

# Exam Winter Semester 2022

## Student Group

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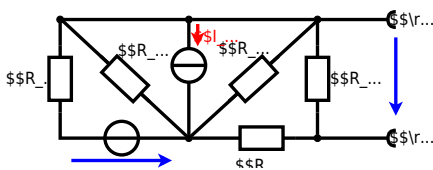
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**Exercise E1 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$\begin{aligned} U_{\text{S}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \Omega \end{aligned}$$



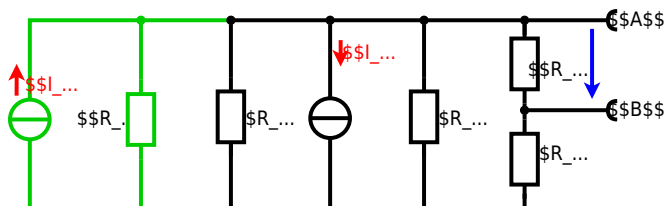
Calculated the internal resistance  $R_{\text{i}}$  and the source voltage  $U_{\text{S}}$  of an equivalent linear voltage source on the connectors  $\text{A}$  and  $\text{B}$ .  
 $R_1=5.0 \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3= 10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$   
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ : 
$$R_{135} = R_1 || R_3 || R_5$$
 
$$I_{24} = I_2 - I_4 = \frac{U_{24}}{R_1} - I_4$$
 The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following: 
$$U_{24}$$

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left( \frac{U_2}{R_1} - I_4 \right) \cdot R_1 || R_3 || R_5$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \left( \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot 15 \Omega \cdot 2.5 \Omega \cdot \frac{1}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

**Exercise E1 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator is explained with the effect of resistance on refrigeration system. The refrigerator has a resistance of  $15 \Omega$  at  $25^\circ \text{C}$  and  $2.5 \Omega$  at  $0^\circ \text{C}$ . Your answer.

Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$

**Result** The temperature inside the refrigeration system can reach down to  $-40^\circ \text{C}$ .

$$R_{25} = 15 \Omega \text{ at } 25^\circ \text{C}$$

The power transfer is  $P = U \cdot I = \frac{U^2}{R}$  and  $Q = P \cdot t$ . Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\text{with } \Delta T = T_{\text{end}} - T_{\text{start}} \implies R = 10 \Omega \cdot \left( 1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2 \right)$$

### Exercise E2 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance  $Z$  of the circuit shown in the figure through the components.  $R$  and  $X_L$  shall be given.

After analysis, the full bridge circuit can be simplified to a series circuit in phasor domain.  $Z = (2 + j4) \parallel (1 + j5) + 5$

Solution  
.. Calculate the physical values of the two components.  
Solution  $R = 2 \Omega$  and  $X_L = 4 \Omega$

Solution  
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \parallel = \frac{50}{(2 + j4) \parallel (1 + j5) + 5}$$
  
The voltage across the capacitor is  $U_C = I \cdot X_C = 50 \cdot \frac{1}{1 + j5}$   
resulting in  $I = 0.24 \angle -90^\circ$  A.  
Therefore, the component  $4.68 \Omega$  is a resistor with the same value  $4.68 \Omega$  in parallel with the capacitor.  $Z = (4.68 \parallel \frac{1}{1 + j5}) + 5$   
$$\underline{Z} = \frac{4.68 \cdot (1 + j5)}{4.68 + 1 + j5} + 5 = \frac{4.68 + j23.4}{5.68 + j5} + 5$$
  
The absolute value  $|Z| = 10.24 \Omega$  is calculated as  $|Z| = \sqrt{10.24^2} = 10.24 \Omega$   
$$|I| = \frac{|U|}{|Z|} = \frac{50}{10.24} = 4.88 \text{ A}$$
  
With the complex part  $\varphi = \arctan(\frac{X}{R}) = \arctan(\frac{4.68}{10.24}) = 24.4^\circ$   
$$\varphi = \arctan(\frac{-4.68}{0.24}) = -89.7^\circ$$
  
