

Exam Winter Semester 2022

Student Group

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Exercise E4 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of nichrome wire with a temperature coefficient of $1.80 \cdot 10^{-5} \text{ K}^{-1}$ is used for electric power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.

Calculate the current I and the operating voltage U for heating elements.

The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .

∴ Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} \\ \sqrt{\frac{P}{R}} &= \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{l}{\frac{1}{4} d^2 \cdot \pi} \quad | \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

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Exercise E1 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator, explained with the effect of constant temperature, is used to cool down food. The food has a resistance of $10 \text{ k}\Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermistor at -40°C .

Resistance transfer relation: $P = U \cdot I$ and $P = \frac{U^2}{R}$. Therefore, a solution is to increase the resistance R up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

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$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator, explained with the effect of constant temperature, is used to cool down food. The food has a resistance of $10 \text{ k}\Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermistor at -40°C .

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$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

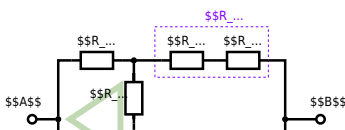
Exercise E6 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at once, $R_2 = R_3 = 100 \Omega$ and the switch shall be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution

$$R_{eq} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

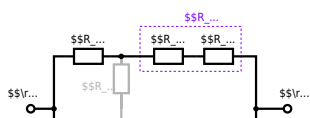
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel (R_Y + R_2)$$

.. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel (500 \Omega) \parallel (200 \Omega) \parallel (500 \Omega \cdot 200 \Omega) / (500 \Omega + 200 \Omega)$$

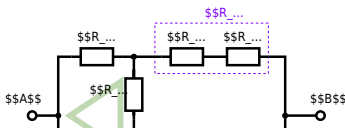
Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with $R_1 = 200 \Omega$, $R_2 = R_3 = 100 \Omega$ and the source $B = 15 \text{ V}$.
 Result given: $R_{\text{eq}} = B$.

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega}$$

**Exercise E1 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



Calculate the internal resistance R_{int} and the source voltage U_{s} of an equivalent linear voltage source on the connectors A and B .
 $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3=10 \Omega$, $I_4=4.2 \text{ A}$,
 $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot (R_6 || R_7)$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - I_4 \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

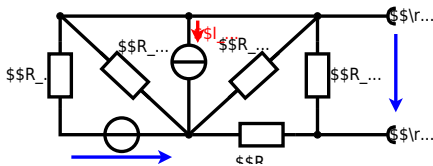
with $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \parallel R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

Exercise E3 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

$$U_s = U_{AB} = 4.5\text{V} \parallel R_i = R_{AB} = 6\Omega$$



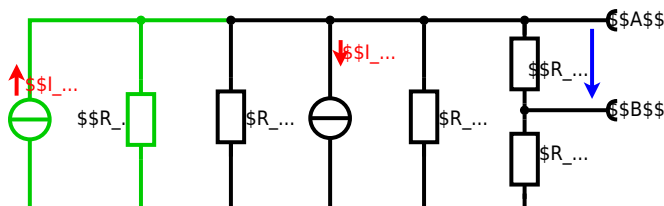
Calculate the internal resistance R_{int} and the source voltage U_{s} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3=10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$. Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_2}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{67} + U_{24} \cdot \frac{R_4}{R_4 + R_{67}}$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} = \left(\frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5}$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} = \left(\frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} || R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E5 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a battery with an internal resistance of $R_1 = 5 \Omega$ and a charging capacitor $C = 2 \mu\text{F}$ connected in parallel with a switch S_1 . The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Result: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution: The ideal voltage source U_{eq} is given by $U_{eq} = \frac{U}{1 + \frac{R_2}{R_1}} = \frac{12 \text{ V}}{1 + 2} = 4 \text{ V}$

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U=12 \text{ V}$, a switch S_1 , a resistor of $R_1=20 \text{ }\Omega$ and a capacitor of $C=100 \text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0)=0 \text{ V}$.



