

Exam Winter Semester 2022

Student Group

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Exam Winter Semester 2022

Additional permitted Aids

- non-programmable calculator,
- formulary (2 DIN A4 pages)

Hits

- The duration of the exam is 60 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

Tasks

Exercise E4 Resistance of a Wire by Resistivity

(written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of nichrome wire with a temperature coefficient of $1.80 \cdot 10^{-4} \text{ K}^{-1}$ is selected. The power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.

Calculate the current I needed to operate it.

The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .

Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \\ \sqrt{\frac{P}{R}} &= \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \text{with } A = r^2 \cdot \pi = \\ \frac{1}{4} d^2 \cdot \pi \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \text{and } R = \\ 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

Exercise E1 Resistance of a Wire by Resistivity

(written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of nichrome wire with a temperature coefficient of $1.80 \cdot 10^{-4} \text{ K}^{-1}$ is selected.

Result power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.
 Calculate the current I needed to operate it for heating elements.
 The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$.
 The heating element is 3 m long and has a diameter of 3.57 mm .
 Solution: $R = 10^{-3} \text{ } \Omega$ (Wrong calculation)

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}}$$

$$R = \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \quad || \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad || \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi}$$

Exercise E1 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A technician explains a temperature sensitive component used in a refrigerator has a resistance of $6.5 \text{ k} \Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ } \frac{1}{\text{K}}$ and $\beta = 71 \cdot 10^{-6} \text{ } \frac{1}{\text{K}^2}$

Result
 The temperature inside the refrigeration system can reach down to -40°C .
 Calculate the resistance of the thermostat at -40°C .
 Resistance of the resistor R depends on the current I and generated heat. Therefore, a solution is to use a heat float up the refrigeration system.
 Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad | \quad \Delta T = T_{\text{end}} - T_{\text{start}} \quad || \quad R = 10 \text{ k} \Omega \cdot \left(1 + 0.01 \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2\right)$$

Exercise E2 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A resistor exhibits a temperature coefficient of resistance of $\alpha = 0.01 \text{ K}^{-1}$ and a nominal resistance of $R_0 = 10 \text{ k}\Omega$ at $T_0 = 25^\circ\text{C}$. Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) = 10 \text{ k}\Omega \cdot (1 + 0.01 \cdot (-15) + 71 \cdot 10^{-6} \cdot (-15)^2) = 6.5 \text{ k}\Omega$$

Resistor transfer resistor R is part of the circuit and generates heat. Therefore, a solution is to use a heat sink up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad | \quad \Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \text{ k}\Omega \cdot (1 + 0.01 \cdot (-40 - 25) + 71 \cdot 10^{-6} \cdot (-40 - 25)^2)$$

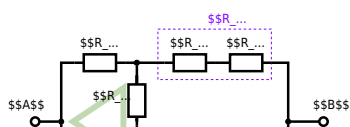
Exercise E6 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall hold: $R_1 = 200 \text{ }\Omega$, $R_2 = R_3 = 100 \text{ }\Omega$, $R_4 = 10 \text{ }\Omega$ and the voltage $U = 10 \text{ V}$.
Result: $R_B = 132.8 \text{ }\Omega$.

Solution

$$R_{\text{B}} = 132.8 \text{ }\Omega$$

Now a wye-delta transformation is necessary.

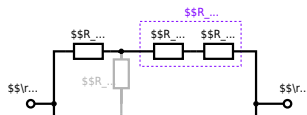


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim\Omega + 200 \sim\Omega + 200 \sim\Omega) \parallel (100 \sim\Omega + 100 \sim\Omega) \parallel R_{\text{eq}} = (500 \sim\Omega) \parallel (200 \sim\Omega) \parallel R_{\text{eq}} = \frac{500 \sim\Omega \cdot 200 \sim\Omega}{500 \sim\Omega + 200 \sim\Omega}$$

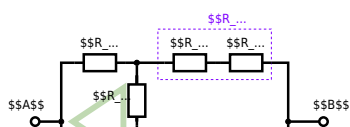
Exercise E3 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with $R_1 = 200 \Omega$, $R_2 = R_3 = 100 \Omega$ and the switch S is given. R_{eq} is given.

