Exam Winter Semester 2022

Student Group

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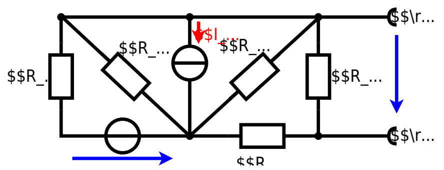
Exercise E1 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

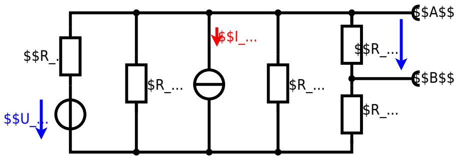
\begin{align*} U_{\rm s} &= U_{\rm AB} &&= 4.5 ~\rm{V}\\ R_{\rm i} &= R_{\rm AB} &&= 6 ~\Omega \end{align*}



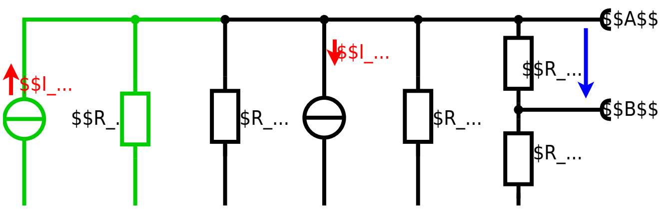
Calculated the internal resistance $R_\rm i$ and the source voltage $U_\rm s$ of an equivalent linear voltage source on the connectors $\rm A$ and $\rm B$. \begin{align*} R_1=5.0 ~\Omega, && U_2=6.0 ~\rm{V}, && R_3= 10 ~\Omega, \\ I_4=4.2 ~\rm{A}, && R_5=10 ~\Omega , && R_6=7.5 ~\Omega, \\ R_7=15 ~\Omega && && \end{align*} Use equivalent sources in order to simplify the circuit!

Solution

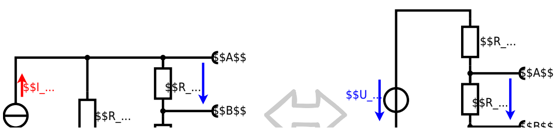
The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2={{U_2}\over{R_1}}$ and R_1:



Now a lot of them can be combined. The resistors R_1, R_3, R_5 are in parallel, like also I_2 and I_4: \begin{align*} R_{135} &= R_1||R_3||R_5\\ I_{24} &= I_2 - I_4 = {{U_2}\over{R_1}} - I_4 \end{align*} The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following: \begin{align*} U_{24}

&= R_{135} \cdot I_{24} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot R_1||R_3||R_5 \end{align*}

On the right side of the last circuit, there is a voltage divider given by R_{135}, R_6, and R_7.
Therefore the voltage between A and B is given as: \begin{align*} U_{\rm AB} &= U_{24} \cdot {{R_7}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot {{R_7 \cdot R_1||R_3||R_5}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ \end{align*}

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit): \begin{align*} R_{\rm AB} &= R_7 || (R_6 + R_1||R_3||R_5) \\ \end{align*}

with $R_1||R_3||R_5 = 5 ~\Omega || 10 ~\Omega || 10 ~\Omega = 5 ~\Omega || 5 ~\Omega = 2.5 ~\Omega$:

\begin{align*} U_{\rm AB} &= ({{6.0 ~\rm{V}}\over{5.0 ~\Omega}} - 4.2 ~\Omega) \cdot {{15 ~\Omega \cdot 2.5 ~\Omega}\over{7.5 ~\Omega + 15 ~\Omega + 2.5 ~\Omega}} \\ R_{\rm AB} &= 15 ~\Omega|| (7.5 ~\Omega + 2.5 ~\Omega) \\ \end{align*}

Exercise E1 Temperature-dependent Resistance
(written test, approx. 6 % of a 60-minute written test, WS2022)

A thermistor is used as a temperature sensor in a refrigeration system. The thermistor has a resistance of $10 ~\rm{k}\Omega$ at $+25 ~°\rm{C}$.
2. Additionally, explain 2 ... Regarding question 2 ... refrigeration system ... Give an explanation for your answer.
Result
Its temperature coefficients are: $\alpha=0.01 ~{{1}\over{\rm{K}}}$ and $\beta=71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}}$
Result
The temperature inside the refrigeration system can reach down to $-40 ~°\rm{C}$.

\begin{align*} R &= 6.5 ~\rm{k}\Omega \\ \end{align*}

1. Calculate the resistance of the thermistor at $-40 ~°\rm{C}$.

The power of the resistor $P = U \cdot I = R \cdot I^2 = {{U^2}\over{R}}$ ... Resistors transfer electrical energy out of the circuit and generate heat. Therefore, a resistor might heat up the refrigeration system. This is equivalent to the heat flow.
Solution
Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \text{with } \Delta T = T_{\rm end} - T_{\rm start}\\ R &= 10 ~\rm{k}\Omega \cdot \left(1 + 0.01 ~{{1}\over{\rm{K}}} \cdot (-40~°\rm{C} - 25~°\rm{C}) + 71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}} \cdot (-40~°\rm{C} - 25~°\rm{C})^2 \right) \\ \end{align*}

Exercise E2 Analyzing complex Impedances
(written test, approx. 14 % of a 60-minute written test, WS2022)

components. (R and $\underline{X}_1$) shall be given.

