task_kricv9fh7haauo6q_with_calculation

Student Group

First Name	Surname	Matrikel Nr.

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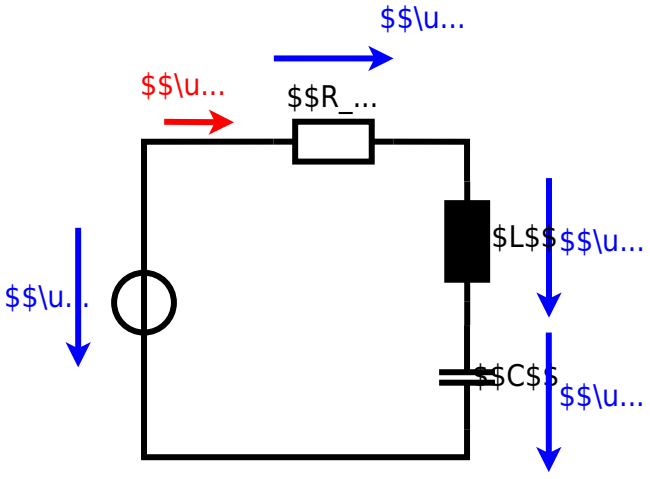
complex impedance, exam ee1 WS2022

Exercise E1.1 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

a. Calculate the complex impedance $\underline{Z}$ of the circuit and the resulting current $\underline{I}$ by the voltage source $u(t) = 3.0 ~{\rm V} \cdot \sin(2\pi \cdot 15 ~{\rm kHz} \cdot t)$ with internal resistance of $10 ~\Omega$.
Solution
This linear source is connected with an inductor of $330 ~ {\rm µH}$ and a capacitor of $0.22 ~{\rm µF}$, all in series.
Result
$$\begin{align*} \underline{Z} &= 19.8 ~\Omega - j \cdot 48.2 ~\Omega \\ Z &= 52.1 ~\Omega \\ \varphi &= -67.7^\circ \end{align*}$$
b. Draw the circuit diagram of the given circuit.
Label all components, voltages, and currents.
Result

c. Calculate the complex impedances $\underline{Z}_R$, $\underline{Z}_L$, and $\underline{Z}_C$ and the resulting $\underline{Z}$ by hand.
Result
$$\begin{align*} Z_R &= R = 10 ~\Omega \\ Z_L &= 2\pi \cdot f \cdot L = 2\pi \cdot 15 ~{\rm kHz} \cdot 330 ~{\rm µH} = 31.1 ~\Omega \\ Z_C &= {{1}\over{2\pi \cdot f \cdot C}} = {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}}} = 48.2 ~\Omega \\ \underline{Z} &= 10 ~\Omega + j \cdot 31.1 ~\Omega - j \cdot 48.2 ~\Omega \\ \underline{Z} &= 10 ~\Omega - j \cdot 17.1 ~\Omega \end{align*}$$
d. Calculate the complex current $\underline{I}$ by hand using the voltage divider rule.
Result
$$\begin{align*} \underline{I} &= {{\underline{U}}\over{\underline{Z}}} \\ \underline{I} &= {{3.0 ~{\rm V}}\over{19.8 ~\Omega - j \cdot 17.1 ~\Omega}} \\ \underline{I} &= 0.115 ~{\rm A} + j \cdot 0.100 ~{\rm A} \\ I &= 0.152 ~{\rm A} \\ \varphi &= 41.0^\circ \end{align*}$$
e. Calculate the complex voltages $\underline{U}_R$, $\underline{U}_L$, and $\underline{U}_C$ by hand using the voltage divider rule.
Result
$$\begin{align*} \underline{U}_R &= {{\underline{Z}_R}\over{\underline{Z}}} \cdot \underline{U} \\ \underline{U}_R &= {{10 ~\Omega}\over{19.8 ~\Omega - j \cdot 17.1 ~\Omega}} \cdot 3.0 ~{\rm V} \\ \underline{U}_R &= 1.15 ~{\rm V} + j \cdot 1.00 ~{\rm V} \\ U_R &= 1.52 ~{\rm V} \\ \varphi &= 41.0^\circ \end{align*}$$
$$\begin{align*} \underline{U}_L &= {{\underline{Z}_L}\over{\underline{Z}}} \cdot \underline{U} \\ \underline{U}_L &= {{j \cdot 31.1 ~\Omega}\over{19.8 ~\Omega - j \cdot 17.1 ~\Omega}} \cdot 3.0 ~{\rm V} \\ \underline{U}_L &= 1.73 ~{\rm V} + j \cdot 1.99 ~{\rm V} \\ U_L &= 2.64 ~{\rm V} \\ \varphi &= 49.0^\circ \end{align*}$$
$$\begin{align*} \underline{U}_C &= {{\underline{Z}_C}\over{\underline{Z}}} \cdot \underline{U} \\ \underline{U}_C &= {{-j \cdot 48.2 ~\Omega}\over{19.8 ~\Omega - j \cdot 17.1 ~\Omega}} \cdot 3.0 ~{\rm V} \\ \underline{U}_C &= -2.88 ~{\rm V} - j \cdot 2.99 ~{\rm V} \\ U_C &= 4.15 ~{\rm V} \\ \varphi &= -133.9^\circ \end{align*}$$
f. Calculate the complex power $\underline{S}$ by hand using the complex current $\underline{I}$ and the complex voltage $\underline{U}$.
Result
$$\begin{align*} \underline{S} &= \underline{U} \cdot \underline{I}^* \\ \underline{S} &= 3.0 ~{\rm V} \cdot (0.115 ~{\rm A} - j \cdot 0.100 ~{\rm A}) \\ \underline{S} &= 0.345 ~{\rm VA} - j \cdot 0.300 ~{\rm VA} \\ S &= 0.457 ~{\rm VA} \\ \varphi &= -41.0^\circ \end{align*}$$
g. Calculate the complex power $\underline{S}$ by hand using the complex voltage $\underline{U}$ and the complex impedance $\underline{Z}$.
Result
$$\begin{align*} \underline{S} &= {{\underline{U}^2}\over{\underline{Z}^*}} \\ \underline{S} &= {{(3.0 ~{\rm V})^2}\over{19.8 ~\Omega + j \cdot 17.1 ~\Omega}} \\ \underline{S} &= 0.345 ~{\rm VA} - j \cdot 0.300 ~{\rm VA} \\ S &= 0.457 ~{\rm VA} \\ \varphi &= -41.0^\circ \end{align*}$$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