

# Exam Winter Semester 2022

## Student Group

First Name	Surname	Matrikel Nr.

## Table of Contents

Exercise E1 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022) ..... 3

Exercise E1 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022) ..... 6

Exercise E2 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022) ..... 7

Exercise E1 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022) ..... 7

Exercise E3 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022) ..... 10

Exercise E4 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022) ..... 10

Exercise E5 Charging Capacitors (written test, approx. 16 % of a 60-minute written test, WS2022) ..... 11

Exercise E6 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022) ..... 12

Exercise E3 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022) ..... 14

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022) ..... 18

Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022) ..... 19

Exercise E7 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022) ..... 19

Exercise E6 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022) ..... 22

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022) ..... 22

Exercise E4 Charging Capacitors (written test, approx. 16 % of a 60-minute written test,

WS2022) ..... 23

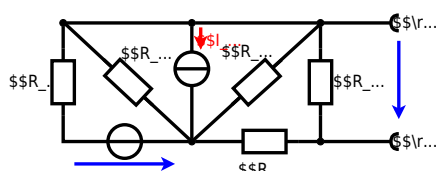
Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute  
written test, WS2022) ..... 24

1

**Exercise E1 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$\begin{aligned} U_{\text{rs}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \text{ } \Omega \end{aligned}$$



Calculated the internal resistance  $R_{\text{i}}$  and the source voltage  $U_{\text{rs}}$  of an equivalent linear voltage source on the connectors  $\text{A}$  and  $\text{B}$ . 
$$R_1=5.0 \text{ } \Omega, \quad U_2=6.0 \text{ V}, \quad R_3= 10 \text{ } \Omega, \quad I_4=4.2 \text{ A}, \quad R_5=10 \text{ } \Omega, \quad R_6=7.5 \text{ } \Omega, \quad R_7=15 \text{ } \Omega$$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left( \frac{U_2}{R_1} - I_4 \right) \cdot \left( \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5} \right)$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \left( \frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \Omega \cdot \left( \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

**Exercise E1 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator is explained with the effect of resistance on refrigeration system. The refrigerator has a resistance of  $15 \Omega$  at  $25^\circ \text{C}$  and  $2.5 \Omega$  at  $0^\circ \text{C}$ . Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

Result: The temperature inside the refrigeration system can reach down to  $-40^\circ \text{C}$ .

Calculate the resistance of the thermistor at  $-40^\circ \text{C}$ .

The power transferred to the heat exchanger and the heat flow. Therefore, a solution is to use the heat flow up the refrigeration system.

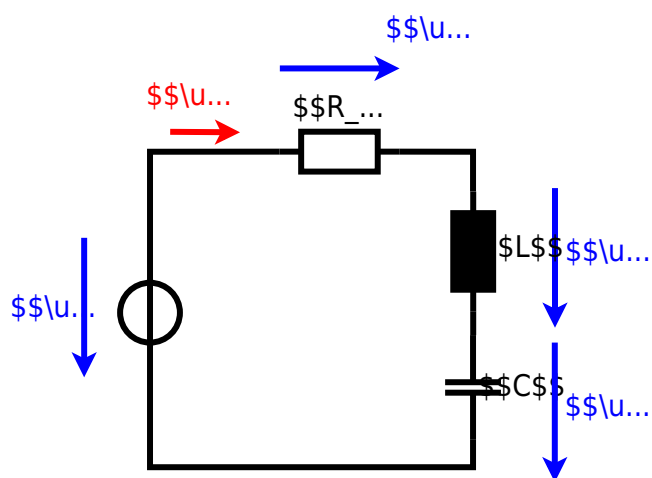
Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\text{with } \Delta T = T_{\text{end}} - T_{\text{start}} \implies R = 10 \Omega \cdot \left( 1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2 \right)$$







**Exercise E3 Impedances at different Frequencies**  
**(written test, approx. 18 % of a 60-minute written test, WS2022)**

2. A series circuit consists of a resistor with a resistance of  $R_1 = 1.00 \text{ k}\Omega$ , a capacitor with a capacitance of  $C_1 = 40 \text{ nF}$ , and an inductor with an inductance of  $L_1 = 4.7 \text{ }\mu\text{H}$ . The circuit is connected to an AC voltage source with a voltage of  $U = 10 \text{ V}$  and a frequency of  $f = 4 \text{ MHz}$ . Calculate the absolute value of the impedance  $|Z|$  of the circuit.