Result

\underline{Z} = \left({{2}\over{3+4{\rm j}}}+5{\rm j} \right) \Omega \end{align*}

Solution

1. Calculate the physical values of the two components.

Solution

\begin{align*} C = 103 ~{\rm mF} \end{align*}

\begin{align*} R = 0.24 ~\Omega \\ L = 2.26 ~{\rm mH} \end{align*}

\begin{align*} \varphi_i &= -87.06° \end{align*}

Solution

\begin{align*} \underline{I} &= {{\underline{U}}\over{\underline{Z}}} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \cdot {{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }\over{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }}

The current and voltage are in phase once there is only a pure ohmic (= pure real) resulting impedance $\underline{Z} + \underline{X}_2$.

Therefore, the component must be a capacitor with the same absolute value of impedance.

\begin{align*} \underline{Z} &= \left({{2}\over{3+4{\rm j}}} + 5{\rm j} \right) ~\Omega \\ &= \left({{2}\over{3+4{\rm j}}} \cdot {{3-4{\rm j}}\over{3-4{\rm j}}} + 5{\rm j} \right) ~\Omega \\ &= \left({{2}\over{9+16}} \cdot (3-4{\rm j}) + 5{\rm j}

The absolute value $|\underline{I}|$ can be calculated as \begin{align*}

With the complex part comes the physical value: \begin{align*} X_L &= \omega L \\ L &= {{X_L}\over{2\pi \cdot f}} \\ &= {{4.68 ~\Omega}\over{2\pi \cdot 300 ~\rm{Hz}}} \\ \end{align*}

The phase φ_i can be calculated as \begin{align*} \varphi_i &= \arctan \left({{\Im()}\over{\Re()}} \right) \\ &= \arctan \left({{-4.68 ~\Omega}\over{0.24 ~\Omega}} \right) \\ \end{align*}

Exercise E1 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

Result

Choose an appropriate scaling factor and write it down.

This linear source is connected with an inductor of $330 ~ {\rm µH}$ and a capacitor of $0.22 ~{\rm µF}$, all in series.

Solution

Result

1. Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.

\begin{align*} Z &= {{\hat{U}}\over{\hat{I}}} \\ \hat{I} &= {{\hat{U}}\over{Z}} \\ \end{align*}

\begin{align*} Z_C &= {{1}\over{2\pi \cdot f \cdot C}}\\ &= {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}}}\\ \end{align*}

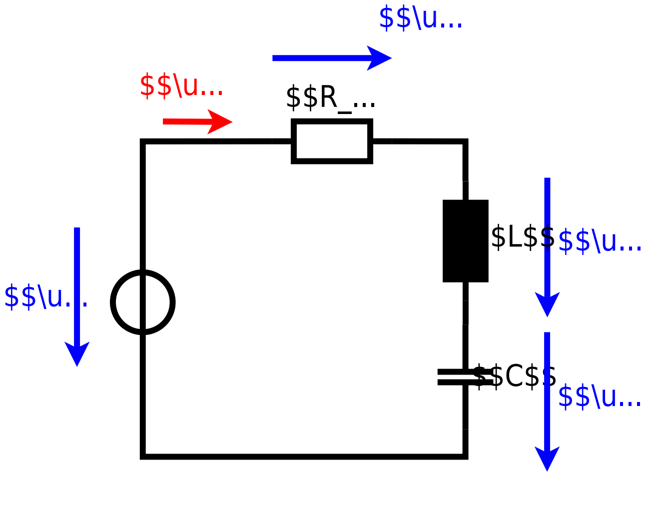
Result

With $I = {{1}\over{\sqrt{2}}}\cdot \hat{I}$: \begin{align*} I &= {{1}\over{\sqrt{2}}}\cdot {{\hat{U}}\over{Z}} \\ &= {{1}\over{\sqrt{2}}}\cdot {{3.0 ~{\rm V}}\over{19.28 ~\Omega}} \\ \end{align*}

\begin{align*} Z_L &= 2\pi \cdot f \cdot L \\ &= 2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}}\\ \end{align*}

\begin{align*} Z_C &= {{1}\over{2\pi \cdot f \cdot C}}\\ &= {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 330 ~{\rm µH}}}\\ \end{align*}

$$\begin{align*} \underline{Z} &= R + \underline{Z}_L + \underline{Z}_C \\ &= R + j \cdot {Z}_L - j \cdot {Z}_C \\ &= R + j \cdot ({Z}_L - {Z}_C) \\ |\underline{Z}| &= \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2 }\\ \end{align*}$$


$$\u...
$$\u...
$$R_...
$L$$
$$\u...
$$\u...
$$C$$
$$\u...

Exercise E3 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

Calculate the resistor values ...

2. A RL series circuit with $L_2=4.7 ~\rm µH$ is used in AC ... $U=1.0 ~\rm V$, $f_2=450 ~\rm kHz$, which generates a current $I_2=60 ~\rm mA$.