The phase  $\varphi$  can be calculated as  $\varphi_i = \arctan(\frac{\text{Im}(I)}{\text{Re}(I)}) = \arctan(\frac{-4.68}{0.24}) = -89.7^\circ$

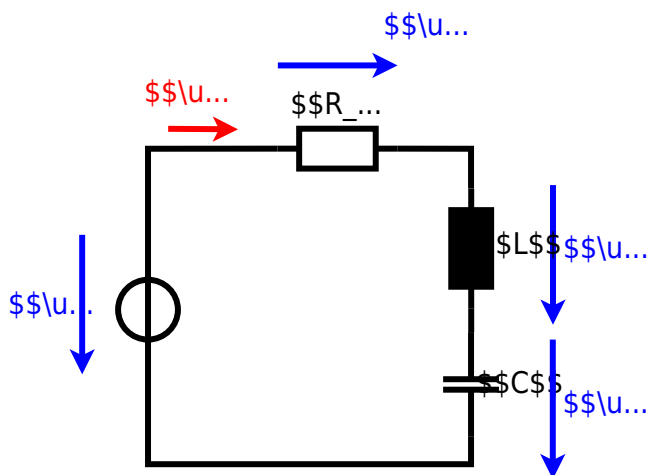
### Exercise E1 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the circuit impedance  $Z$  for a source  $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V. The circuit consists of a voltage source  $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V, a resistor  $R = 1 \Omega$ , an inductor  $L = 330 \mu\text{H}$ , and a capacitor  $C = 0.22 \mu\text{F}$ , all in series.

Solution  
Result  
.. Draw the circuit diagram of the given circuit and label all components, voltages, and currents.

$$Z = R + j\omega L - j\omega C = 1 + j2\pi \cdot 15 \cdot 330 \cdot 10^{-6} - j2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}$$
  
$$Z = 1 + j0.0314 - j0.0022 = 1 + j0.0292 \Omega$$
  
Result  $|Z| = 1.0004 \Omega$   
With  $\varphi = \arctan(\frac{0.0292}{1}) = 1.67^\circ$   
$$Z = 1.0004 \angle 1.67^\circ \Omega$$





**Exercise E3 Impedances at different Frequencies**  
**(written test, approx. 18 % of a 60-minute written test, WS2022)**

**Question:** A series circuit consists of a resistor with a resistance of  $R_1 = 1.00 \text{ k}\Omega$  and a capacitor with a capacitance of  $C_1 = 40 \text{ nF}$ . The circuit is connected to an AC voltage source with a peak voltage of  $U_0 = 10 \text{ V}$  and a frequency of  $f = 4 \text{ MHz}$ . Calculate the absolute value of the impedance  $|Z|$  of the circuit.

**Solution:**

The impedance of a resistor is  $Z_R = R$  and the impedance of a capacitor is  $Z_C = -j / (\omega C)$ . The total impedance of a series circuit is  $Z = Z_R + Z_C$ .

$$|Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

Substituting the given values:

$$|Z| = \sqrt{(1000)^2 + \left(\frac{1}{2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}}\right)^2}$$

$$|Z| \approx 1000 \text{ }\Omega$$

**Exercise E4 Resistance of a Wire by Resistivity**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

**Question:** A heating element made of nichrome wire is used to heat water with a temperature rise of  $\Delta T = 180 \text{ K}$ . The electric power dissipation (= heat flow) of  $P = 40 \text{ W}$  is necessary. Calculate the current  $I$  needed to operate the heating element.

