

Exam Winter Semester 2022

Student Group

First Name	Surname	Matrikel Nr.

Table of Contents

Exercise E1 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022) 3

Exercise E1 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022) 6

Exercise E2 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022) 7

Exercise E1 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022) 7

Exercise E3 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022) 10

Exercise E4 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022) 10

Exercise E5 Charging Capacitors (written test, approx. 16 % of a 60-minute written test, WS2022) 11

Exercise E6 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022) 12

Exercise E3 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022) 14

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022) 18

Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022) 19

Exercise E7 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022) 19

Exercise E6 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022) 22

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022) 22

Exercise E4 Charging Capacitors (written test, approx. 16 % of a 60-minute written test,

WS2022) 23

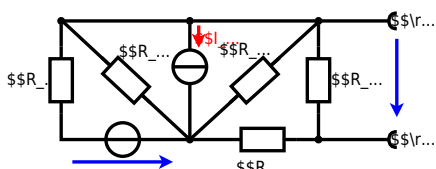
Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute
written test, WS2022) 24

1

**Exercise E1 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$\begin{aligned} U_{\text{rs}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} \\ &= 6 \text{ } \Omega \end{aligned}$$



Calculated the internal resistance R_{i} and the source voltage U_{rs} of an equivalent linear voltage source on the connectors A and B .
$$R_1=5.0 \text{ } \Omega, \quad U_2=6.0 \text{ V}, \quad R_3= 10 \text{ } \Omega, \quad I_4=4.2 \text{ A}, \quad R_5=10 \text{ } \Omega, \quad R_6=7.5 \text{ } \Omega, \quad R_7=15 \text{ } \Omega$$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \left(R_1 || R_3 || R_5 \right)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \Omega \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E1 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator is explained with the effect of resistance on refrigeration system. The circuit has a resistance of 15Ω and a voltage of 6 V at 25°C for your answer.

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$

Result The temperature inside the refrigeration system can reach down to -40°C .

$$R_{25} = 15 \Omega$$

The power of the resistor is $P = U \cdot I$ and $Q = P \cdot t$. Therefore, a solution is to increase the resistance of the resistor to reduce the heat flow.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 15 \Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2 \right)$$

Exercise E2 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex power \underline{S} (W and VA) of the load \underline{Z} through the components. \underline{S}_R and \underline{S}_L shall be given.

After analysis, the full load impedance \underline{Z} can be extracted and legitimated in phasor notation $\underline{Z} = |Z| \cdot e^{j\varphi}$ with $\varphi = \varphi(\omega)$.

Solution
 .. Calculate the physical values of the load components.
 Solution $R = 10 \Omega$, $L = 10 \text{ mH}$, $C = 10 \text{ nF}$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \text{ V}}{0.24 - j4.68 \text{ } \Omega} = 208.33 \text{ A} \cdot e^{j(90^\circ - \varphi)}$$
 The voltage \underline{U} is 50 V and the current \underline{I} is 208.33 A .
 The resulting complex power $\underline{S} = \underline{U} \cdot \underline{I}^*$ is $\underline{S} = 50 \text{ V} \cdot 208.33 \text{ A} \cdot e^{j(90^\circ - \varphi - \varphi)} = 10416.67 \text{ VA} \cdot e^{-j2\varphi}$.
 The real power P is $10416.67 \text{ VA} \cdot \cos(2\varphi)$ and the reactive power Q is $10416.67 \text{ VA} \cdot \sin(2\varphi)$.
 With the complex power $\underline{S} = P + jQ$ we can calculate the phase φ as $\varphi = \arctan\left(\frac{Q}{P}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -10.8^\circ$.
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(\underline{Z})}{\text{Re}(\underline{Z})}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -10.8^\circ$.

Exercise E1 Complex Impedance Circuit
 (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the circuit impedance \underline{Z} for a signal $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$ V. The circuit consists of a voltage source $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$ V, a resistor $R = 10 \text{ } \Omega$, an inductor $L = 10 \text{ mH}$, and a capacitor $C = 10 \text{ nF}$.

