task_6tqttque1e2nf2c7_with_calculation

Student Group

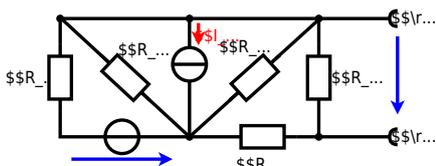| First Name	Surname	Matrikel Nr.

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dc network analysis, pure resistor network simplification, delta wye transformation, exam ee1 WS2022

Exercise 1.1 : Equivalent Linear Source

(written test, approx. 14% of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.

Calculated the internal resistance R_i and the source voltage U_s of an equivalent linear voltage source on the connectors A and B. \begin{align*} R_1=5.0 \Omega, && U_2=6.0 V, && R_3=10 \Omega, \\ I_4=4.2 A, && R_5=10 \Omega , && R_6=7.5 \Omega, \\ R_7=15 \Omega && && \end{align*} Use equivalent sources in order to simplify the circuit!

Solution

Best thing is to re-think the wiring like rubber bands and adjust them:
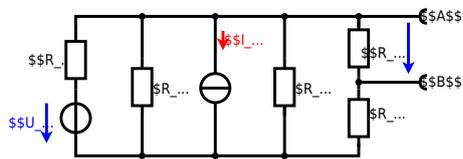
The linear voltage source of U_2 and R_1 can be transormed into a current source $I_2={{U_2}\over{R_1}}$ and R_1:
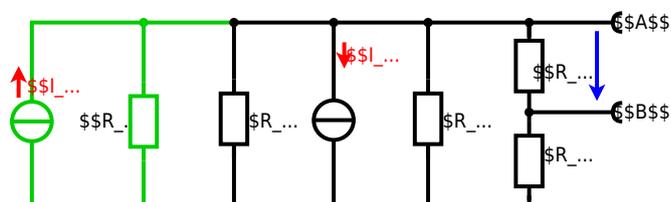
Now a lot of can be combined. The resistors R_1, R_3, R_5 are in parallel, like also I_2 and I_4: \begin{align*} R_{135} &= R_1||R_3||R_5\\ I_{24} &= I_2 - I_4 = {{U_2}\over{R_1}} - I_4 \end{align*} The resulting circuit can again be transformed:

Here, the U_{24} is calculated by I_{24} as the following: \begin{align*} U_{24} &= R_{135} \cdot I_{24} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot R_1||R_3||R_5 \end{align*}

On the right side of the last circuit there is a voltage divider given by R_{135}, R_6 and R_7.
Therefore the voltage between A and B is given as: \begin{align*} U_{AB} &= U_{24} \cdot {{R_7}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot {{R_7 \cdot R_1||R_3||R_5}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ \end{align*}

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit): \begin{align*} R_{AB} &= R_7 || (R_6 + R_1||R_3||R_5) \\ \end{align*}

with $R_1||R_3||R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

\begin{align*} U_{AB} &= ({{6.0 V}\over{5.0 \Omega}} - 4.2 \Omega) \cdot {{15 \Omega \cdot 2.5 \Omega}\over{7.5 \Omega + 15 \Omega + 2.5 \Omega}} \\ R_{AB} &= 15 \Omega|| (7.5 \Omega + 2.5 \Omega) \\ \end{align*}

Final result

\begin{align*} U_{AB} &= 4.5 V\\ R_{AB} &= 6 \Omega \end{align*}