Solution

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$|Z| = \sqrt{1000^2 + (2\pi \cdot 4 \cdot 10^6 \cdot 4.7 \cdot 10^{-6} - \frac{1}{2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}})^2}$$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $L$  combined is given by  $Z = R + jX_L$   
 Parallel circuit means that the voltage is the same on  $R$  and  $C$   $Z = \frac{R \cdot X_C}{R + jX_C}$   
 $|Z| = \sqrt{R^2 + X_L^2}$  since  $X_L$  and  $X_C$  are perpendicular  
 $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$  since  $X_L$  and  $X_C$  are perpendicular  
 $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$  (It has to, since  $R$  is perpendicular to  $X_L$  and  $X_C$  is perpendicular to  $X_L$ )  
 $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$   
 Therefore, the resulting current of the parallel circuit is given as:  
 $I = \frac{U}{|Z|}$   
 $I = \frac{10 \text{ V}}{\sqrt{1000^2 + (2\pi \cdot 4 \cdot 10^6 \cdot 4.7 \cdot 10^{-6} - \frac{1}{2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}})^2}}$   
 Back to the first formula:  $R \cdot I = X_C \cdot I$   
 $R \cdot I = X_C \cdot I$   
 $R \cdot I = X_C \cdot I$

**Exercise E4 Resistance of a Wire by Resistivity**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. Heating elements are used to heat the water in a domestic use of  $1.80 \text{ m}^3$  of water. The electric power dissipation (= heat flow) of  $P = 40 \text{ W}$  is necessary. Calculate the current  $I$  needed to operate for heating elements.  
 The Nichrome wire has a resistivity of  $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$ .  
 The heating element is  $3 \text{ m}$  long and has a diameter of  $3.57 \text{ mm}$ .  
 Calculate the resistance  $R$  of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad \rho = \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

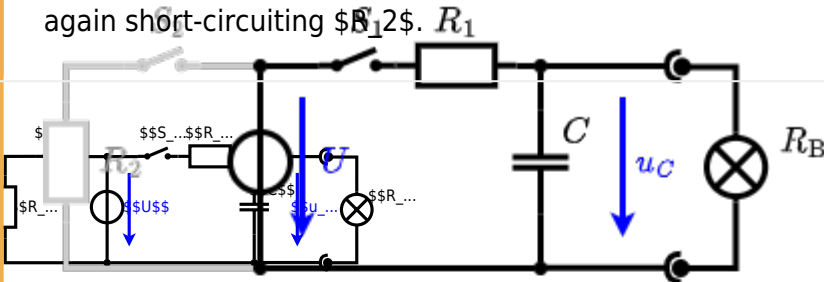
**Exercise E5 Charging Capacitors**  
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) is a series of  $R_1$  and  $R_2$  and a capacitor  $C$  and a switch  $S_1$  and a switch  $S_2$ . The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

The ideal voltage source  $U$  and the voltage source  $U_1 = U \cdot \frac{R_2}{R_1 + R_2}$  are independent of this circuit.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $S_2$ .