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1)=0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1)=0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$
 An equivalent linear voltage source can be given with U_s , R_1 and R_B as seen in yellow:

$$\ln(0.5) = -t/\tau \Rightarrow t = -\tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$$

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$
 The internal resistance is given by substituting the ideal voltage source with its resistance ($R_i = 0 \text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ ms} / (10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

Exercise E4 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (see the solution) consists of a 12 V DC voltage source, a $20 \text{ }\Omega$ resistor, a $100 \text{ }\mu\text{F}$ capacitor, a $10 \text{ }\Omega$ resistor, and a light bulb. The voltage across the capacitor is again 0 V at the moment $t_0=0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1 \text{ ms}$ after closing the switch.

Solution To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

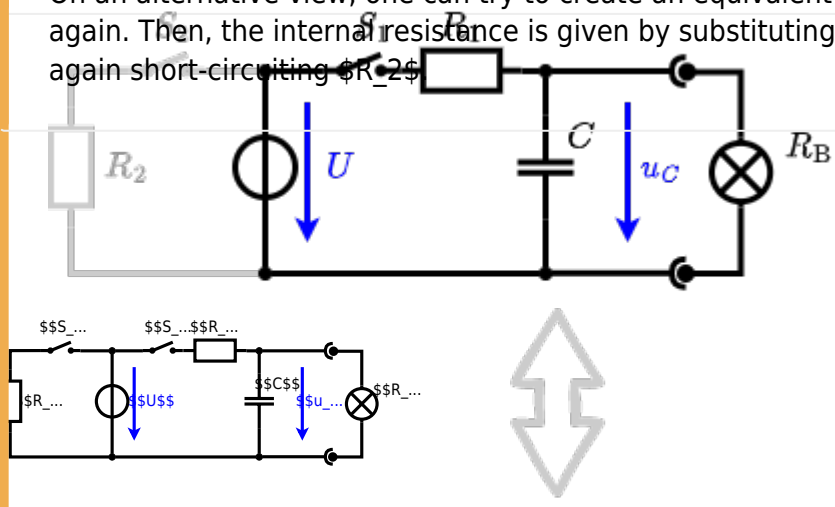
$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U = 6 \text{ V}$$

$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

Solution

The ideal voltage source is $U = 12 \text{ V}$. The internal resistance is $R_1 = 20 \text{ }\Omega$. The voltage across the capacitor is u_C . The voltage across the light bulb is u_B . The voltage across the resistor R_2 is u_{R_2} . The voltage across the capacitor is u_C . The voltage across the light bulb is u_B . The voltage across the resistor R_2 is u_{R_2} .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

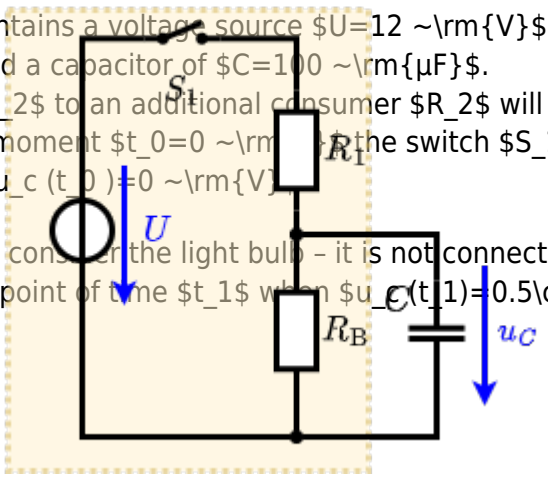


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0) = 0$.

First do not consider the light bulb - it is not connected to the RC circuit.

Calculate the point of time t_1 when $u_C(t_1) = 0.5 \cdot U$.



Solution

An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \text{ }\Omega$, short-circuit). $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$.