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.

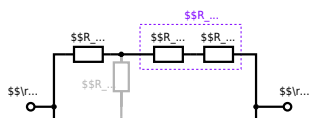


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



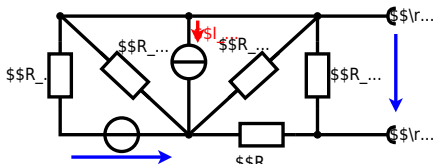
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E1 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

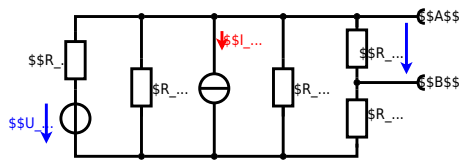
$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



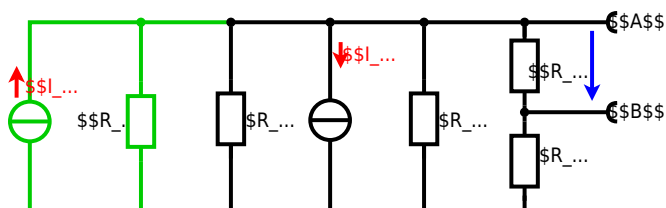
Calculated the internal resistance R_{int} and the source voltage U_{oc} of an equivalent linear voltage source on the connectors A and B .
$$R_1=5.0 \text{ } \Omega, \quad U_2=6.0 \text{ V}, \quad R_3= 10 \text{ } \Omega, \quad I_4=4.2 \text{ A}, \quad R_5=10 \text{ } \Omega, \quad R_6=7.5 \text{ } \Omega, \quad R_7=15 \text{ } \Omega$$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot (R_6 || R_7)$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - I_4 \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - (U_2 \cdot \frac{R_7}{R_1} - I_4) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

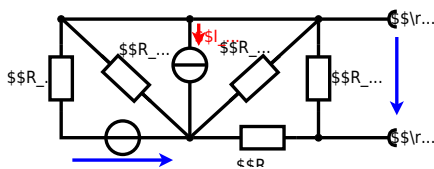
with $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \parallel R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

Exercise E3 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

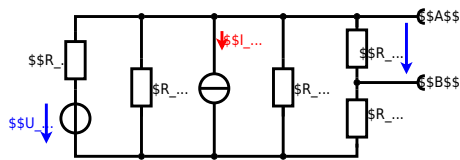
$$U_s = U_{AB} = 4.5\text{V} \parallel R_i = R_{AB} = 6\Omega$$



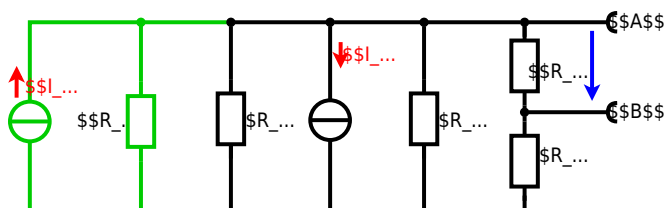
Calculate the internal resistance R_{int} and the source voltage U_s of an equivalent linear voltage source on the connectors A and B .
 $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \Omega$, $I_4=4.2 \text{ A}$,
 $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{64}$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - (U_2 \over R_1 - I_4) \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E5 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

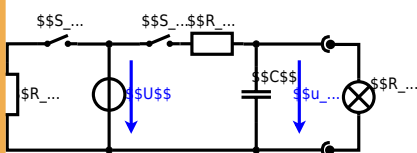
The circuit below is a RC circuit consisting of a DC voltage source U , a resistor R_1 , a resistor R_2 , a capacitor C , and a switch S_1 . The switch S_1 is initially open. The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