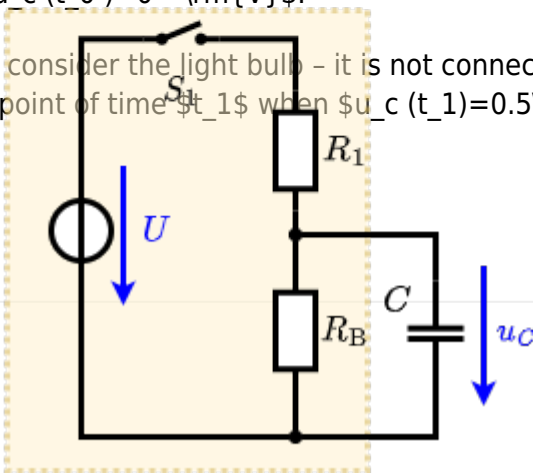


The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \Omega$  and a capacitor of  $C = 100 \mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first tasks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

**Solution**



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \Omega$ , short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ :  

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



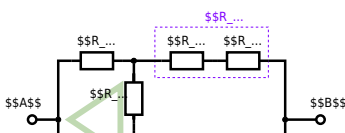
**Exercise E6 Pure Resistor Network Simplification**  
 (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0.10 A,  $R_1 = R_2 = R_3 = 10 \Omega$  and the voltage  $U = 10 \text{ V}$  is given.  $R_B$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

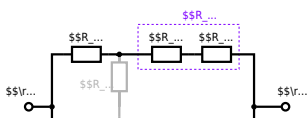
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{\text{eq}}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E3 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



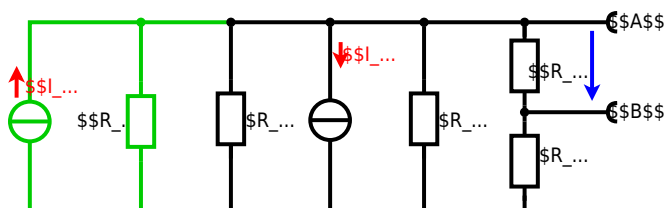
Calculate the internal resistance  $R_i$  and the source voltage  $U_s$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3= 10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$  Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{67}$$

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left( \frac{U_2}{R_1} - I_4 \right) \cdot \left( \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5} \right)$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \left( \frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \Omega \cdot \left( \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

**Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator is explained with the effect of resistance on the refrigeration system. The circuit has a resistance of  $15 \Omega$  and a voltage of  $6 \text{ V}$ . Your answer:

Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$

**Result**  
The temperature inside the refrigeration system can reach down to  $-40 \text{ }^\circ\text{C}$ .

$$R = 6.5 \text{ } \Omega \text{ at } -40 \text{ }^\circ\text{C}$$

The power of the resistor is  $P = U \cdot I$  and  $P = \frac{U^2}{R}$ . Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with  $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ } \Omega \cdot \left( 1 + 0.01 \text{ K}^{-1} \cdot (-40 \text{ }^\circ\text{C} - 25 \text{ }^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40 \text{ }^\circ\text{C} - 25 \text{ }^\circ\text{C})^2 \right)$$

**Exercise E5 Analyzing complex Impedances**  
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex power  $\underline{S}$  (W) and the complex power factor  $\cos \varphi$  in the circuit shown in the figure. The voltage  $\underline{u}(t) = 30 \sqrt{2} \sin(2\pi \cdot 150 t)$  V and the current  $\underline{i}(t) = 2 \sqrt{2} \sin(2\pi \cdot 150 t + \varphi)$  A shall be given.

After analysis, the following dimensions must be indicated:  $\underline{S}$  in VA and  $\cos \varphi$  in dimensionless. The phase angle  $\varphi$  must be indicated in degrees.

Solution  

$$\underline{S} = \underline{U} \cdot \underline{I}^* = 30 \sqrt{2} \cdot 2 \sqrt{2} \cdot e^{j(\varphi - 0)} = 120 e^{j\varphi} \text{ VA}$$

Solution  

$$\underline{S} = P + jQ = 120 \cos \varphi + j120 \sin \varphi \text{ VA}$$

The voltage  $\underline{u}(t) = 30 \sqrt{2} \sin(2\pi \cdot 150 t)$  V and the current  $\underline{i}(t) = 2 \sqrt{2} \sin(2\pi \cdot 150 t + \varphi)$  A shall be given. The resulting complex power  $\underline{S}$  (W) and the complex power factor  $\cos \varphi$  must be indicated.