3. A RC parallel circuit ... $I_{3R}=1.0 ~\rm A$ through R_3.

Result

1. A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1=40 ~{\rm nF}$ at $f_1=4 ~{\rm MHz}$.

Solution

Solution \begin{align*} R_1 &= 1.00 ~\Omega \\ \end{align*}

Solution \begin{align*} R_2 &= 10.0 ~\Omega \\ \end{align*}

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by \begin{align*} {{\underline{U}}\over{\underline{I}}} &= R_2 + \underline{X}_{L2} \\ &= R_2 + {\rm j} \cdot \omega L \end{align*} Since ${\rm j} \cdot \omega L$ is perpendicular to R_2 this can be simplified to: \begin{align*} \left| {{\underline{U}}\over{\underline{I}}} \right|^2 &= |R_2|^2 + |\underline{X}_{L2}|^2 \\ \left({{U}\over{I}} \right)^2 &= {R_2}^2 + {X_{L2}}^2 \\ \end{align*}

This can be rearranged to get R_2: \begin{align*} R_2 &= \sqrt{ \left({{U}\over{I}} \right)^2 - X_{L2}^2 } \\ &= \sqrt{ \left({{1~{\rm V}}\over{60~\rm mA}} \right)^2 - (2\pi \cdot 450~{\rm kHz} \cdot 4.7 ~ {\rm µH})^2 } \\ \end{align*}

Parallel circuit means that the voltage is the same on R_3 and C_3.

\begin{align*} R_1 &= |\underline{X}_{C1}| \\ &= {{1}\over{2\pi \cdot f \cdot C_1}} \\ &= {{1}\over{2\pi \cdot 4 ~{\rm MHz} \cdot 40 ~\rm nF}} \\ \end{align*}

... it gets clear, that $\underline{I}_{3R}$ is perpendicular to $\underline{I}_{3C}$ (It has to, since R_3 is perpendicular to ${\rm j}\cdot{X}_{3C}$, too).

Therefore, the resulting current of the parallel circuit is given as: \begin{align*} \underline{I}_{3} &= \underline{I}_{3R} + \underline{I}_{3C} \\ |\underline{I}_{3}|^2 &= |\underline{I}_{3R}|^2 + |\underline{I}_{3C}|^2 \\ {I}_{3C} &= \sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2} \end{align*}

Back to the first formula: \begin{align*} R_3 \cdot {I}_{3R} &= {X}_{3C} \cdot {I}_{3C} \\ R_3 &= {X}_{3C} \cdot {{{I}_{3C}}\over{{I}_{3R}}} \\ &= {{1}\over{2\pi \cdot f \cdot C_3}} \cdot {{\sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2}}\over{{I}_{3R}}} \\ \end{align*}

Exercise E4 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of a Nichrome wire with a round cross-section is used in an electric oven; a power dissipation (= heat flow) of $P=40 ~\rm{W}$ is necessary.

2. The heating element is to heat the oven to a temperature of $180 ~°\rm{C}$. For this, Nichrome is a common Nickel-Chrome alloy for heating elements. Calculate the current I needed to operate it.

Result

The Nichrome wire has a resistivity of $1.10\cdot 10^{-6} ~\Omega \rm{m}$.

The heating element is $3 ~\rm{m}$ long and has a diameter of $3.57 ~\rm{mm}$.

1. Calculate the resistance R of the heating element.

Solution \begin{align*} R=&1.10 ~\rm{\Omega} \end{align*}

Solution

\begin{align*} P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I=

\sqrt{\frac{P}{R}} = \sqrt{\frac{40 ~\rm{W}}{0.33 ~\Omega}} \end{align*}

\begin{align*} R &= \rho \cdot \frac{l}{A} && | \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} && \\ R &= 1.10\cdot 10^{-6} ~\Omega \rm{m} \cdot \frac{4 \cdot 3~\rm{m}}{(3.57\cdot 10^{-3}~\rm{m})^2 \cdot \pi} && \\ \end{align*}

Exercise E5 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

[illegible]
[illegible] The voltage across the capacitor is again $0 ~\rm{V}$ at the moment $t_0=0 ~\rm{s}$ when the switch S_1 is closed. Calculate the voltage $u_c (t_2)$ across the capacitor at $t_2=1 ~\rm{ms}$ after closing the switch.

[illegible]

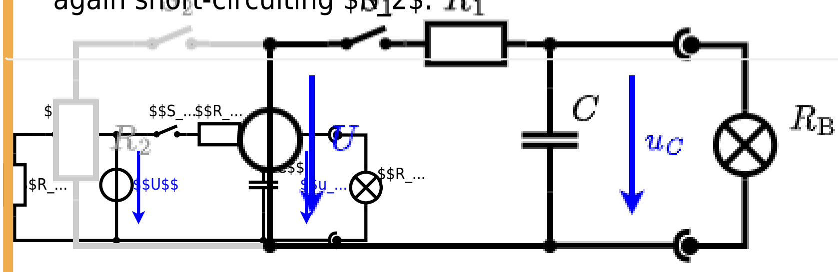
Hint: To solve this, first create an equivalent linear voltage source from U, R_1, and $R_\rm B$.

[illegible]

[illegible]

[illegible]

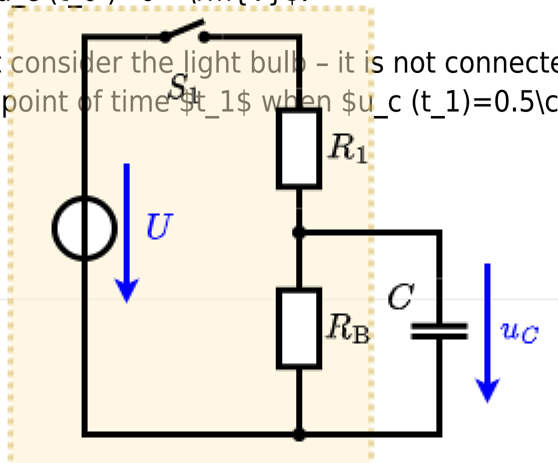
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2.

