

# Exam Winter Semester 2022

## Student Group

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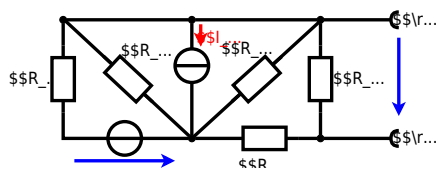
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**Exercise E1 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$\begin{aligned} U_{\text{rs}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \text{ } \Omega \end{aligned}$$



Calculate the internal resistance  $R_{\text{i}}$  and the source voltage  $U_{\text{rs}}$  of an equivalent linear voltage source on the connectors  $\text{A}$  and  $\text{B}$ .  
 $R_1=5.0 \text{ } \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3= 10 \text{ } \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \text{ } \Omega$ ,  $R_6=7.5 \text{ } \Omega$ ,  $R_7=15 \text{ } \Omega$   
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{135} + I_{24} \cdot R_4$$

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left( \frac{U_2}{R_1} - I_4 \right) \cdot \left( R_1 || R_3 || R_5 \right)$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \left( \frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \text{ A} \cdot \left( \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

**Exercise E1 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)**

2. The diagram explains the effect of resistance on refrigeration systems. The circuit has a resistance of  $10 \text{ k}\Omega$  at  $25^\circ\text{C}$  and a  $25 \text{ W}$  power source.

Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

**Result**  
The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

$$R = 10 \text{ k}\Omega$$

The power of the resistor is  $P = U \cdot I$  and  $Q = P \cdot t$ . Therefore, a solution is to use a heat sink to cool the resistor.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \text{ k}\Omega \cdot \left( 1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

### Exercise E2 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance  $Z$  of the circuit shown in the figure through the components.  $R$  and  $X_L$  shall be given.

After analysis, the full bridge circuit can be simplified to a series circuit in phasor domain.  $Z = (2 + j4) \parallel (1 + j5) + 5$

Solution  
.. Calculate the physical values of the two components.  
Solution  $R = 2 \Omega$   $X_L = 4 \Omega$

Solution  
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \parallel \&= \{ \{50 \text{ V} \} \over \{ 2 + j4 \} \parallel \{ 1 + j5 \} + 5 \}$$
  
The voltage across the capacitor is  $U_C = I \cdot X_C = 50 \cdot \frac{1}{j5} = -j10 \text{ V}$   
The voltage across the inductor is  $U_L = I \cdot X_L = 50 \cdot j4 = j200 \text{ V}$   
The voltage across the resistor is  $U_R = I \cdot R = 50 \cdot 2 = 100 \text{ V}$   
The phase angle  $\varphi$  can be calculated as  $\varphi = \arctan \left( \frac{\text{Im}(Z)}{\text{Re}(Z)} \right) = \arctan \left( \frac{4.68}{0.24} \right)$

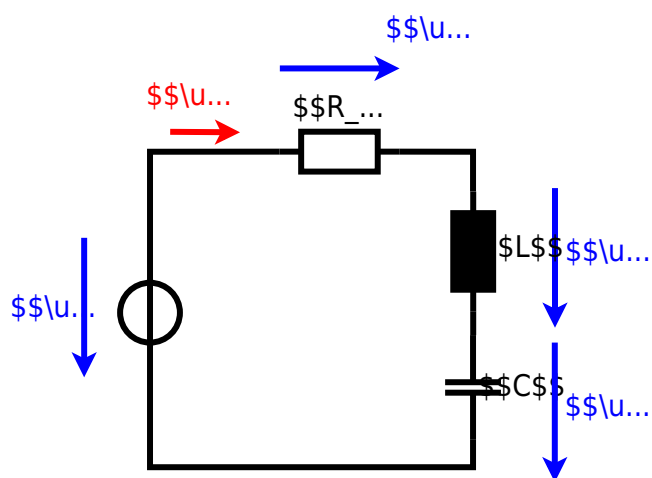
### Exercise E1 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance  $Z$  of the circuit shown in the figure. The voltage source  $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V is connected to a series circuit of an inductor of  $330 \mu\text{H}$  and a capacitor of  $0.22 \mu\text{F}$ .

Solution  
Result  
.. Draw the circuit diagram of the bridge circuit.