The real power  $P$  is the average power  $P = \text{Re}\{\underline{S}\} = 120 \cos \varphi$  W. The reactive power  $Q$  is the imaginary part  $Q = \text{Im}\{\underline{S}\} = 120 \sin \varphi$  var. The complex power factor  $\cos \varphi$  is the ratio of real power to complex power  $\cos \varphi = P / |\underline{S}| = \cos \varphi$ . The phase angle  $\varphi$  is the angle between the voltage and current phasors  $\varphi = \arg(\underline{u}) - \arg(\underline{i})$ .

With the complex power  $\underline{S}$  (W) and the complex power factor  $\cos \varphi$  (dimensionless) the phase angle  $\varphi$  (degrees) can be calculated as  $\varphi = \arccos(\cos \varphi)$ .

**Exercise E7 Complex Impedance Circuit**  
 (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the complex power  $\underline{S}$  (W) and the complex power factor  $\cos \varphi$  in the circuit shown in the figure. The voltage  $\underline{u}(t) = 30 \sqrt{2} \sin(2\pi \cdot 150 t)$  V and the current  $\underline{i}(t) = 2 \sqrt{2} \sin(2\pi \cdot 150 t + \varphi)$  A shall be given.

After analysis, the following dimensions must be indicated:  $\underline{S}$  in VA and  $\cos \varphi$  in dimensionless. The phase angle  $\varphi$  must be indicated in degrees.

Solution  

$$\underline{S} = \underline{U} \cdot \underline{I}^* = 30 \sqrt{2} \cdot 2 \sqrt{2} \cdot e^{j(\varphi - 0)} = 120 e^{j\varphi} \text{ VA}$$

The real power  $P$  is the average power  $P = \text{Re}\{\underline{S}\} = 120 \cos \varphi$  W. The reactive power  $Q$  is the imaginary part  $Q = \text{Im}\{\underline{S}\} = 120 \sin \varphi$  var. The complex power factor  $\cos \varphi$  is the ratio of real power to complex power  $\cos \varphi = P / |\underline{S}| = \cos \varphi$ .

With the complex power  $\underline{S}$  (W) and the complex power factor  $\cos \varphi$  (dimensionless) the phase angle  $\varphi$  (degrees) can be calculated as  $\varphi = \arccos(\cos \varphi)$ .





**Exercise E6 Impedances at different Frequencies**  
**(written test, approx. 18 % of a 60-minute written test, WS2022)**

2. A series circuit consists of a resistor with a resistance of  $R_1 = 1.00 \text{ k}\Omega$ , a capacitor with a capacitance of  $C_1 = 40 \text{ nF}$ , and an inductor with an inductance of  $L_1 = 4.7 \text{ }\mu\text{H}$ . The circuit is connected to an AC voltage source with a voltage of  $U = 10 \text{ V}$  and a frequency of  $f = 4 \text{ MHz}$ . Calculate the absolute value of the impedance  $|Z|$  of the circuit.

Solution

$$R_1 = 1.00 \text{ k}\Omega$$

$$R_2 = 10.0 \text{ }\Omega$$

A series circuit means that the current is constant on every component.

The equivalent impedance for  $R_1$  and  $R_2$  combined is given by 
$$R_{\text{parallel}} = R_1 + R_2$$

Parallel circuit means that the voltage is the same on  $R_1$  and  $R_2$ . 
$$U = I \cdot (R_1 + R_2)$$

Since  $U$  and  $R_1 + R_2$  are perpendicular to  $R_2$ , this can be simplified to 
$$I = \frac{U}{\sqrt{R_1^2 + R_2^2}}$$

$R_2$  is perpendicular to  $R_1$  (It has to, since  $R_1$  is perpendicular to  $U$  and  $U$  is perpendicular to  $R_2$ ).