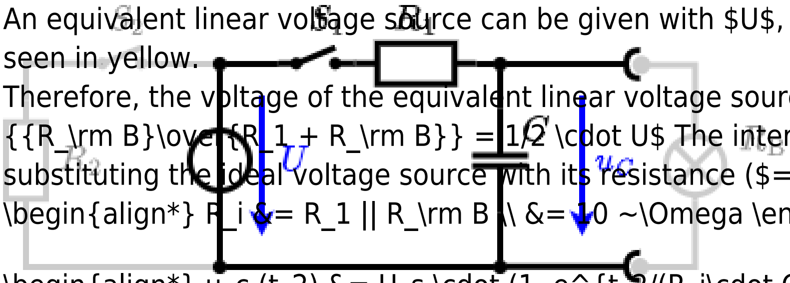


The circuit contains a voltage source $U=12 ~\rm{V}$, a switch S_1, a resistor of $R_1=20 ~\Omega$ and a capacitor of $C=100 ~\rm{µF}$.
The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0=0 ~\rm{s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c (t_0)=0 ~\rm{V}$.

1. First do not consider the light bulb – it is not connected to the RC circuit.
Calculate the point of time t_1 when $u_c (t_1)=0.5\cdot U$.

Solution





An equivalent linear voltage source can be given with U, R_1, and $R_\rm B$ as seen in yellow.
Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot {{R_\rm B}\over{R_1 + R_\rm B}} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 ~\Omega$, short-circuit).

$$\begin{align*} R_i &= R_1 || R_\rm B \\ &= 10 ~\Omega \end{align*}$$

So, here only R_1 and C gives the time constant: $\tau = R_1\cdot C$

$$\begin{align*} u_c (t_2) &= U_s \cdot (1- e^{t_2/(R_i\cdot C)}) \\ &= {{1}\over{2}} \cdot U \cdot (1- e^{1~\rm{ms}/(10 ~\Omega \cdot 100 ~\rm{µF})}) \end{align*}$$

The following formula describes the time course of $u_C(t)$ which has to be $u_c (t_1)=0.5\cdot U$:

$$\begin{align*} u_c (t) = U \cdot (1- e^{t/\tau}) = 0.5\cdot U \end{align*}$$

It has to be rearranged to t

$$\begin{align*} (1- e^{t/\tau}) &= 0.5 \\ e^{t/\tau} &= 0.5 \\ t/\tau &= \ln(0.5) \\ t &= \tau \cdot \ln(0.5) \\ t &= R_1 \cdot C \cdot \ln(0.5) \end{align*}$$

Exercise E6 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following circuit with $R_1=200 ~\Omega$, $R_2=R_3=100 ~\Omega$ ... R_5 is given.

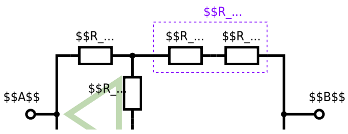
2. The switch shall now be closed. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Result

Solution

$$\begin{align*} R_{\rm eq} &= 142.8 ~\Omega \\ \end{align*}$$

Now a wye-delta transformation is necessary.

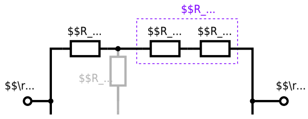


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as: \begin{align*} R_{Y} &= {{R_2 \cdot R_2}\over{R_2 + R_2 + R_2}} \\ &= {{(100 ~\Omega)^2}\over{3 \cdot 100 ~\Omega}} \\ &= {{1}\over{3}} \cdot 100 ~\Omega = 33.33 ~\Omega \end{align*}

The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= R_Y + (R_Y + R_1 + R_1)||(R_Y + R_2)\\ R_{\rm eq} &= 33.33 ~\Omega + (33.33 ~\Omega + 400 ~\Omega)||(33.33 ~\Omega + 100 ~\Omega)\\ \end{align*}

1. The switch shall now be open. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Solution

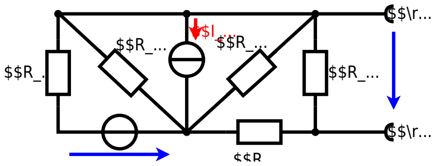


The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= (R_2 + R_1 + R_1)||(R_2 + R_2)\\ R_{\rm eq} &= (100 ~\Omega + 200 ~\Omega + 200 ~\Omega)&&||(100 ~\Omega + 100 ~\Omega) &&\\ R_{\rm eq} &= (500 ~\Omega)&&||(200 ~\Omega)&&\\ R_{\rm eq} &= {{500 ~\Omega \cdot 200 ~\Omega }\over{500 ~\Omega + 200 ~\Omega}}&&\\ \end{align*}

Exercise E3 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

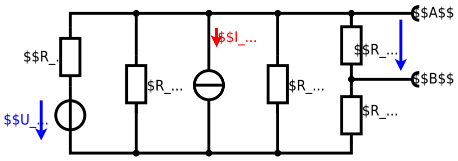
\begin{align*} U_{\rm s} &= U_{\rm AB} &&= 4.5 ~\rm{V}\\ R_{\rm i} &= R_{\rm AB} &&= 6 ~\Omega \end{align*}



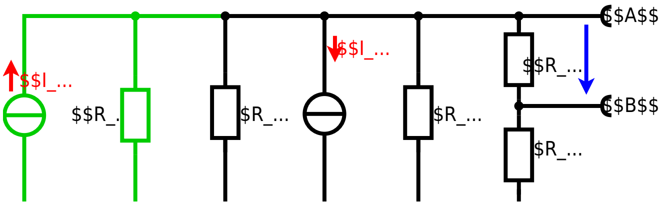
Calculated the internal resistance $R_\rm i$ and the source voltage $U_\rm s$ of an equivalent linear voltage source on the connectors $\rm A$ and $\rm B$. \begin{align*} R_1=5.0 ~\Omega, && U_2=6.0 ~\rm{V}, && R_3= 10 ~\Omega, \\ I_4=4.2 ~\rm{A}, && R_5=10 ~\Omega , && R_6=7.5 ~\Omega, \\ R_7=15 ~\Omega && && \end{align*} Use equivalent sources in order to simplify the circuit!