Solution  
Result  
$$Z = (330 \cdot 10^{-6} \cdot j\omega) \parallel \left( \frac{1}{0.22 \cdot 10^{-6} \cdot (-j\omega)} \right) + 3$$
  
With  $\omega = 2\pi \cdot 15 = 94.2 \text{ rad/s}$   
$$Z = (330 \cdot 10^{-6} \cdot j94.2) \parallel \left( \frac{1}{0.22 \cdot 10^{-6} \cdot (-j94.2)} \right) + 3$$





**Exercise E3 Impedances at different Frequencies**  
**(written test, approx. 18 % of a 60-minute written test, WS2022)**

2. A series circuit consists of a resistor with a resistance of  $R_1 = 1.00 \text{ k}\Omega$  and a capacitor with a capacitance of  $C_1 = 40 \text{ nF}$ . The voltage across the resistor is  $U_{R_1} = 100 \text{ V}$  at a frequency of  $f = 4 \text{ MHz}$ . Calculate the absolute value of the impedance of the capacitor  $Z_C$  and the total impedance  $Z_{\text{total}}$  of the circuit.

Solution

$$Z_C = \frac{1}{j\omega C_1} = \frac{1}{j \cdot 2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}} = -j0.995 \text{ k}\Omega$$

$$Z_{\text{total}} = R_1 + Z_C = 1.00 \text{ k}\Omega - j0.995 \text{ k}\Omega$$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R_1$  and  $Z_C$  combined is given by  $Z_{\text{total}} = R_1 + Z_C$ .  
 Parallel circuit means that the voltage is the same on  $R_1$  and  $Z_C$ .  

$$\frac{U_{R_1}}{R_1} = \frac{U_{Z_C}}{Z_C} \Rightarrow I = \frac{U_{R_1}}{R_1} = \frac{U_{Z_C}}{Z_C}$$
  
 Since  $U_{R_1} = U_{Z_C} = 100 \text{ V}$ , we have  $I = \frac{100}{R_1} = \frac{100}{1000} = 0.1 \text{ A}$ .  

$$I = \frac{U_{Z_C}}{Z_C} \Rightarrow Z_C = \frac{U_{Z_C}}{I} = \frac{100}{0.1} = 1000 \text{ }\Omega = 1 \text{ k}\Omega$$
  
 Therefore, the resulting current of the parallel circuit is given as:  

$$I_{\text{total}} = I_{R_1} + I_{Z_C} = 0.1 \text{ A} + 0.1 \text{ A} = 0.2 \text{ A}$$
  
 This current is the same as the current through  $R_1$ .  

$$I = \frac{U_{R_1}}{R_1} = \frac{100}{1000} = 0.1 \text{ A}$$
  
 Back to the first formula: 
$$R_1 \cdot I = U_{R_1} \Rightarrow R_1 = \frac{U_{R_1}}{I} = \frac{100}{0.1} = 1000 \text{ }\Omega = 1 \text{ k}\Omega$$

**Exercise E4 Resistance of a Wire by Resistivity**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. For a heating element, it is required to heat the water in a tank with a volume of  $V = 180 \text{ l}$  by electric power dissipation (= heat flow) of  $P = 40 \text{ W}$ . Calculate the current  $I$  needed to operate the heating element.  
 The Nichrome wire has a resistivity of  $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$ .  
 The heating element is  $l = 3 \text{ m}$  long and has a diameter of  $d = 3.57 \text{ mm}$ .  
 Calculate the resistance  $R$  of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \Rightarrow I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

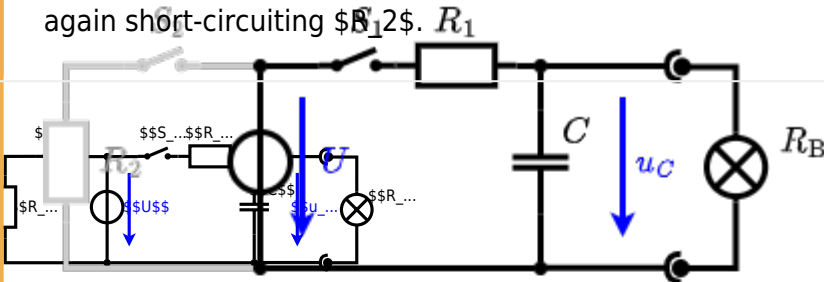
**Exercise E5 Charging Capacitors**  
**(written test, approx. 16 % of a 60-minute written test, WS2022)**

The circuit (with the light bulb) is a series of  $R_1$  and  $R_2$  and a capacitor  $C$  as indicated in Figure 5.2. Initially the voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

The ideal voltage source  $U$  is in series with  $R_1$  and  $R_2$ . The voltage  $u_c$  is independent of this series combination.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $S_2$ .