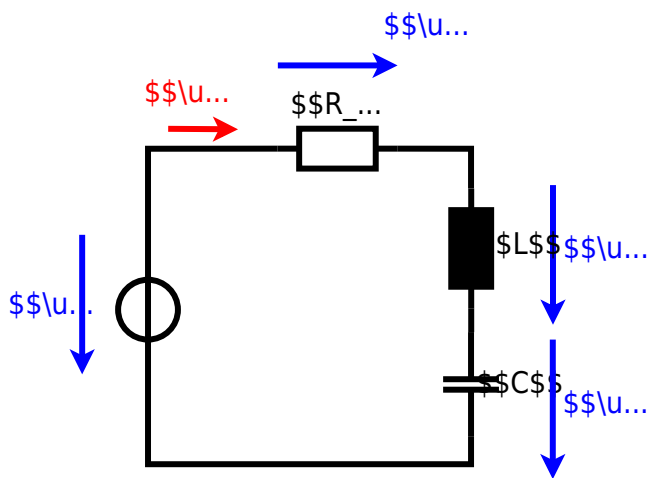
Solution
 .. Draw the circuit diagram of the given circuit.

Result

$$\underline{Z} = R + j\omega L - j\omega C = 10 \text{ } \Omega + j(2\pi \cdot 15 \cdot 10 \text{ mH}) - j(2\pi \cdot 15 \cdot 10 \text{ nF}) = 10 \text{ } \Omega + j0.942 \text{ } \Omega - j0.00314 \text{ } \Omega = 10 \text{ } \Omega + j0.939 \text{ } \Omega$$

$$|\underline{Z}| = \sqrt{10^2 + 0.939^2} = 10.045 \text{ } \Omega$$

$$\varphi = \arctan\left(\frac{0.939}{10}\right) = 5.3^\circ$$



Exercise E3 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit consists of a resistor with a resistance of $R_1 = 1.00 \text{ k}\Omega$, a capacitor with a capacitance of $C_1 = 40 \text{ nF}$, and an inductor with an inductance of $L_1 = 4.7 \text{ }\mu\text{H}$. The circuit is connected to an AC voltage source with a voltage of $U = 10 \text{ V}$ and a frequency of $f = 4 \text{ MHz}$. Calculate the absolute value of the impedance $|Z|$ of the circuit.

Solution

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$|Z| = \sqrt{(1.00 \text{ k}\Omega)^2 + (2\pi \cdot 4 \text{ MHz} \cdot 4.7 \text{ }\mu\text{H} - \frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}})^2}$$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z = R + jX_L$
 Parallel circuit means that the voltage is the same on R and C $Z = \frac{R \cdot X_C}{R + jX_C}$
 $|Z| = \sqrt{R^2 + X_L^2}$ since X_L and X_C are perpendicular
 $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$ since X_L and X_C are perpendicular
 $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$ (It has to, since R is perpendicular to X_L and X_C is perpendicular to X_L)
 $|Z| = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (2\pi f L - \frac{1}{2\pi f C})^2}$
 Therefore, the resulting current of the parallel circuit is given as:

$$I = \frac{U}{|Z|} = \frac{10 \text{ V}}{\sqrt{(1.00 \text{ k}\Omega)^2 + (2\pi \cdot 4 \text{ MHz} \cdot 4.7 \text{ }\mu\text{H} - \frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}})^2}}$$

Exercise E4 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. For heating elements used to heat the oven at a temperature of $180 \text{ }^\circ\text{C}$, an electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.
 Calculate the current I needed to operate the heating elements.
 The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$.

The heating element is $l = 3 \text{ m}$ long and has a diameter of $d = 3.57 \text{ mm}$.
 Calculate the resistance R of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

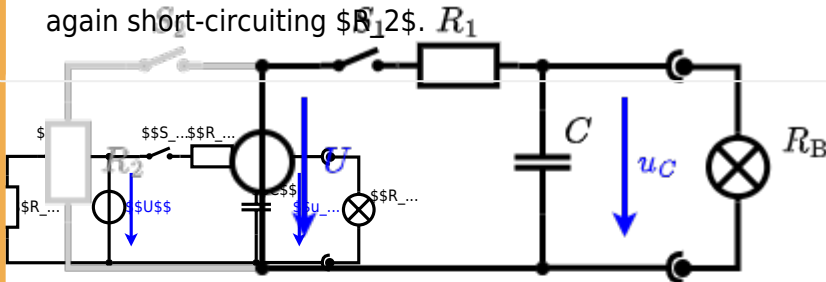
Exercise E5 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) is a series of R_1 and R_2 and a capacitor C and a switch S_1 and a switch S_2 . The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
 Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution
 The ideal voltage source U is in series with R_1 and R_2 . The voltage u_c is independent of the choice of R_1 and R_2 .