Solution

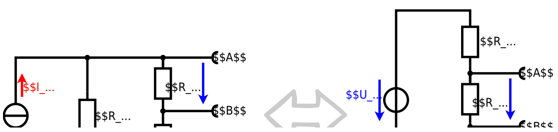
The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2={{U_2}\over{R_1}}$ and R_1:



Now a lot of them can be combined. The resistors R_1, R_3, R_5 are in parallel, like also I_2 and I_4: \begin{align*} R_{135} &= R_1||R_3||R_5\\ I_{24} &= I_2 - I_4 = {{U_2}\over{R_1}} - I_4 \end{align*} The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following: \begin{align*} U_{24}

&= R_{135} \cdot I_{24} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot R_1||R_3||R_5 \end{align*}

On the right side of the last circuit, there is a voltage divider given by R_{135}, R_6, and R_7.
Therefore the voltage between A and B is given as: \begin{align*} U_{\rm AB} &= U_{24} \cdot {{R_7}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot {{R_7 \cdot R_1||R_3||R_5}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ \end{align*}

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit): \begin{align*} R_{\rm AB} &= R_7 || (R_6 + R_1||R_3||R_5) \\ \end{align*}

with $R_1||R_3||R_5 = 5 ~\Omega || 10 ~\Omega || 10 ~\Omega = 5 ~\Omega || 5 ~\Omega = 2.5 ~\Omega$:

\begin{align*} U_{\rm AB} &= ({{6.0 ~\rm{V}}\over{5.0 ~\Omega}} - 4.2 ~\Omega) \cdot {{15 ~\Omega \cdot 2.5 ~\Omega}\over{7.5 ~\Omega + 15 ~\Omega + 2.5 ~\Omega}} \\ R_{\rm AB} &= 15 ~\Omega|| (7.5 ~\Omega + 2.5 ~\Omega) \\ \end{align*}

Exercise E2 Temperature-dependent Resistance
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Its temperature coefficients are: $\alpha=0.01 ~{{1}\over{\rm{K}}}$ and $\beta=71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}}$
The temperature inside the refrigeration system can reach down to $-40 ~°\rm{C}$.

2. Additionally, explain ... Give an explanation for your answer.

Result

1. Calculate the resistance of the thermistor at $-40 ~°\rm{C}$.

Result

\begin{align*} R &= 6.5 ~\rm{k}\Omega \\ \end{align*}

Resistors transfer electrical energy out of the circuit and generate heat. Therefore, a resistor can heat up the refrigeration system.

Solution

The power of the resistor $P = U \cdot I = R \cdot I^2 = {{U^2}\over{R}}$ is equivalent to the heat flow.
Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \text{with } \Delta T = T_{\rm end} - T_{\rm start}\\ R &= 10 ~\rm{k}\Omega \cdot \left(1 + 0.01 ~{{1}\over{\rm{K}}} \cdot (-40~°\rm{C} - 25~°\rm{C}) + 71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}} \cdot (-40~°\rm{C} - 25~°\rm{C})^2 \right) \\ \end{align*}

Exercise E5 Analyzing complex Impedances
(written test, approx. 14 % of a 60-minute written test, WS2022)

A complex impedance ... components. (R and $\underline{X}_1$) shall be given.

After analysis, the following formula for the impedance was extracted:

\begin{align*} \underline{Z} = \left({{2}\over{3+4{\rm j}}}+5{\rm j} \right) \Omega \end{align*}

1. Calculate the physical values of the two components.

Solution

\begin{align*} \underline{Z} &= \left({{2}\over{3+4{\rm j}}} + 5{\rm j} \right) ~\Omega \\ &= \left({{2}\over{3+4{\rm j}}} \cdot {{3-4{\rm j}}\over{3-4j}} + 5{\rm j} \right) ~\Omega \\ &= \left({{2}\over{9+16 }} \cdot (3-4{\rm j}) + 5{\rm j} \right) ~\Omega \\ &= 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega \end{align*}

With the complex part comes the physical value: \begin{align*} X_L &= \omega L \\ L &= {{X_L}\over{2\pi \cdot f}} \\ &= {{4.68 ~\Omega}\over{2\pi \cdot 300 ~\rm{Hz}}} \\ \end{align*}

Result

\begin{align*} R &= 0.24 ~\Omega \\ L &= 2.26 ~\rm{mH} \end{align*}

2. Now a voltage source with $\underline{U} = 50 ~\rm V$ and $f = 300 ~\rm Hz$ is applied. Calculate the current and phase through the circuit.