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2 / (R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2 / (10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$: $u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$. It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

Exercise E2 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source $\underline{U} = 50 \angle 0^\circ \text{ V}$ and the admittance $\underline{Y} = 0.24 \text{ S}$ are both in the components. ($\$R\$ and $\$X_L\$ shall be given.$$

After analysis, the full width dimensioned complex impedance \underline{Z} shall be extracted and given in polar form. $\underline{Z} = \left| Z \right| \angle \varphi \text{ } \Omega$

Solution
.. Calculation of physical values of the two components.
Solution $\underline{R} = \frac{1}{\text{Re}\{\underline{Y}\}} = \frac{1}{0.24} = 4.167 \text{ } \Omega$ and $\underline{X}_L = \frac{1}{\text{Im}\{\underline{Y}\}} = \frac{1}{-0.07} = -14.286 \text{ } \Omega$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \text{with } \underline{Z} = \underline{R} + j\underline{X}_L = 4.167 - j14.286 \text{ } \Omega$$

The current \underline{I} is the voltage \underline{U} divided by the complex impedance \underline{Z} .
resulting in $\underline{I} = \frac{50 \angle 0^\circ}{4.167 - j14.286} = 10.3 \angle 73.7^\circ \text{ A}$
Therefore, the component \underline{R} has a voltage drop $\underline{U}_R = \underline{I} \cdot \underline{R} = 42.8 \angle 73.7^\circ \text{ V}$
impedance $\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle 0^\circ}{10.3 \angle 73.7^\circ} = 4.85 \angle -73.7^\circ \text{ } \Omega$
$$\underline{Z} = \sqrt{4.167^2 + (-14.286)^2} \angle \arctan\left(\frac{-14.286}{4.167}\right) = 14.8 \angle -73.7^\circ \text{ } \Omega$$

The phase angle φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}\{\underline{Z}\}}{\text{Re}\{\underline{Z}\}}\right) = \arctan\left(\frac{-14.286}{4.167}\right) = -73.7^\circ$

Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source $\underline{U} = 50 \angle 0^\circ \text{ V}$ and the admittance $\underline{Y} = 0.24 \text{ S}$ are both in the components. ($\$R\$ and $\$X_L\$ shall be given.$$

After analysis, the full width dimensioned complex impedance \underline{Z} shall be extracted and given in polar form. $\underline{Z} = \left| Z \right| \angle \varphi \text{ } \Omega$

Solution
.. Calculation of physical values of the two components.
Solution $\underline{R} = \frac{1}{\text{Re}\{\underline{Y}\}} = \frac{1}{0.24} = 4.167 \text{ } \Omega$ and $\underline{X}_L = \frac{1}{\text{Im}\{\underline{Y}\}} = \frac{1}{-0.07} = -14.286 \text{ } \Omega$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \text{with } \underline{Z} = \underline{R} + j\underline{X}_L = 4.167 - j14.286 \text{ } \Omega$$

The current \underline{I} is the voltage \underline{U} divided by the complex impedance \underline{Z} .
resulting in $\underline{I} = \frac{50 \angle 0^\circ}{4.167 - j14.286} = 10.3 \angle 73.7^\circ \text{ A}$
Therefore, the component \underline{R} has a voltage drop $\underline{U}_R = \underline{I} \cdot \underline{R} = 42.8 \angle 73.7^\circ \text{ V}$
impedance $\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle 0^\circ}{10.3 \angle 73.7^\circ} = 4.85 \angle -73.7^\circ \text{ } \Omega$
$$\underline{Z} = \sqrt{4.167^2 + (-14.286)^2} \angle \arctan\left(\frac{-14.286}{4.167}\right) = 14.8 \angle -73.7^\circ \text{ } \Omega$$

The phase angle φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}\{\underline{Z}\}}{\text{Re}\{\underline{Z}\}}\right) = \arctan\left(\frac{-14.286}{4.167}\right) = -73.7^\circ$

The absolute value of the impedance is $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$ and the phase angle is $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$.
 With the complex part comes the physical value: $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$.
 The phase ϕ is given by $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$.