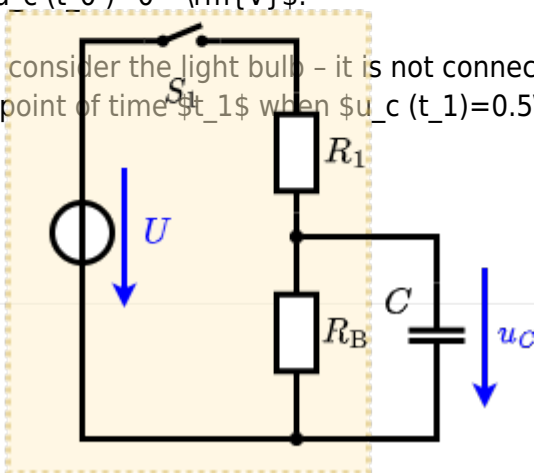
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting S_1 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \Omega$ and a capacitor of $C = 100 \mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R_1 = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



Exercise E6 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0.10 A, $R_1 = R_2 = R_3 = 10 \Omega$ and the voltage between B_1 and B_2 is given.

Solution

$$R_{eq} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



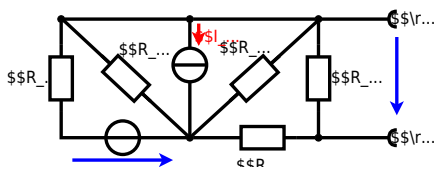
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E3 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



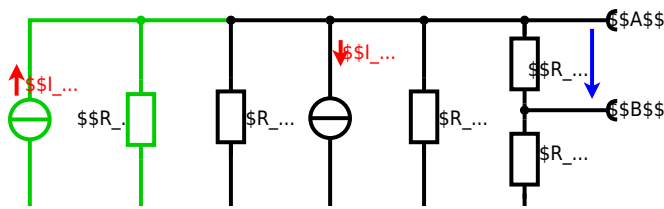
Calculate the internal resistance R_{in} and the source voltage U_{s} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3=10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$. Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot (R_2 + R_4 || R_6)$$

$$U_{24} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) \cdot \left(\frac{U_2}{R_1} - I_4 \right) \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) \cdot \left(\frac{U_2}{R_1} - I_4 \right) \cdot (R_1 || R_3 || R_5)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot (15 \Omega \cdot 2.5 \Omega) / (7.5 \Omega + 15 \Omega + 2.5 \Omega)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator is explained with the effect of resistance on the refrigeration system. The circuit has a resistance of 15Ω and a voltage of 6 V at 25°C .

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result The temperature inside the refrigeration system can reach down to -40°C .

$$R_0 = 15 \Omega$$

The power of the resistor is $P = U^2 / R$ and the heat flow is $Q = P \cdot t$. Therefore, a solution is to increase the resistance of the resistor.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$R = 15 \Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2 \right)$$

Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

1. Calculate the complex impedance Z of the circuit shown in the figure. The voltage $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$ V and the current $i(t) = 0.24 \cdot \sin(2\pi \cdot 15 \cdot t - \varphi)$ A shall be given.

After analysis, the following complex impedance Z shall be extracted and the phase angle φ in degrees shall be determined. The result shall be given in the form $Z = |Z| \cdot e^{j\varphi}$ and $\varphi = \varphi^\circ$.