Solution

\begin{align*} \underline{I} &= {{\underline{U}}\over{\underline{Z}}} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \cdot {{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }\over{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }} \\ &= {{50 ~\rm{V}\cdot (0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega)}\over{ (0.24 ~\Omega)^2 + (4.68 ~\Omega)^2 }} \end{align*}

The absolute value $|\underline{I}|$ can be calculated as \begin{align*} |\underline{I}| &= {{|\underline{U}|}\over{|\underline{Z}|}} \\ &= {{50 ~\rm{V}}\over{| 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega |}} \\ &= {{50 ~\rm{V}}\over{\sqrt{ (0.24 ~\Omega)^2 + (4.68 ~\Omega)^2 }}} \end{align*}

The phase φ_i can be calculated as \begin{align*} \varphi_i &= \arctan \left({{\Im()}\over{\Re()}} \right) \\ &= \arctan \left({{-4.68 ~\Omega}\over{0.24 ~\Omega}} \right) \\ \end{align*}

Result

\begin{align*} |\underline{I}| &= 10.67 ~\rm{A} \\ \varphi_i &= -87.06° \end{align*}

3. Now add a third component so that the current and voltage are in phase. Calculate the value.

Solution

The current and voltage are in phase once there is only a pure ohmic (= pure real) resulting impedance $\underline{Z} + \underline{X}_2$.
Therefore, the component mus be a capacitor with the same absolute value of impedance: $|\underline{X}_C| = |\underline{X}_L|$ \begin{align*} X_C &= {{1}\over{\omega \cdot C}} = X_L \\ C &= {{1}\over{\omega \cdot X_L}} \\ &= {{1}\over{2\pi \cdot 300 ~\rm{Hz} \cdot 4.68 ~\Omega}} \\ \end{align*}

Result

\begin{align*} C = 103 ~\rm{µF} \end{align*}

Exercise E7 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

A circuit is driven by a voltage source $u(t) = 3.0 ~{\rm V} \cdot \sin(2\pi \cdot 15 ~{\rm kHz} \cdot t)$ with an internal resistance of $10 ~\Omega$.
This linear source is connected with an inductor of $330 ~ {\rm µH}$ and a capacitor of $0.22 ~{\rm µF}$, all in series.

1. Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.

2. Calculate the complex impedances $\underline{Z}_C$, $\underline{Z}_L$ and $\underline{Z}$. Choose an appropriate scaling factor and write it down.

Solution

\begin{align*} Z_C &= {{1}\over{2\pi \cdot f \cdot C}}\\ &= {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}}}\\ \end{align*}

\begin{align*} Z_L &= 2\pi \cdot f \cdot L\\ &= 2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}}\\ \end{align*}

Result

\begin{align*} Z_L &= 31.1 ~\Omega \\ Z_C &= 48.2 ~\Omega \\ Z &= 19.8 ~\Omega \end{align*}

3. Calculate the current through the circuit.

Solution

\begin{align*} Z &= {{\hat{U}}\over{\hat{I}}} \\ \hat{I} &= {{\hat{U}}\over{Z}} \\ \end{align*}

With $I = {{1}\over{\sqrt{2}}}\cdot \hat{I}$: \begin{align*} I &= {{1}\over{\sqrt{2}}}\cdot {{\hat{U}}\over{Z}} \\ &= {{1}\over{\sqrt{2}}}\cdot {{3.0 ~{\rm V}}\over{19.28 ~\Omega}} \\ \end{align*}

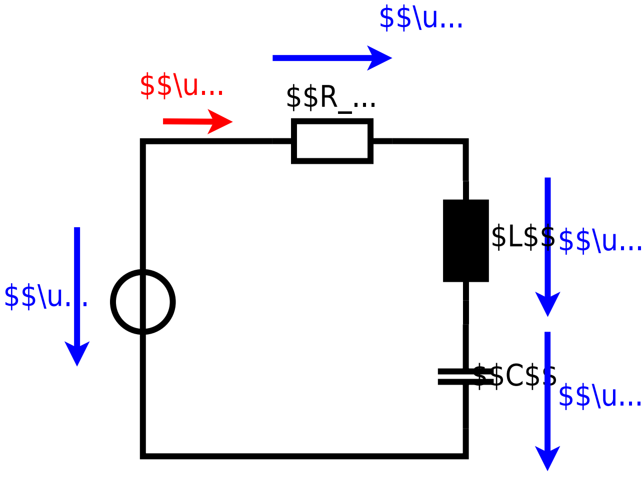
Result

\begin{align*} I = 107.3 ~{\rm mA} \end{align*}

\begin{align*} Z_C &= {{1}\over{2\pi \cdot f \cdot C}}\\ &= {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 330 ~{\rm µH}}}\\ \end{align*}

$$\begin{align*} \underline{Z} &= R + \underline{Z}_L + \underline{Z}_C \\ &= R + j \cdot {Z}_L - j \cdot {Z}_C \\ &= R + j \cdot ({Z}_L - {Z}_C) \\ |\underline{Z}| &= \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2 }\\ \end{align*}$$

`$$10...`


$$\u...
$$\u...
$$R_...
$L$$
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Exercise E6 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

Calculate the resistor values in the following circuits:

1. A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1=40 ~{\rm nF}$ at $f_1=4 ~{\rm MHz}$.
2. A RL series circuit with $L_2=4.7 ~\rm µH$, used in an AC voltage source $U_2=1.0 ~\rm V$, $f_2=450 ~\rm kHz$, which generates a current $I_2=60 ~\rm mA$.
3. A RC parallel circuit with $f_3=200 ~\rm kHz$ generates a current $I_3=1.3 ~\rm A$ and a current $I_{3R}=1.0 ~\rm A$ through R_3.

