task_x357drkaqv84jnsc_with_calculation

Student Group

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Exercise 1 : Pure Resistor Network Simplification
(written test, approx. 13% of a 60-minute written test, WS2022)

The following circuit with $R_1=200 \Omega$, $R_2=R_3=100 \Omega$ and the switch S is given.

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution

With the switch open the resistor R_3 dies not take part into the resulting resistor.
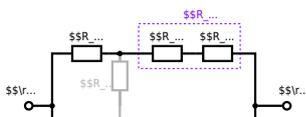
The equivalent resistor is given by a parallel configuation of resistors in series:

$$\begin{align*} R_{eq} &= (R_2 + R_1 + R_1)||(R_2 + R_2)\\ R_{eq} &= (100 \Omega + 200 \Omega + 200 \Omega)&&||(100 \Omega + 100 \Omega) &&\\ R_{eq} &= (500 \Omega)&&||(200 \Omega)&&\\ R_{eq} &= {{500 \Omega \cdot 200 \Omega }\over{500 \Omega + 200 \Omega}}&&\\ \end{align*}$$

Final result

$$\begin{align*} R_{eq} &= 142.8 \Omega \\ \end{align*}$$

2. The switch shall now be closed. Calculate the equivalent resistance R_{eq} between A and B.

Solution

Now a wye-delta transformation is necessary.
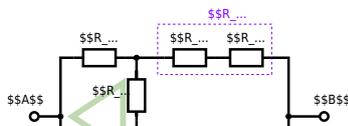
Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
\begin{align*} R_{Y} &= {{R_2 \cdot R_2}\over{R_2 + R_2 + R_2}} \\ &= {{(100 \Omega)^2}\over{3 \cdot 100 \Omega}} \\ &= {{1}\over{3}} \cdot 100 \Omega = 33.33 \Omega \end{align*}

The equivalent resistor is given by a parallel configuation of resistors in series:
\begin{align*} R_{eq} &= R_Y + (R_Y + R_1 + R_1)||(R_Y + R_2)\\ R_{eq} &= 33.33 \Omega + (33.33 \Omega + 400 \Omega)||(33.33 \Omega + 100 \Omega)\\ \end{align*}

Final result

\begin{align*} R_{eq} &= 135.3 \Omega \\ \end{align*}