Solution
.. Calculate the physical values of the two components.
Solution $R = 10 \Omega$ and $X_L = 2\pi \cdot 15 \cdot 0.01 = 0.942 \Omega$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \text{with } \underline{U} = 3.0 \cdot e^{j\omega t} \text{ and } \underline{I} = 0.24 \cdot e^{j(\omega t - \varphi)}$$

The voltage U and current I are in phase with the voltage U (pure real) resulting in a phase angle $\varphi = 0^\circ$.
Therefore, the component R and X_L are in series with the same current I .
Impedance $Z = R + jX_L = 10 + j0.942 \Omega$
$$\underline{Z} = R + jX_L = 10 + j0.942 \Omega$$

The phase angle φ can be calculated as $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{0.942}{10}\right) = 5.3^\circ$
With the complex part $Z = 10 + j0.942 \Omega$ and $\varphi = 5.3^\circ$
$$\underline{Z} = |Z| \cdot e^{j\varphi} = 10.04 \cdot e^{j5.3^\circ}$$

The phase angle φ can be calculated as $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{0.942}{10}\right) = 5.3^\circ$

Exercise E7 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the complex impedance Z of the circuit shown in the figure. The voltage $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$ V and the current $i(t) = 0.24 \cdot \sin(2\pi \cdot 15 \cdot t - \varphi)$ A shall be given.

After analysis, the following complex impedance Z shall be extracted and the phase angle φ in degrees shall be determined. The result shall be given in the form $Z = |Z| \cdot e^{j\varphi}$ and $\varphi = \varphi^\circ$.

Solution
.. Draw the circuit diagram of the given circuit.
Solution $Z = 10 + j0.942 \Omega$

Solution
$$\underline{Z} = R + jX_L = 10 + j0.942 \Omega$$

The phase angle φ can be calculated as $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{0.942}{10}\right) = 5.3^\circ$
With the complex part $Z = 10 + j0.942 \Omega$ and $\varphi = 5.3^\circ$
$$\underline{Z} = |Z| \cdot e^{j\varphi} = 10.04 \cdot e^{j5.3^\circ}$$

The phase angle φ can be calculated as $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{0.942}{10}\right) = 5.3^\circ$



Exercise E6 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit consists of a resistor with a resistance of $R_1 = 1.00 \text{ k}\Omega$ and a capacitor with a capacitance of $C_1 = 40 \text{ nF}$. The voltage across the resistor is $U_{R_1} = 100 \text{ V}$ at a frequency of $f = 4 \text{ MHz}$. Calculate the absolute value of the impedance of the capacitor Z_C and the total impedance Z_{total} of the circuit.

Solution

$$Z_C = \frac{1}{j\omega C_1} = \frac{1}{j \cdot 2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}} = -j \cdot 0.995 \text{ k}\Omega$$

$$Z_{\text{total}} = R_1 + Z_C = 1.00 \text{ k}\Omega - j \cdot 0.995 \text{ k}\Omega$$

$$|Z_{\text{total}}| = \sqrt{1.00^2 + 0.995^2} \text{ k}\Omega = 1.41 \text{ k}\Omega$$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R_1 and Z_C combined is given by $Z_{\text{total}} = R_1 + Z_C$.
 Parallel circuit means that the voltage is the same on R_1 and Z_C .

$$U_{R_1} = I \cdot R_1 \Rightarrow I = \frac{U_{R_1}}{R_1} = \frac{100 \text{ V}}{1.00 \text{ k}\Omega} = 0.1 \text{ A}$$

 Since Z_C is perpendicular to R_1 , the resulting current of the parallel circuit is given as:

$$I_{\text{total}} = \sqrt{I_{R_1}^2 + I_{Z_C}^2} = \sqrt{0.1^2 + \left(\frac{U_{R_1}}{|Z_C|}\right)^2}$$

 Therefore, the resulting current of the parallel circuit is given as:

$$I_{\text{total}} = \sqrt{0.1^2 + \left(\frac{100 \text{ V}}{0.995 \text{ k}\Omega}\right)^2} = 0.1005 \text{ A}$$

 This current is the same as the current through R_1 .

$$U_{\text{total}} = I_{\text{total}} \cdot |Z_{\text{total}}| = 0.1005 \text{ A} \cdot 1.41 \text{ k}\Omega = 141.7 \text{ V}$$