Result

Solution

$$\begin{align*} R_1 &= 1.00 ~\Omega \\ \end{align*}$$

Solution

$$\begin{align*} R_2 &= 10.0 ~\Omega \\ \end{align*}$$

Solution

$$\begin{align*} R_3 &= 70.0 ~\Omega \\ \end{align*}$$

$$\begin{align*} R_1 &= |\underline{X}_{C1}| \\ &= {{1}\over{2\pi \cdot f\cdot C_1}} \\ &= {{1}\over{2\pi \cdot 4 ~{\rm MHz} \cdot 40 ~\rm nF}} \\ \end{align*}$$

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by $$\begin{align*} {{\underline{U}}\over{\underline{I}}} &= R_2 + \underline{X}_{L2} \\ &= R_2 + {\rm j} \cdot \omega L \end{align*}$$ Since ${\rm j} \cdot \omega L$ is perpendicular to R_2 this can be simplified to: $$\begin{align*} \left| {{\underline{U}}\over{\underline{I}}} \right|^2 &= |R_2|^2 + |\underline{X}_{L2}|^2 \\ \left({{U}\over{I}} \right)^2 &= {R_2}^2 + {X_{L2}}^2 \\ \end{align*}$$

This can be rearranged to get R_2: $$\begin{align*} R_2 &= \sqrt{ \left({{U}\over{I}} \right)^2 - {X_{L2}}^2 } \\ &= \sqrt{ \left({{1~\rm V}\over{60~\rm mA}} \right)^2 - (2\pi \cdot 450~{\rm kHz} \cdot 4.7 ~ {\rm µH})^2 } \\ \end{align*}$$

Parallel circuit means that the voltage is the same on R_3 and C_3. It gets clear, that $\underline{I}_{3R}$ is perpendicular to $\underline{I}_{3C}$ (It has to, since R_3 is perpendicular to ${\rm j}\cdot{X}_{3C}$, too).

Therefore, the resulting current of the parallel circuit is given as: $$\begin{align*} \underline{I}_{3} &= \underline{I}_{3R} + \underline{I}_{3C} \\ |\underline{I}_{3}|^2 &= |\underline{I}_{3R}|^2 + |\underline{I}_{3C}|^2 \\ {I}_{3C} &= \sqrt{ {I}_{3}^2 - {I}_{3R}^2 } \\ \end{align*}$$

Back to the first formula: $$\begin{align*} R_3 \cdot {I}_{3R} &= {X}_{3C} \cdot {I}_{3C} \\ R_3 &= {X}_{3C} \cdot {{{I}_{3C}}\over{{I}_{3R}}} \\ &= {{1}\over{2\pi \cdot f \cdot C_3}} \cdot {{\sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2}}\over{{I}_{3R}}} \\ \end{align*}$$

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of a Nichrome wire with a round cross-section is used in an electric oven. For this a power dissipation (= heat flow) of $P=40 ~\rm{W}$ is necessary.

Nichrome is a common Nickel-Chrome alloy for heating elements.

The Nichrome wire has a resistivity of $1.10\cdot 10^{-6} ~\Omega \rm{m}$.

The heating element is $3 ~\rm{m}$ long and has a diameter of $3.57 ~\rm{mm}$.

1. Calculate the resistance R of the heating element.
2. The heating element needs to heat the oven to a temperature of $180 ~°\rm{C}$. Calculate the current I needed to operate it.

Result

Solution

$$\begin{align*} R=&0.330 ~\rm{\Omega} \end{align*}$$

Solution

\begin{align*} P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I=

\sqrt{\frac{P}{R}} = \sqrt{\frac{40 ~\rm{W}}{0.33 ~\Omega}} \end{align*}

\begin{align*} R &= \rho \cdot \frac{l}{A} && | \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} && \\ R &= 1.10\cdot 10^{-6} ~\Omega \rm{m} \cdot \frac{4 \cdot 3~\rm{m}}{(3.57\cdot 10^{-3}~\rm{m})^2 \cdot \pi} && \\ \end{align*}

Exercise E4 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

4. Now, a consumer (the light bulb) is connected as $R_\rm B=20 ~\Omega$ to the capacitor; S_2 is still open. The voltage across the capacitor is again $0 ~\rm{V}$ at the moment $t_0=0 ~\rm{s}$ when the switch S_1 is closed. Calculate the voltage $u_c (t_2)$ across the capacitor at $t_2=1 ~\rm{ms}$ after closing the switch.

Result

Hint: To solve this, first create an equivalent linear voltage source from U, R_1, and $R_\rm B$.

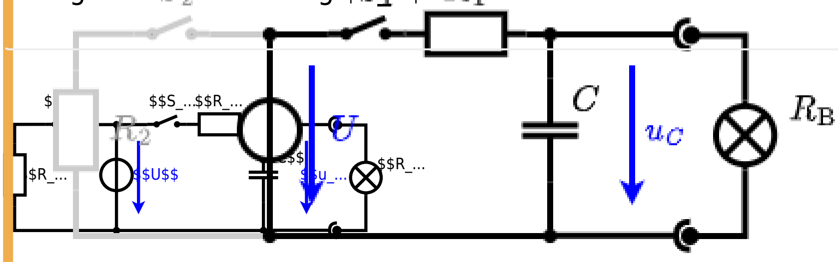
\begin{align*} t_1 = 3.79 ~\rm{ms} \end{align*}

Solution

It does not change anything. The ideal voltage source provides the voltage of $U=12~\rm{V}$ independent of this resistor.