Exercise E3 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with $R_1 = 1.00 \text{ k}\Omega$, $R_2 = 4.70 \text{ k}\Omega$, $L = 4.70 \text{ mH}$, and $C = 100 \text{ nF}$, the current is $I = 100 \text{ mA}$.
 Result: $R_1 = 1.00 \text{ k}\Omega$, $R_2 = 4.70 \text{ k}\Omega$, $L = 4.70 \text{ mH}$, $C = 100 \text{ nF}$, $I = 100 \text{ mA}$.
 A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution
 Solution $R_1 = 1.00 \text{ k}\Omega$
 Solution $R_2 = 4.70 \text{ k}\Omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z = \sqrt{R^2 + X_L^2}$.
 Parallel circuit means that the voltage is the same on R_2 and C_1 .
 The equivalent impedance for R_2 and C_1 combined is given by $Z_{parallel} = \frac{R_2 \cdot X_C}{R_2 + X_C}$.
 The total impedance is $Z_{total} = Z + Z_{parallel}$.
 The resulting current is given as: $I = \frac{U}{Z_{total}}$.
 Back to the first formula: $R_1 \cdot I = Z_{parallel} \cdot I$.

Exercise E6 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

Resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution

$$R_1 = 1.00 \text{ } \Omega$$

$$R_2 = 10.0 \text{ } \Omega$$

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by

$$Z = R + j\omega L$$

Parallel circuit means that the voltage is the same on R_1 and C_1

$$\frac{1}{Z} = \frac{1}{R_2} + \frac{1}{j\omega C_1}$$

Since R_2 and C_1 are perpendicular to each other, the resulting current of the parallel circuit is given as:

$$I_3 = I_{R2} + I_{C1}$$

This can be simplified to

$$I_3 = \frac{U}{R_2} + \frac{U}{j\omega C_1}$$

Back to the first formula:

$$R_3 \cdot I_3 = X_{C1} \cdot I_3$$

$$R_3 = \frac{X_{C1} \cdot I_3}{I_3} = \frac{1}{2\pi \cdot f \cdot C_1} \cdot \frac{I_3}{I_3} = \frac{1}{2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}} = 1.00 \text{ } \Omega$$

Exercise E1 Complex Impedance Circuit
 (written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the current $i(t)$ through the resistor R in the circuit shown in the figure. The voltage source is $u(t) = 3.0 \text{ V} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$.

Solution

Result

$$Z = 48.2 \text{ } \Omega \quad Z_C = 19.8 \text{ } \Omega$$

Draw the circuit diagram of the given circuit

$$Z = \frac{U}{I} \quad I = \frac{U}{Z}$$

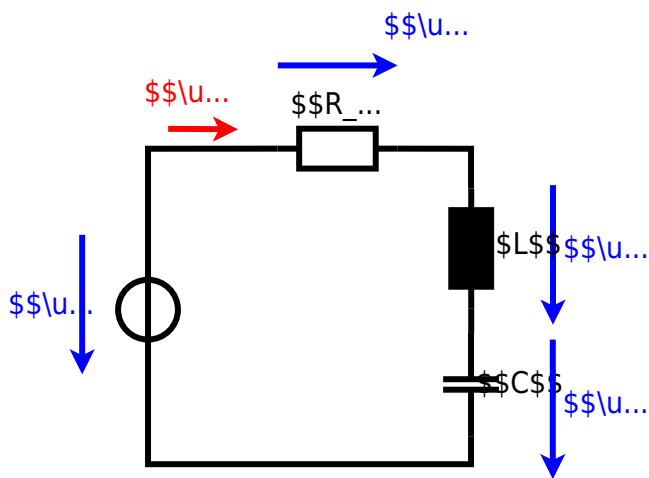
$$Z_C = \frac{1}{2\pi \cdot f \cdot C} = \frac{1}{2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ } \mu\text{F}} = 19.8 \text{ } \Omega$$

With $Z = \sqrt{R^2 + (Z_L - Z_C)^2}$

$$Z = \sqrt{10^2 + (29.8 - 19.8)^2} = 19.8 \text{ } \Omega$$

$$I = \frac{U}{Z} = \frac{3.0 \text{ V}}{19.8 \text{ } \Omega} = 0.1515 \text{ A}$$

$$i(t) = 0.1515 \text{ A} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$$





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