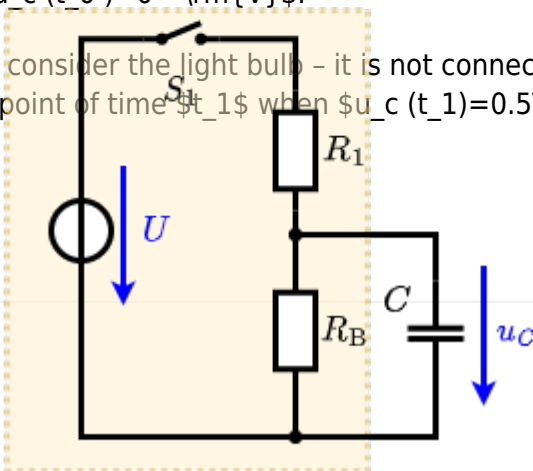


The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ } \Omega$  and a capacitor of  $C = 100 \text{ } \mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first tasks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

**Solution**



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \Omega$ , short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ : 
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



**Exercise E6 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved at once,  $R_1 = R_2 = R_3 = 10 \Omega$  and the voltage  $U = 10V$  is given.  $R_B$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

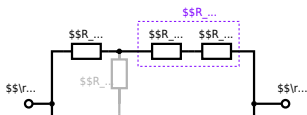
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{\text{eq}}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E3 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



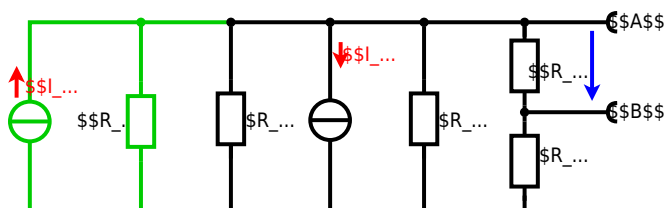
Calculated the internal resistance  $R_{\text{int}}$  and the source voltage  $U_{\text{oc}}$  of an equivalent linear voltage source on the connectors  $\text{A}$  and  $\text{B}$ .  $R_1=5.0 \text{ } \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3= 10 \text{ } \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \text{ } \Omega$ ,  $R_6=7.5 \text{ } \Omega$ ,  $R_7=15 \text{ } \Omega$  Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ : 
$$R_{135} = R_1 || R_3 || R_5$$
 
$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following: 
$$U_{24} = I_{24} \cdot (R_6 || R_7)$$

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left( \frac{U_2}{R_1} - I_4 \right) \cdot \left( R_1 || R_3 || R_5 \right)$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \left( \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot 15 \Omega \cdot 2.5 \Omega \cdot \frac{1}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

**Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator, explained in the effect of resistance on refrigeration systems, has a resistance of  $15 \Omega$  at  $25^\circ\text{C}$  and a temperature coefficient of  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

**Result** The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

Calculate the resistance of the thermistor at  $-40^\circ\text{C}$ .

The power transferred to the resistor is  $P = U^2 / R$  and  $Q = P \cdot t$ . Therefore, a solution is to increase the resistance of the thermistor to reduce the heat flow.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\text{with } \Delta T = T_{\text{end}} - T_{\text{start}} \implies R = 15 \Omega \cdot \left( 1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

### Exercise E5 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance  $Z$  of the circuit shown in the figure. The voltage  $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V and the current  $i(t) = 0.24 \cdot \sin(2\pi \cdot 15 \cdot t - \varphi)$  A shall be given.

After analysis, the following complex impedance  $Z$  shall be extracted and the phase  $\varphi$  in phase angle  $^\circ$  shall be given.