\begin{align*} \Delta W_R &= \Delta W_C \\ &= {{1}\over{2}} \cdot C \cdot U^2 \\ &= {{1}\over{2}} \cdot 100 ~\rm{µF} \cdot (12 ~\rm{V})^2 \end{align*}

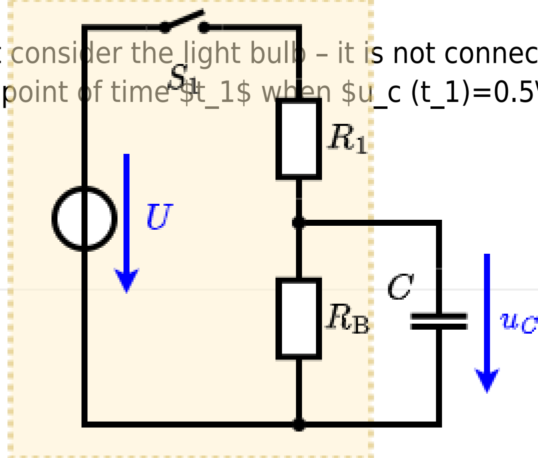
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2.



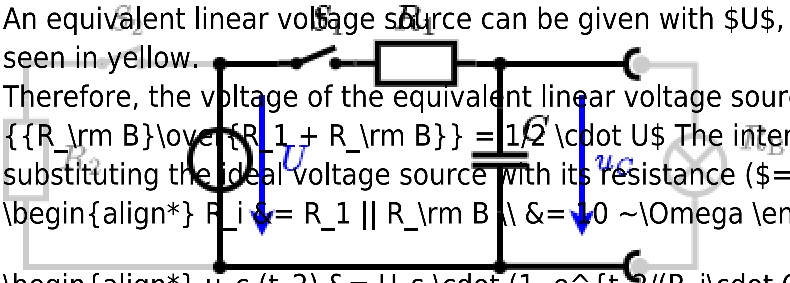
The circuit contains a voltage source $U=12 ~\rm{V}$, a switch S_1, a resistor of $R_1=20 ~\Omega$ and a capacitor of $C=100 ~\rm{µF}$.
The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0=0 ~\rm{s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c (t_0)=0 ~\rm{V}$.

1. First do not consider the light bulb – it is not connected to the RC circuit.
Calculate the point of time t_1 when $u_c (t_1)=0.5\cdot U$.

Solution



An equivalent linear voltage source can be given with U, R_1, and $R_\rm B$ as seen in yellow.
Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot {{R_\rm B}\over{R_1 + R_\rm B}} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 ~\Omega$, short-circuit). \begin{align*} R_i &= R_1 || R_\rm B \\ &= 10 ~\Omega \end{align*}



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

\begin{align*} u_c (t_2) &= U_s \cdot (1- e^{t_2/(R_i\cdot C)}) \\ &= {{1}\over{2}} \cdot U \cdot (1- e^{1~\rm{ms}/(10 ~\Omega \cdot 100 ~\rm{µF})}) \end{align*}

The following formula describes the time course of $u_C(t)$ which has to be $u_c (t_1)=0.5\cdot U$: \begin{align*} u_c (t) = U \cdot (1- e^{t/\tau}) = 0.5\cdot U \end{align*} It has to be rearranged to t \begin{align*} (1- e^{t/\tau}) &= 0.5 \\ e^{t/\tau} &= 0.5 \\ t/\tau &= ln(0.5) \\ t &= \tau \cdot ln(0.5) \\ t &= R_1 \cdot C \cdot ln(0.5) \end{align*}

Exercise E3 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

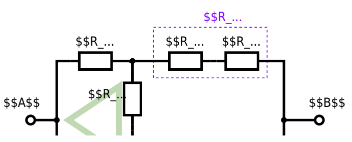
2. The switch shall now be closed. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.
The following circuit with $R_1=200 ~\Omega$, $R_2=R_3=100 ~\Omega$ ... $$$ is given.

Result

Solution

\begin{align*} R_{\rm eq} &= 142.8 ~\Omega \\ \end{align*}

Now a wye-delta transformation is necessary.

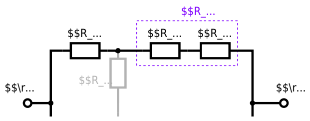


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as: \begin{align*} R_{Y} &= {{R_2 \cdot R_2}\over{R_2 + R_2 + R_2}} \\ &= {{(100 ~\Omega)^2}\over{3 \cdot 100 ~\Omega}} \\ &= {{1}\over{3}} \cdot 100 ~\Omega = 33.33 ~\Omega \end{align*}

The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= R_Y + (R_Y + R_1 + R_1)||(R_Y + R_2)\\ R_{\rm eq} &= 33.33 ~\Omega + (33.33 ~\Omega + 400 ~\Omega)||(33.33 ~\Omega + 100 ~\Omega)\\ \end{align*}

1. The switch shall now be open. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= (R_2 + R_1 + R_1)||(R_2 + R_2)\\ R_{\rm eq} &= (100 ~\Omega + 200 ~\Omega + 200 ~\Omega)&&||(100 ~\Omega + 100 ~\Omega) &&\\ R_{\rm eq} &= (500 ~\Omega)&&||(200 ~\Omega)&&\\ R_{\rm eq} &= {{500 ~\Omega \cdot 200 ~\Omega }\over{500 ~\Omega + 200 ~\Omega}}&&\\ \end{align*}