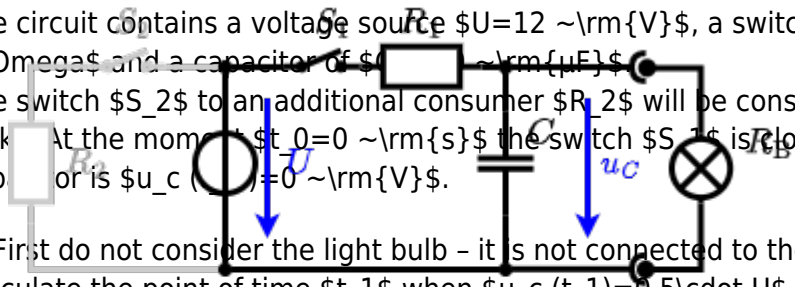
The ideal voltage source U_{eq} is given by:

$$U_{eq} = U \cdot \frac{R_2}{R_1 + R_2}$$

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

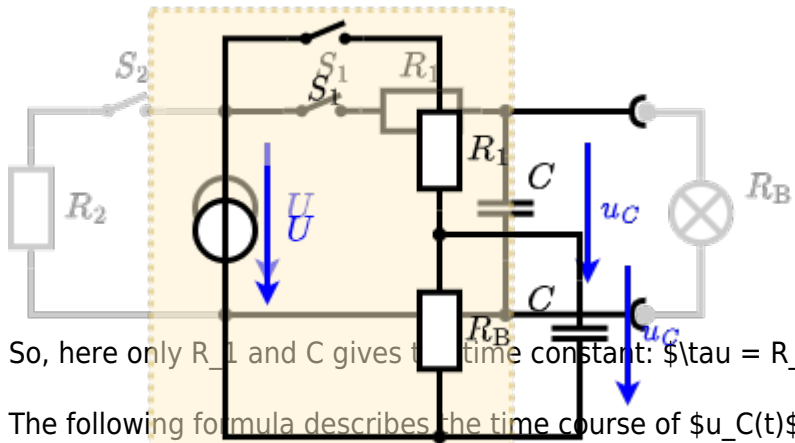


The circuit contains a voltage source $U=12\text{ V}$, a switch S_1 , a resistor of $R_1=20\text{ }\Omega$ and a capacitor of $C=100\text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0\text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0)=0\text{ V}$.



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1)=0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1)=0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$
 An equivalent linear voltage source can be given with U_s , R_1 and R_B as seen in yellow:

$$t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$$

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$
 The internal resistance is given by substituting the ideal voltage source with its resistance ($r=0\text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1\text{ ms}/(10\text{ }\Omega \cdot 100\text{ }\mu\text{F})})$$

Exercise E4 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (as shown in the figure) consists of a DC voltage source $U=6\text{ V}$, a resistor $R_1=20\text{ }\Omega$, a capacitor $C=20\text{ }\mu\text{F}$ and a light bulb $R_B=10\text{ }\Omega$. The voltage across the capacitor is again 0 V at the moment $t_0=0\text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1\text{ ms}$ after closing the switch.

Solution To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

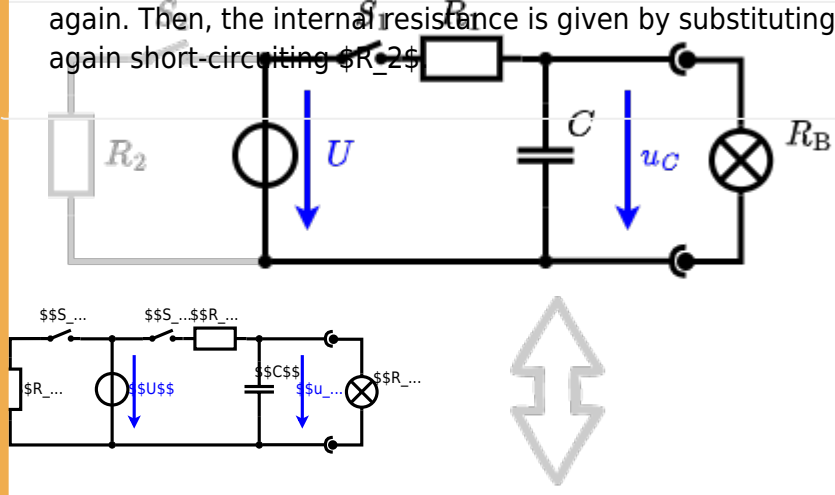
$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 2\text{ V}$$

$$R_i = R_1 \parallel R_B = 13.33\text{ }\Omega$$

Solution

The ideal voltage source is $U = 12 \text{ V}$. The internal resistance is $R_1 = 20 \text{ }\Omega$. The voltage across the capacitor is $u_C(t)$. The voltage across the light bulb is $u_B(t)$. The voltage across the resistor R_2 is $u_{R_2}(t)$. The voltage across the capacitor is $u_C(t)$. The voltage across the light bulb is $u_B(t)$. The voltage across the resistor R_2 is $u_{R_2}(t)$.

