task_pdkggtyexxy1ktu3_with_calculation

Student Group

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Table of Contents

complex impedance, exam ee1 WS2022

Exercise 1.1 : Impedances at Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

Calculate the **resistor values** which have to be used in the following circuits.

1. A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1=40 ~{\rm nF}$ at $f_1=4 ~{\rm MHz}$.

Solution

> \begin{align*} R_1 &= |\underline{X}_{C1}| \\ &= {{1}\over{2\pi \cdot f \cdot C_1}} \\ &= {{1}\over{2\pi \cdot 4 ~{\rm MHz} \cdot 40 ~\rm nF}} \\ \end{align*}

Final result

> \begin{align*} R_1 &= 1.00 ~\Omega \\ \end{align*}

2. A RL series circuit with $L_2=4.7 ~\rm µH$, where an AC voltage source of $U_2=1.0 ~\rm V$ with $f_2=450 ~\rm kHz$ generates a current $I_2=60 ~\rm mA$.

Solution

> Series circuit means that the current is constant on every component.
> The equivalent impedance for R and L combined is given by \begin{align*} {{\underline{U}}\over{\underline{I}}} &= R_2 + \underline{X}_{L2} \\ &= R_2 + {\rm j} \cdot \omega L \end{align*} Since ${\rm j} \cdot \omega L $ is perpendicular to R_2 this can be simplified to: \begin{align*} \left| {{\underline{U}}\over{\underline{I}}} \right|^2 &= |R_2|^2 + |\underline{X}_{L2}|^2 \\ \left({{U}\over{I}} \right)^2 &= {R_2}^2 + {X_{L2}}^2 \\ \end{align*}
>
> This can be rearranged to get R_2: \begin{align*} R_2 &= \sqrt{ \left({{U }\over{I }} \right)^2 - X_{L2}^2 } \\ &= \sqrt{ \left({{1~{\rm V}}\over{60~\rm mA}} \right)^2 - (2\pi \cdot 450~{\rm kHz} \cdot 4.7 ~ {\rm µH})^2 } \\ \end{align*}

Final result

\begin{align*} R_2 &= 10.0 ~\Omega \\ \end{align*}

3. A RC parallel circuit with $C_3=4.7 ~\rm nF$ on an AC current source ($I_{3S}=1.3 ~\rm A$,$f_3=200 ~\rm kHz$), which generates a current of $I_{3R}=1.0 ~\rm A$ through R_3.

Solution

Parallel circuit means that the voltage is the same on R_3 and C_3: \begin{align*} \underline{U}_3 = R_3 \cdot \underline{I}_{3R} = -{\rm j}\cdot {X}_{3C} \cdot \underline{I}_{3C} \end{align*} So it gets clear, that $\underline{I}_{3R}$ is perpendicular to $\underline{I}_{3C}$ (It has to, since R_3 is perpendicular to $-{\rm j}\cdot {X}_{3C}$, too).
Therefore, the resulting current of the parallel circuit is given as: \begin{align*} \underline{I}_{3} &= \underline{I}_{3R} + \underline{I}_{3C} \\ |\underline{I}_{3}|^2 &= |\underline{I}_{3R}|^2 + |\underline{I}_{3C}|^2 \\ {I}_{3C} &= \sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2} \end{align*}

Back in the first formula: \begin{align*} R_3 \cdot {I}_{3R} &= {X}_{3C} \cdot {I}_{3C} \\ R_3 &= {X}_{3C} \cdot {{{I}_{3C}}\over{{I}_{3R}}} \\ &= {{1}\over{2\pi \cdot f \cdot C_3}} \cdot {{\sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2}}\over{{I}_{3R}}} \\ \end{align*}

Final result

\begin{align*} R_3 &= 70.0 ~\Omega \\ \end{align*}