt{R^2 + (X_L - X_C)^2}$  with  $R = 5 \Omega$ ,  $X_L = \omega L = 2\pi \cdot 50 \text{ Hz} \cdot 0.24 \text{ mH} = 0.754 \Omega$  and  $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \cdot 50 \text{ Hz} \cdot 4.68 \text{ nF}} = 680 \Omega$ .  
 With the complex part comes the physical value:  $X_L - X_C = \omega L - \frac{1}{\omega C} = 0.754 \Omega - 680 \Omega = -679.246 \Omega$ .  
 The phase  $\phi$  is  $\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{-679.246 \Omega}{5 \Omega}\right) = -89.98^\circ$ .

**Exercise E3 Impedances at different Frequencies**  
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with  $R_1 = 1 \text{ k}\Omega$ ,  $R_2 = 10 \text{ k}\Omega$ ,  $L = 4.7 \mu\text{H}$ ,  $C = 40 \text{ nF}$  and  $f = 4 \text{ MHz}$ , the voltage across the capacitor  $U_C = 10 \text{ V}$  is measured.  
 Result:  $I = 0.14 \text{ mA}$ ,  $R_3 = 10 \text{ k}\Omega$ .  
 A resistor  $R_1$  shall have the same absolute value of the impedance as a capacitor  $C_1 = 40 \text{ nF}$  at  $f_1 = 4 \text{ MHz}$ .

Solution  
 Solution:  $R_1 = 1.00 \Omega$   
 Solution:  $R_3 = 10.0 \Omega$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $L$  combined is given by  $Z = \sqrt{R^2 + X_L^2}$ .  
 Parallel circuit means that the voltage is the same on  $R_2$  and  $C_1$ .  
 $Z_{parallel} = \frac{R_2 \cdot X_C}{R_2 + X_C}$ . Since  $X_C$  is perpendicular to  $R_2$ , this can be simplified to  $Z_{parallel} = \frac{R_2 \cdot X_C}{\sqrt{R_2^2 + X_C^2}}$ .  
 $X_C$  is perpendicular to  $Z_{parallel}$ . (It has to, since  $R_2$  is perpendicular to  $X_C$ .)  
 Therefore, the resulting current of the parallel circuit is given as:  
 $I_{parallel} = \frac{U_C}{Z_{parallel}} = \frac{U_C \cdot \sqrt{R_2^2 + X_C^2}}{R_2 \cdot X_C}$   
 This can be rearranged to  $R_2 = \frac{I_{parallel} \cdot X_C}{U_C} \cdot \sqrt{R_2^2 + X_C^2}$ .  
 $R_2^2 = \frac{I_{parallel}^2 \cdot X_C^2}{U_C^2} \cdot (R_2^2 + X_C^2)$   
 $R_2^2 \cdot \left(1 - \frac{I_{parallel}^2 \cdot X_C^2}{U_C^2}\right) = \frac{I_{parallel}^2 \cdot X_C^4}{U_C^2}$   
 $R_2 = \frac{I_{parallel} \cdot X_C^2}{U_C \cdot \sqrt{1 - \frac{I_{parallel}^2 \cdot X_C^2}{U_C^2}}}$   
 Back to the first formula:  $R_3 \cdot I_{parallel} = X_C \cdot I_{parallel} \cdot \frac{R_2}{\sqrt{R_2^2 + X_C^2}}$   
 $R_3 = \frac{X_C \cdot R_2}{\sqrt{R_2^2 + X_C^2}} = \frac{X_C \cdot \frac{I_{parallel} \cdot X_C^2}{U_C \cdot \sqrt{1 - \frac{I_{parallel}^2 \cdot X_C^2}{U_C^2}}}}{\sqrt{R_2^2 + X_C^2}}$

**Exercise E6 Impedances at different Frequencies**  
 (written test, approx. 18 % of a 60-minute written test, WS2022)



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**Exercise E7 Complex Impedance Circuit**  
**(written test, approx. 15 % of a 60-minute written test, WS2022)**

1. Calculate the current  $i(t)$  in the circuit shown in Fig. 1. The voltage source is  $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V. The circuit consists of a resistor of  $10 \mu\Omega$ , an inductor of  $330 \mu\text{H}$ , and a capacitor of  $0.22 \mu\text{F}$ , all in series.

Result

$$Z = 19.8 \cdot 10^{-6} \Omega \quad |Z| = 19.8 \cdot 10^{-6} \Omega$$

Draw the circuit diagram of the given circuit.

$$Z = \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} \quad Z_C = \frac{1}{2\pi \cdot f \cdot C} = \frac{1}{2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}}$$

Result  $15 \text{ kHz} \cdot 0.22 \mu\text{F}$

$$\hat{I} = \frac{\hat{U}}{\sqrt{2}} = \frac{3.0}{\sqrt{2}} = 2.12 \text{ A} \quad \hat{I} = 2.12 \text{ A} \cdot \frac{1}{\sqrt{2}} = 1.5 \text{ A}$$

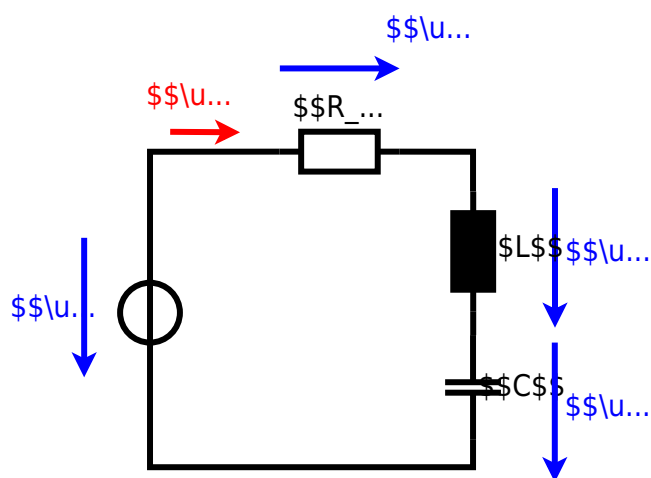
$$Z = \frac{1}{19.8 \cdot 10^{-6} \Omega} = 50505 \Omega \quad \hat{I} = \frac{3.0}{50505} = 5.94 \cdot 10^{-5} \text{ A}$$

$$\underline{Z} = R + \underline{Z}_L + \underline{Z}_C = R + j\omega L - j\omega C = R + j(\omega L - \omega C)$$

$$|\underline{Z}| = \sqrt{R^2 + (\omega L - \omega C)^2}$$

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