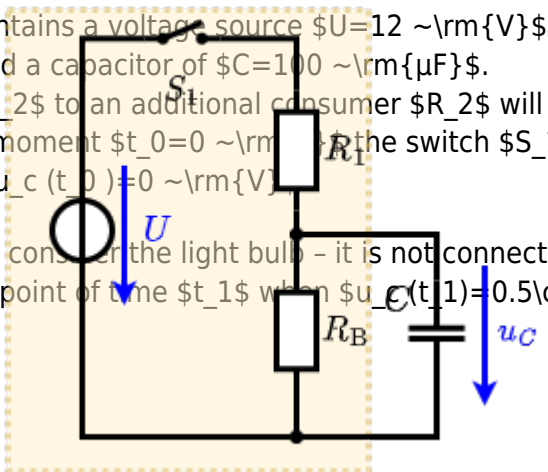
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0) = 0 \text{ V}$.

First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_C(t_1) = 0.5 \cdot U$.



Solution

An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \text{ }\Omega$, short-circuit). $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$

$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2/(10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$
So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$: $u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$
It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5)$

Exercise E2 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source $\underline{U} = 50 \angle 0^\circ \text{ V}$ and the admittance $\underline{Y} = 0.24 \text{ S}$ are both in the components. ($\$R\$ and $\$X_1\$) shall be given.$$

After analysis, the full width dimensioned complex impedance \underline{Z} shall be extracted and the magnitude $|Z|$ in phase φ shall be given. $\underline{Z} = (2 + j4) \Omega$

Solution
 .. Calculation of physical values of the two components.
 Solution $\begin{aligned} R &= \frac{1}{\text{Re}\{\underline{Y}\}} = \frac{1}{0.24} = 4.167 \Omega \\ X_1 &= \frac{1}{\text{Im}\{\underline{Y}\}} = \frac{1}{-0.06} = -16.667 \Omega \end{aligned}$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{(2 + j4) \Omega} = 12.5 \angle -63.4^\circ \text{ A}$$
 The current I has a magnitude of 12.5 A and a phase angle of -63.4° .
 The voltage U is 50 V and the admittance Y is 0.24 S .
 The resulting impedance Z is $\underline{Z} = \frac{1}{\underline{Y}} = \frac{1}{0.24 \text{ S}} = 4.167 - j16.667 \Omega$.
 The magnitude of Z is $|Z| = \sqrt{4.167^2 + 16.667^2} = 17.17 \Omega$.
 The phase angle of Z is $\varphi = \arctan\left(\frac{-16.667}{4.167}\right) = -75.96^\circ$.
 With the complex part $\varphi = -75.96^\circ$ the physical value φ is 75.96° .

$$\varphi = \arctan\left(\frac{\text{Im}\{\underline{Z}\}}{\text{Re}\{\underline{Z}\}}\right) = \arctan\left(\frac{-16.667}{4.167}\right) = -75.96^\circ$$

Exercise E5 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage source $\underline{U} = 50 \angle 0^\circ \text{ V}$ and the admittance $\underline{Y} = 0.24 \text{ S}$ are both in the components. ($\$R\$ and $\$X_1\$) shall be given.$$

After analysis, the full width dimensioned complex impedance \underline{Z} shall be extracted and the magnitude $|Z|$ in phase φ shall be given. $\underline{Z} = (2 + j4) \Omega$