**Solution:**

$$P = U \cdot I \Rightarrow I = \frac{P}{U}$$

$$I = \frac{40 \text{ W}}{230 \text{ V}} \approx 0.174 \text{ A}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} \cdot d^2 \cdot \pi \quad \& \quad \rho = \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

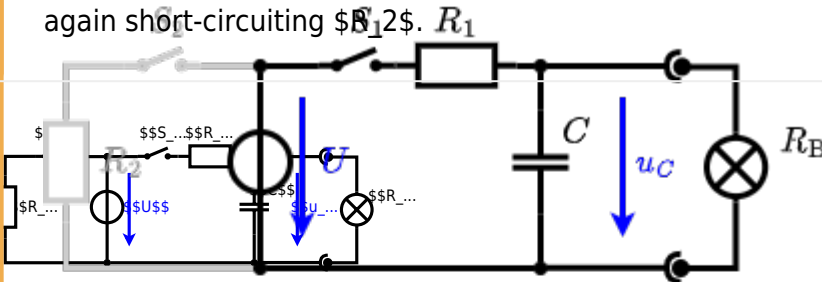
**Exercise E5 Charging Capacitors**  
**(written test, approx. 16 % of a 60-minute written test, WS2022)**

The circuit (with the light bulb) is a series of  $R_1$  and  $R_2$  and a capacitor  $C$  and a switch  $S_1$  and a switch  $S_2$ . The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution**  
 Hint: To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

**Solution**  
 The ideal voltage source  $U$  and the voltage  $u_c$  are independent of this circuit.

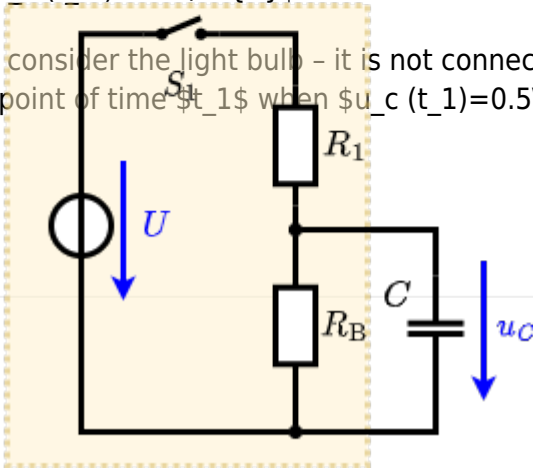
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $S_2$ .



The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \Omega$  and a capacitor of  $C = 100 \mu\text{F}$ . The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first tasks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

**Solution**



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \Omega$ , short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ :  

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $t$ :  

$$(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) \implies t = R_1 \cdot C \cdot \ln(0.5)$$



**Exercise E6 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved at 0V and  $R_1 = R_2 = R_3 = 10 \Omega$  and the switch shall be given.  $R_B$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{\text{eq}}$  between  $A$  and  $B$ .

Solution



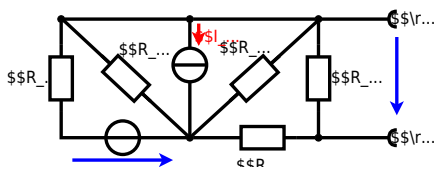
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E3 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



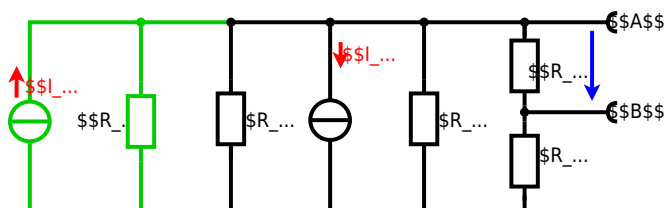
Calculate the internal resistance  $R_{int}$  and the source voltage  $U_s$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3=10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$ . Use equivalent sources in order to simplify the circuit!

### Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ : 
$$R_{135} = R_1 || R_3 || R_5$$
 
$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following: 
$$U_{24}$$

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left( \frac{U_2}{R_1} - I_4 \right) \cdot \left( R_1 || R_3 || R_5 \right)$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \left( \frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \text{ A} \cdot \left( \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

**Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator is explained with the effect of resistance on refrigeration systems. The circuit has a resistance of  $10 \Omega$  and a voltage of  $6 \text{ V}$ . Your answer.

Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$

**Result**  
The temperature inside the refrigeration system can reach down to  $-40 \text{ }^\circ\text{C}$ .

$$R = 10 \cdot (1 + 0.01 \cdot \Delta T + 71 \cdot 10^{-6} \cdot \Delta T^2)$$

The power of the resistor is  $P = U \cdot I$  and  $P = \frac{U^2}{R}$ . Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\text{with } \Delta T = T_{\text{end}} - T_{\text{start}} \implies R = 10 \Omega \cdot \left( 1 + 0.01 \cdot (-40 \text{ }^\circ\text{C} - 25 \text{ }^\circ\text{C}) + 71 \cdot 10^{-6} \cdot (-40 \text{ }^\circ\text{C} - 25 \text{ }^\circ\text{C})^2 \right)$$

**Exercise E5 Analyzing complex Impedances**  
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex power  $\underline{S}$  and the real power  $P$  in the circuit shown in the figure. The voltage  $\underline{u}$  and the current  $\underline{i}$  shall be given.

After analysis, the following complex power  $\underline{S}$  and real power  $P$  shall be extracted and given in the table. The voltage  $\underline{u}$  and the current  $\underline{i}$  shall be given in the table.

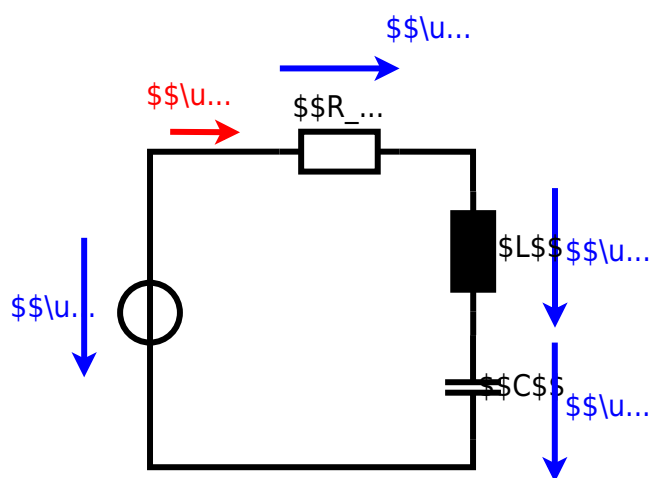
Solution
.. Calculate the physical values of the two components.
Solution $R = 10 \Omega \quad L = 10 \text{ mH} \quad C = 10 \text{ nF}$
Solution
$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \text{ V}}{10 \Omega + j 0.68 \text{ m}\Omega - j 0.24 \text{ m}\Omega} = \frac{50 \text{ V}}{10 \Omega + j 0.44 \text{ m}\Omega}$
The voltage $\underline{u}$ and the current $\underline{i}$ shall be given in the table.
resulting impedance $\underline{Z} = 10 \Omega + j 0.44 \text{ m}\Omega$
Therefore, the component $10 \Omega$ is a resistor with the same absolute value $10 \Omega$ and the component $j 0.44 \text{ m}\Omega$ is an inductor with the same absolute value $0.44 \text{ m}\Omega$ .
$\underline{S} = \underline{U} \cdot \underline{I}^* = 50 \text{ V} \cdot \left( \frac{50 \text{ V}}{10 \Omega + j 0.44 \text{ m}\Omega} \right)^* = \frac{2500 \text{ VA}}{10 \Omega + j 0.44 \text{ m}\Omega} = 250 \text{ VA} - j 11.5 \text{ var}$
$P = \text{Re}(\underline{S}) = 250 \text{ W} \quad Q = \text{Im}(\underline{S}) = -11.5 \text{ var}$
With the complex part $\underline{S} = 250 \text{ VA} - j 11.5 \text{ var}$ and $\underline{U} = 50 \text{ V}$ and $\underline{I} = \frac{50 \text{ V}}{10 \Omega + j 0.44 \text{ m}\Omega}$ the phase $\varphi$ shall be calculated as $\varphi = \arctan \left( \frac{\text{Im}(\underline{S})}{\text{Re}(\underline{S})} \right) = \arctan \left( \frac{-11.5}{250} \right) = -0.026 \text{ rad}$

**Exercise E7 Complex Impedance Circuit**  
 (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the complex power  $\underline{S}$  and the real power  $P$  in the circuit shown in the figure. The voltage  $\underline{u}$  and the current  $\underline{i}$  shall be given. The voltage  $\underline{u}$  and the current  $\underline{i}$  shall be given in the table.