Solution  
.. Calculate the physical values of the two components.  
Solution  $R = 10 \Omega$  and  $X_L = 2\pi \cdot 15 \cdot 0.01 = 0.942 \Omega$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \Rightarrow \quad \underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{3.0 \angle 0^\circ}{0.24 \angle -\varphi} = 12.5 \angle \varphi$$

The voltage  $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V and the current  $i(t) = 0.24 \cdot \sin(2\pi \cdot 15 \cdot t - \varphi)$  A shall be given. The resulting impedance  $Z$  shall be extracted and the phase  $\varphi$  in phase angle  $^\circ$  shall be given.

Therefore, the component  $R$  shall be  $R = \frac{U}{I} = \frac{3.0}{0.24} = 12.5 \Omega$  and the component  $X_L$  shall be  $X_L = \frac{U}{I} = \frac{3.0}{0.24} = 12.5 \Omega$ .

The phase  $\varphi$  shall be calculated as  $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{12.5}{12.5}\right) = 45^\circ$ .

With the complex part  $Z = 12.5 \angle 45^\circ \Omega$  and the voltage  $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V, the current  $i(t) = 0.24 \cdot \sin(2\pi \cdot 15 \cdot t - 45^\circ)$  A shall be given.

The phase  $\varphi$  shall be calculated as  $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{12.5}{12.5}\right) = 45^\circ$ .

The phase  $\varphi$  shall be calculated as  $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{12.5}{12.5}\right) = 45^\circ$ .

### Exercise E7 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance  $Z$  of the circuit shown in the figure. The voltage  $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V and the current  $i(t) = 0.24 \cdot \sin(2\pi \cdot 15 \cdot t - \varphi)$  A shall be given.

After analysis, the following complex impedance  $Z$  shall be extracted and the phase  $\varphi$  in phase angle  $^\circ$  shall be given.

Solution  
Result  $Z = 19.8 \angle 48.2^\circ \Omega$

.. Draw the circuit diagram of the given circuit and label all components, voltages, and currents.

$$Z = \frac{U}{I} = \frac{3.0 \angle 0^\circ}{0.24 \angle -48.2^\circ} = 12.5 \angle 48.2^\circ \Omega$$

With the complex part  $Z = 12.5 \angle 48.2^\circ \Omega$  and the voltage  $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V, the current  $i(t) = 0.24 \cdot \sin(2\pi \cdot 15 \cdot t - 48.2^\circ)$  A shall be given.

The phase  $\varphi$  shall be calculated as  $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{12.5}{12.5}\right) = 45^\circ$ .





**Exercise E6 Impedances at different Frequencies**  
**(written test, approx. 18 % of a 60-minute written test, WS2022)**

2. A series circuit consists of a resistor with a resistance of  $R_1 = 1.00 \text{ k}\Omega$ , a capacitor with a capacitance of  $C_1 = 40 \text{ nF}$ , and an inductor with an inductance of  $L_1 = 4.7 \text{ }\mu\text{H}$ . The circuit is connected to an AC voltage source with a voltage of  $U = 10 \text{ V}$  and a frequency of  $f = 4 \text{ MHz}$ . Calculate the absolute value of the impedance  $|Z|$  of the circuit.

Solution

$$|Z| = \sqrt{R_1^2 + (X_L - X_C)^2}$$

$$|Z| = \sqrt{(1.00 \text{ k}\Omega)^2 + (2\pi \cdot 4 \text{ MHz} \cdot 4.7 \text{ }\mu\text{H} - \frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}})^2}$$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $L$  combined is given by  $Z = R + jX_L$   
 Parallel circuit means that the voltage is the same on  $R_2$  and  $C_2$   $\frac{1}{Z} = \frac{1}{R_2} + \frac{1}{jX_C}$   
 $\frac{1}{Z} = \frac{1}{R_2} - \frac{j}{X_C}$  since  $\frac{1}{j} = -j$   
 $Z = \frac{R_2 \cdot jX_C}{R_2 - jX_C}$  multiply numerator and denominator by  $R_2 + jX_C$   
 $Z = \frac{jR_2 X_C (R_2 + jX_C)}{R_2^2 - X_C^2}$   
 $Z = \frac{jR_2 X_C R_2 - X_C^2 R_2}{R_2^2 - X_C^2}$   
 $Z = \frac{jR_2^2 X_C - R_2 X_C^2}{R_2^2 - X_C^2}$   
 Therefore, the resulting current of the parallel circuit is given as:  