Solution
 .. Calculation of physical values of the two components.
 Solution $\begin{aligned} R &= \frac{1}{\text{Re}\{\underline{Y}\}} = \frac{1}{0.24} = 4.167 \Omega \\ X_1 &= \frac{1}{\text{Im}\{\underline{Y}\}} = \frac{1}{-0.06} = -16.667 \Omega \end{aligned}$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{(2 + j4) \Omega} = 12.5 \angle -63.4^\circ \text{ A}$$
 The current I has a magnitude of 12.5 A and a phase angle of -63.4° .
 The voltage U is 50 V and the admittance Y is 0.24 S .
 The resulting impedance Z is $\underline{Z} = \frac{1}{\underline{Y}} = \frac{1}{0.24 \text{ S}} = 4.167 - j16.667 \Omega$.
 The magnitude of Z is $|Z| = \sqrt{4.167^2 + 16.667^2} = 17.17 \Omega$.
 The phase angle of Z is $\varphi = \arctan\left(\frac{-16.667}{4.167}\right) = -75.96^\circ$.
 With the complex part $\varphi = -75.96^\circ$ the physical value φ is 75.96° .

$$\varphi = \arctan\left(\frac{\text{Im}\{\underline{Z}\}}{\text{Re}\{\underline{Z}\}}\right) = \arctan\left(\frac{-16.667}{4.167}\right) = -75.96^\circ$$

The absolute value of the impedance is $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$ and the phase is $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$.
 With the complex part comes the physical value: $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$.
 The phase ϕ is given by $\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$.

Exercise E3 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with $R_1 = 100 \Omega$, $R_2 = 450 \Omega$, $L = 4.7 \mu H$ and $C = 40 nF$ at $f = 4 MHz$.
 Result: $Z = 100 - j450 + j100 - j100 = 100 - j450 \Omega$.
 A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 nF$ at $f_1 = 4 MHz$.

Solution
 $Z = R_1 + jX_L - jX_C = 100 + j100 - j450 = 100 - j350 \Omega$
 $Z = R_2 + jX_L - jX_C = 450 + j100 - j100 = 450 \Omega$
 A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z_{RL} = R + jX_L$.
 Parallel circuit means that the voltage is the same on R_2 and C_1 .
 $Z_{RC} = \frac{R_2 \cdot (-jX_C)}{R_2 - jX_C}$. Since X_L and X_C are perpendicular to R_2 , this can be simplified to $Z_{RC} = \frac{R_2 \cdot X_C}{R_2 + jX_C}$.
 Z_{RC} is perpendicular to Z_{RL} . (It has to, since R_2 is perpendicular to X_C .)
 Therefore, the resulting current of the parallel circuit is given as:
 $I_{RC} = \frac{U}{Z_{RC}} = \frac{U}{\frac{R_2 \cdot X_C}{R_2 + jX_C}} = \frac{U \cdot (R_2 + jX_C)}{R_2 \cdot X_C}$
 This can be written as $I_{RC} = \frac{U}{R_2} + \frac{jU \cdot X_C}{R_2 \cdot X_C} = \frac{U}{R_2} + \frac{jU}{R_2}$.
 Back to the first formula: $I_{RC} \cdot Z_{RC} = I_{RL} \cdot Z_{RL}$
 $\left(\frac{U}{R_2} + \frac{jU}{R_2}\right) \cdot \frac{R_2 \cdot X_C}{R_2 + jX_C} = \left(\frac{U}{R_1 + jX_L} + \frac{jU}{R_1 + jX_L}\right) \cdot (R_1 + jX_L)$

Exercise E6 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

Resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution

$$R_1 = 1.00 \text{ } \Omega$$

$$R_2 = 10.0 \text{ } \Omega$$

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by

$$Z_{RL} = R + j\omega L$$

Parallel circuit means that the voltage is the same on R_2 and C_1

$$Z_{RC} = \frac{R_2 \cdot (-j/\omega C_1)}{R_2 - j/\omega C_1}$$

Since R_2 and C_1 are perpendicular to R_1 , this can be simplified to

$$Z_{RC} = \frac{R_2}{1 + \omega^2 R_2^2 C_1^2}$$

(It has to, since R_3 is perpendicular to $j\omega L$ and $-j/\omega C_1$)