Solution
Result
.. Draw the circuit diagram of the given circuit.
able all components, voltages, and currents.
$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{10 \text{ V}}{0.22 \text{ A}} = 45.45 \Omega$
$\underline{Z}_C = \frac{1}{j \omega C} = \frac{1}{j 2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ }\mu\text{F}} = -j 1.67 \text{ }\Omega$
Result $\underline{Z} = 45.45 \Omega - j 1.67 \text{ }\Omega$
With $\underline{S} = \underline{U} \cdot \underline{I}^* = 10 \text{ V} \cdot \left( \frac{10 \text{ V}}{45.45 \Omega - j 1.67 \text{ }\Omega} \right)^* = \frac{100 \text{ VA}}{45.45 \Omega - j 1.67 \text{ }\Omega} = 2.2 \text{ VA} + j 0.08 \text{ var}$
$P = \text{Re}(\underline{S}) = 2.2 \text{ W} \quad Q = \text{Im}(\underline{S}) = 0.08 \text{ var}$





**Exercise E6 Impedances at different Frequencies**  
**(written test, approx. 18 % of a 60-minute written test, WS2022)**

2. A series circuit consists of a resistor with a resistance of  $R_1 = 1.00 \text{ k}\Omega$ , a capacitor with a capacitance of  $C_1 = 40 \text{ nF}$ , and an inductor with an inductance of  $L_1 = 4.7 \text{ }\mu\text{H}$ . The circuit is connected to an AC voltage source with a voltage of  $U = 10 \text{ V}$  and a frequency of  $f = 4 \text{ MHz}$ . Calculate the absolute value of the impedance  $|Z|$  of the circuit.

Solution

$$|Z| = \sqrt{R_1^2 + (X_L - X_C)^2}$$

$$|Z| = \sqrt{(1.00 \text{ k}\Omega)^2 + (2\pi \cdot 4 \text{ MHz} \cdot 4.7 \text{ }\mu\text{H} - \frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}})^2}$$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $L$  combined is given by  $Z = R + jX_L$   
 Parallel circuit means that the voltage is the same on  $R$  and  $C$   $Z = \frac{R \cdot X_C}{R + jX_C}$   
 $|Z| = \frac{R \cdot X_C}{\sqrt{R^2 + X_C^2}}$  since  $\underline{U} \perp \underline{I}$  and  $\underline{U} \perp \underline{I}$  is perpendicular to  $\underline{R}$  this can be simplified to  $|Z| = \frac{R \cdot X_C}{\sqrt{R^2 + X_C^2}}$  (It has to, since  $R$  is perpendicular to  $X_C$ )  
 $|Z| = \frac{R \cdot X_C}{\sqrt{R^2 + X_C^2}}$   
 Therefore, the resulting current of the parallel circuit is given as:  
 $I = \frac{U}{|Z|} = \frac{U \cdot \sqrt{R^2 + X_C^2}}{R \cdot X_C}$   
 $I = \frac{10 \text{ V} \cdot \sqrt{(1.00 \text{ k}\Omega)^2 + (2\pi \cdot 4 \text{ MHz} \cdot 4.7 \text{ }\mu\text{H})^2}}{1.00 \text{ k}\Omega \cdot 2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}$   
 Back to the first formula:  $|Z| = \frac{U}{I} = \frac{10 \text{ V}}{0.00018 \text{ A}} = 55.5 \text{ k}\Omega$

**Exercise E1 Resistance of a Wire by Resistivity**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. Heating elements are used to heat the oven with a temperature of  $180 \text{ }^\circ\text{C}$ . The electric power dissipation (= heat flow) of  $P = 40 \text{ W}$  is necessary.  
 Calculate the current  $I$  needed to operate the heating elements.