Therefore, the resulting current of the parallel circuit is given as: 
$$I_{\text{parallel}} = I_{R_1} + I_{R_2}$$

This can be rearranged to 
$$I_{\text{parallel}} = \frac{U}{\sqrt{R_1^2 + R_2^2}}$$

Back to the first formula: 
$$R_3 \cdot I_{\text{parallel}} = X_{L_1} \cdot I_{\text{parallel}}$$

$$R_3 = X_{L_1} \cdot \frac{\sqrt{R_1^2 + R_2^2}}{U}$$

**Exercise E1 Resistance of a Wire by Resistivity**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. For a heating element used to heat the oven at a temperature of  $180 \text{ }^\circ\text{C}$ , an electric power dissipation (= heat flow) of  $P = 40 \text{ W}$  is necessary. Calculate the current  $I$  needed to operate the heating elements.

The Nichrome wire has a resistivity of  $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$ .

The heating element is  $l = 3 \text{ m}$  long and has a diameter of  $d = 3.57 \text{ mm}$ .

Solution

Calculate the resistance  $R$  of the heating element.

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

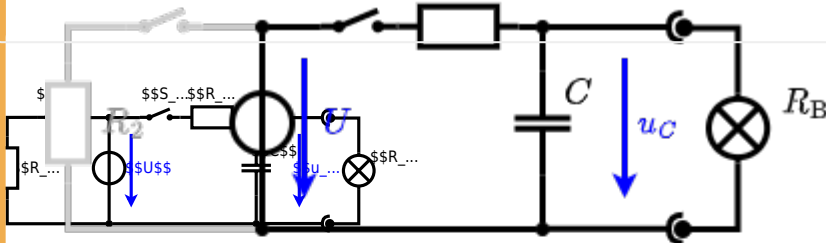
**Exercise E4 Charging Capacitors**  
**(written test, approx. 16 % of a 60-minute written test, WS2022)**

The circuit (with the light bulb) is a series of  $R_1$  and  $R_2$  and a capacitor  $C$  and a light bulb  $R_B$ . The switch  $S_1$  is open. The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution:** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

The ideal voltage source  $U$  is in series with  $R_1$  and  $R_2$ . The voltage  $u_c$  is independent of this series combination.

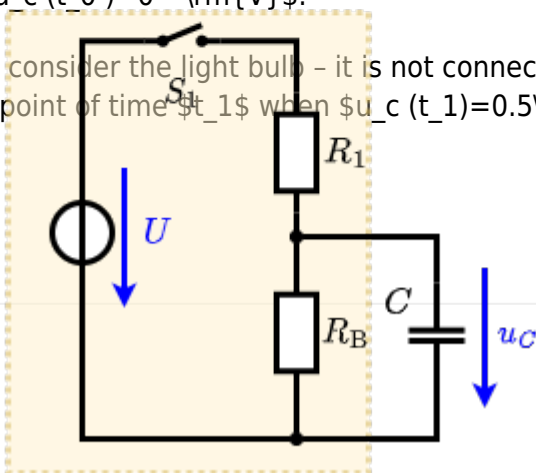
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $S_2$ .



The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ }\Omega$  and a capacitor of  $C = 100 \text{ }\mu\text{F}$ . The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first tasks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

**Solution**



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \Omega$ , short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ :  

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



**Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be simplified with  $R_1 = 200 \Omega$ ,  $R_2 = R_3 = 150 \Omega$ ,  $R_4 = 100 \Omega$  and the voltage  $U = 10 \text{ V}$ .  
 Result:  $R_{eq}$ .

Solution

$$R_{eq} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

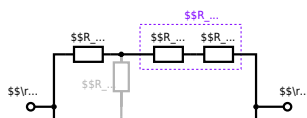
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{\text{eq}}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = \{500 \sim \Omega\} \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \{500 \sim \Omega \cdot 200 \sim \Omega\} \over {500 \sim \Omega + 200 \sim \Omega}$$

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