 Back to the first formula:
$$R_3 \cdot I_{\text{total}} = U_{\text{total}} \Rightarrow R_3 = \frac{U_{\text{total}}}{I_{\text{total}}} = \frac{141.7 \text{ V}}{0.1005 \text{ A}} = 1.41 \text{ k}\Omega$$

Exercise E1 Resistance of a Wire by Resistivity
 (written test, approx. 6 % of a 60-minute written test, WS2022)

2. Heating elements are used to heat the water in a domestic use electric boiler. The power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary. Calculate the current I and the resistance R of the heating element.
 The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$.
 The heating element is $l = 3 \text{ m}$ long and has a diameter of $d = 3.57 \text{ mm}$.

Solution

$$P = U \cdot I = R \cdot I^2 \Rightarrow I = \sqrt{\frac{P}{R}}$$

$$R = \rho \cdot \frac{l}{A} = \rho \cdot \frac{l}{\pi \cdot \left(\frac{d}{2}\right)^2}$$

$$I = \sqrt{\frac{P}{\rho \cdot \frac{l}{\pi \cdot \left(\frac{d}{2}\right)^2}}} = \sqrt{\frac{40 \text{ W} \cdot \pi \cdot \left(\frac{3.57 \text{ mm}}{2}\right)^2}{1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m} \cdot 3 \text{ m}}} = 1.0 \text{ A}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

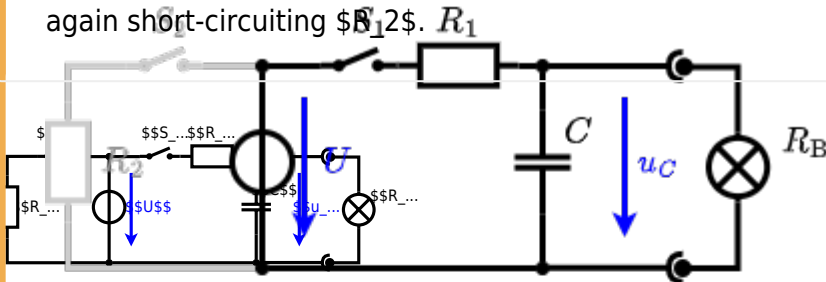
Exercise E4 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) is a series of R_1 and R_2 and a capacitor C and a switch S_1 and a switch S_2 . The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

The ideal voltage source U is in series with R_1 and R_2 . The voltage u_c is independent of this series.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting S_2 .

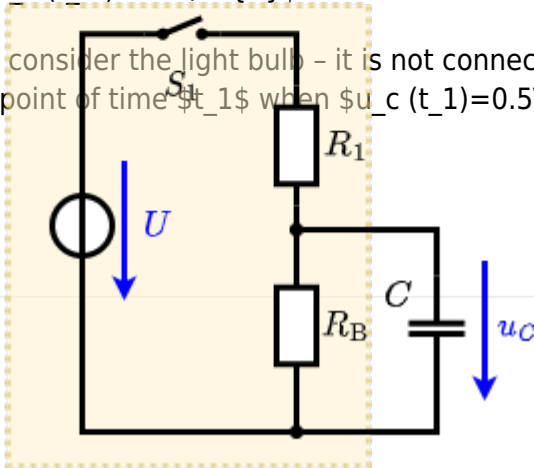


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \Omega$ and a capacitor of $C = 100 \mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5)$



Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0 degree, $R_1 = R_2 = R_3 = 1.5 \Omega$ and the voltage $U = 10V$ is given. R_B .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

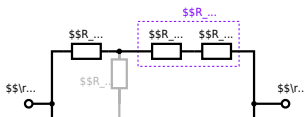
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = (500 \sim \Omega) \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \frac{500 \sim \Omega \cdot 200 \sim \Omega}{500 \sim \Omega + 200 \sim \Omega}$$

From:

<https://mexle.te.hs-heilbronn.de/> - MEXLE Wiki

Permanent link:

https://mexle.te.hs-heilbronn.de/electrical_engineering_1/ws2022_exam?rev=1680241446

Last update: **2023/03/31 07:44**