Therefore, the resulting current of the parallel circuit is given as:

$$I_3 = I_{R2} + I_{C1}$$

This can be rearranged to get R_3

$$R_3 = \frac{U}{I_3} = \frac{U}{I_{R2} + I_{C1}} = \frac{U}{\frac{U}{R_2} + \frac{U}{Z_{RC}}}$$

$$R_3 = \frac{R_2 \cdot Z_{RC}}{R_2 + Z_{RC}}$$

Back to the first formula:

$$R_3 \cdot I_3 = X_{C1} \cdot I_3$$

$$R_3 = \frac{X_{C1} \cdot I_3}{I_3} = \frac{1}{2\pi \cdot f \cdot C_1} \cdot \frac{I_3}{I_3} = \frac{1}{2\pi \cdot f \cdot C_1} = 1.00 \text{ } \Omega$$

Exercise E1 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the current $i(t)$ through the series combination of Z_L and Z_C and draw the equivalent circuit diagram of the given circuit. $u(t) = 3.0 \text{ V} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$

Solution

Result

$$Z = 48.2 \text{ } \Omega \quad Z_C = 19.8 \text{ } \Omega$$

Draw the circuit diagram of the given circuit

$$Z = \frac{U}{I} \quad I = \frac{U}{Z}$$

$$Z_C = \frac{1}{2\pi \cdot f \cdot C} = \frac{1}{2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ } \mu\text{F}} = 19.8 \text{ } \Omega$$

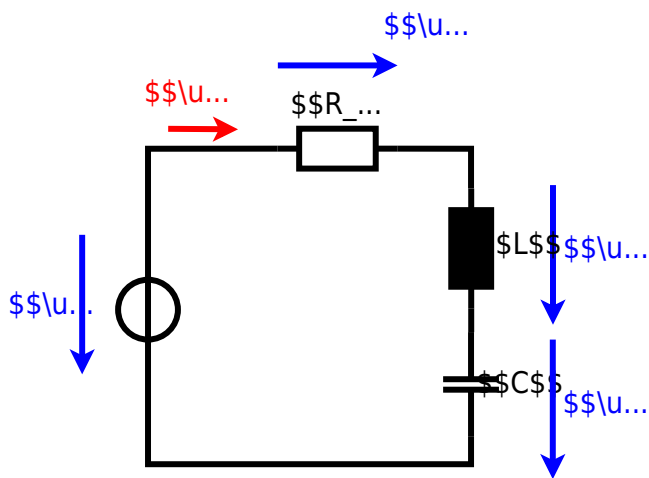
With $I = \frac{U}{Z}$

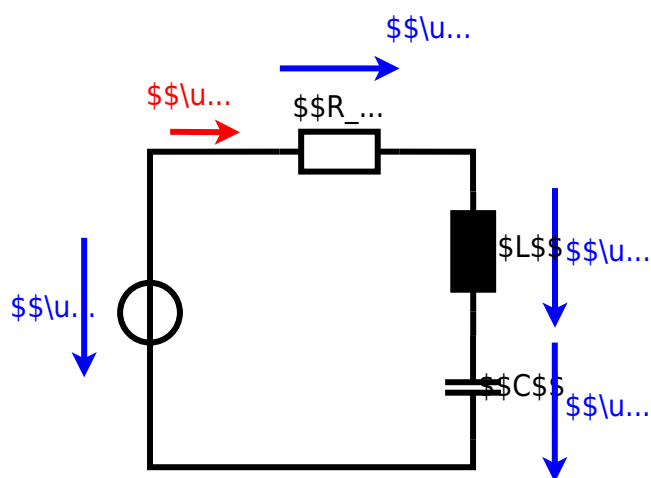
$$I = \frac{3.0 \text{ V}}{48.2 \text{ } \Omega} = 62.2 \text{ mA}$$

$$Z = R + jZ_L - jZ_C = R + j(2\pi \cdot 15 \text{ kHz} \cdot 330 \text{ } \mu\text{H}) - j19.8 \text{ } \Omega$$

$$Z = R + j(31.4 - 19.8) \text{ } \Omega = R + j11.6 \text{ } \Omega$$

$$|Z| = \sqrt{R^2 + (11.6 \text{ } \Omega)^2} = 48.2 \text{ } \Omega$$





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