$$I = \frac{U}{Z} = \frac{U (R_2^2 - X_C^2)}{jR_2^2 X_C - R_2 X_C^2}$$
  

$$|I| = \frac{U \sqrt{R_2^2 - X_C^2}}{\sqrt{R_2^4 X_C^2 + R_2^2 X_C^4}}$$
  
 This can be simplified to  $|I| = \frac{U \sqrt{R_2^2 - X_C^2}}{R_2 X_C \sqrt{R_2^2 + X_C^2}}$   
 Back to the first formula:  $|Z| = \frac{U}{|I|} = \frac{R_2 X_C \sqrt{R_2^2 + X_C^2}}{\sqrt{R_2^2 - X_C^2}}$

**Exercise E1 Resistance of a Wire by Resistivity**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. For heating elements used to heat the oven at a temperature of  $180 \text{ }^\circ\text{C}$ , an electric power dissipation (= heat flow) of  $P = 40 \text{ W}$  is necessary.  
 Calculate the current  $I$  needed to operate the heating elements.

The Nichrome wire has a resistivity of  $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$ .  
 The heating element is  $l = 3 \text{ m}$  long and has a diameter of  $d = 3.57 \text{ mm}$ .  
 Calculate the resistance  $R$  of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

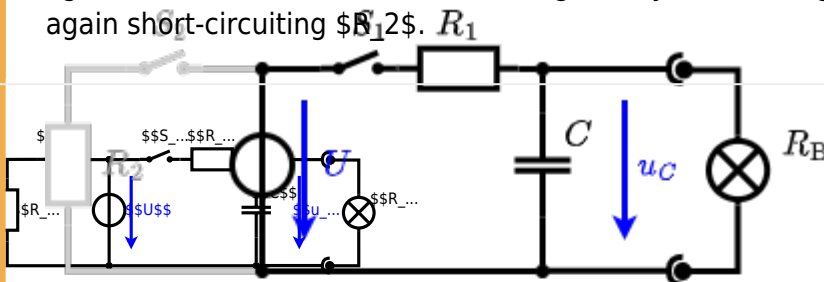
**Exercise E4 Charging Capacitors**  
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) is a series of  $R_1$  and  $R_2$  and a capacitor  $C$  and a switch  $S_1$  and a switch  $S_2$ . The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution:** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

The ideal voltage source  $U$  is in series with  $R_1$  and  $R_2$ . The voltage  $u_c$  is independent of this series  $(U - I \cdot R_2)^2$ .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $S_2$ .

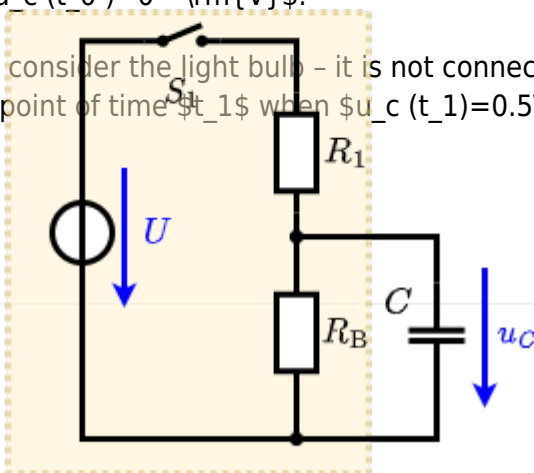


The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \Omega$  and a capacitor of  $C = 100 \mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first tasks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

**Solution**



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \Omega$ , short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ : 
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



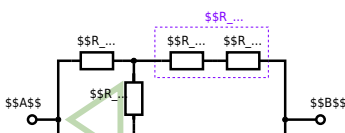
**Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved at 0.1 A.  $R_1 = R_2 = R_3 = 10 \Omega$  and the voltage  $U = 10 \text{ V}$  is given.  $R_B$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

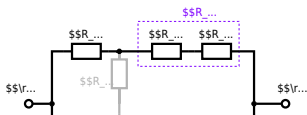
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{\text{eq}}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = (500 \sim \Omega) \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \frac{500 \sim \Omega \cdot 200 \sim \Omega}{500 \sim \Omega + 200 \sim \Omega}$$

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