The Nichrome wire has a resistivity of  $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$ .  
 The heating element is  $l = 3 \text{ m}$  long and has a diameter of  $d = 3.57 \text{ mm}$ .  
 Calculate the resistance  $R$  of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

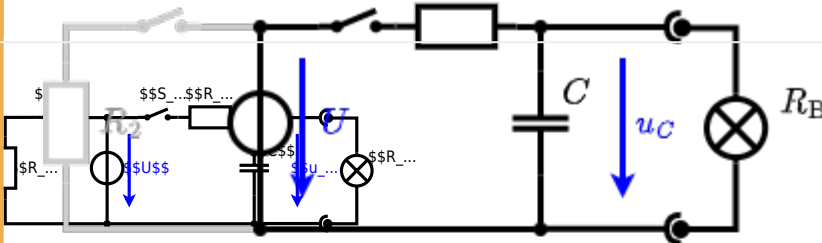
**Exercise E4 Charging Capacitors**  
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) is a series RC circuit. The capacitor is initially uncharged. At  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution:** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

The ideal voltage source  $U = 12 \text{ V}$  is in series with the resistor  $R_1 = 20 \Omega$ . The voltage  $u_c$  is independent of the choice of  $R_2$ .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .



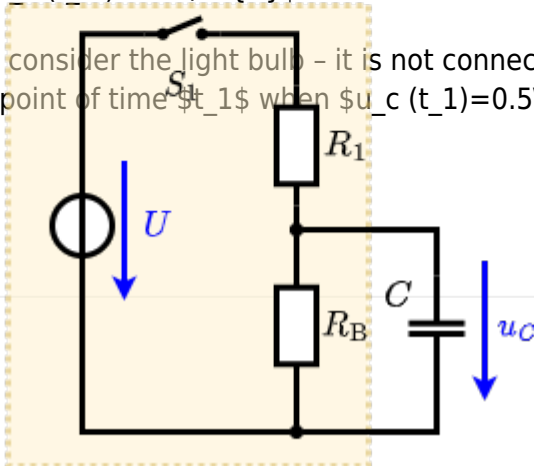
The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \Omega$  and a capacitor of  $C = 100 \mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first tasks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

.. First do not consider the light bulb - it is not connected to the RC circuit.

Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

**Solution**



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

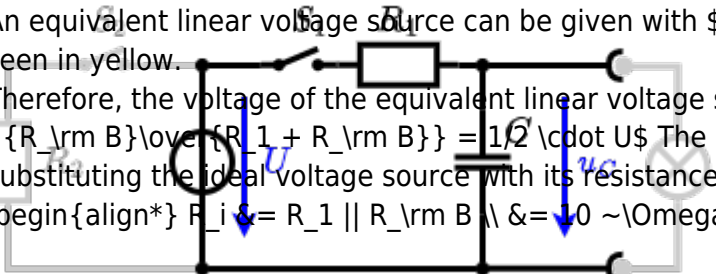
Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \Omega$ , short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_C(t)$  which has to be  $u_C(t_1) = 0.5 \cdot U$ :  $u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$   
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



**Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved at once,  $R_1 = R_2 = R_3 = 10 \Omega$  and the voltage  $U = 10V$  is given.  $R_B$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.

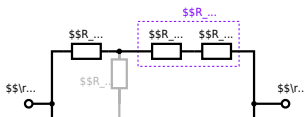


Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as: 
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series: 
$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{\text{eq}}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = (500 \sim \Omega) \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \frac{500 \sim \Omega \cdot 200 \sim \Omega}{500 \sim \Omega + 200 \sim \Omega}$$

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