

# Exam Summer Semester 2023

## Student Group

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**Exercise E4 Resistance of a Wire by Resistivity**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

A heating element made of solid nichrome wire with a diameter of  $d = 0.357 \text{ mm}$  is used in an electric power dissipation (= heat flow) of  $P = 40 \text{ W}$  is necessary. Determine the current  $I$  needed to operate for heating elements. The Nichrome wire has a resistivity of  $\rho = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m}$ .

The heating element is  $l = 3 \text{ m}$  long and has a diameter of  $d = 0.357 \text{ mm}$ .  
 Solution:  $R = \rho \cdot \frac{l}{A}$   
 ∴ Calculate the resistance  $R$  of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}}$$

$$R = \rho \cdot \frac{l}{A} \quad | \quad \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \quad \parallel \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \parallel \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi}$$

**Exercise E1 Resistance of a Wire by Resistivity**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

A heating element made of solid nichrome wire with a diameter of  $d = 0.357 \text{ mm}$  is used in an electric power dissipation (= heat flow) of  $P = 40 \text{ W}$  is necessary. Determine the current  $I$  needed to operate for heating elements. The Nichrome wire has a resistivity of  $\rho = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m}$ .

The heating element is  $l = 3 \text{ m}$  long and has a diameter of  $d = 0.357 \text{ mm}$ .  
 Solution:  $R = \rho \cdot \frac{l}{A}$   
 ∴ Calculate the resistance  $R$  of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}}$$

$$R = \rho \cdot \frac{l}{A} \quad | \quad \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \quad \parallel \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \parallel \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi}$$

**Exercise E1 Temperature-dependent Resistance**

**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator exhibits a temperature coefficient of resistance in a refrigeration system. The refrigerator has a resistance of  $10 \text{ k}\Omega$  at  $25^\circ\text{C}$ . Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

Result: The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

Calculate the resistance of the thermal sensor at  $-40^\circ\text{C}$ .

The power transfer resistor  $P$  is a part of the circuit and generates heat. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right)$$

**Exercise E2 Temperature-dependent Resistance**

**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator exhibits a temperature coefficient of resistance in a refrigeration system. The refrigerator has a resistance of  $10 \text{ k}\Omega$  at  $25^\circ\text{C}$ . Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

Result: The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

Calculate the resistance of the thermal sensor at  $-40^\circ\text{C}$ .

The power transfer resistor  $P$  is a part of the circuit and generates heat. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right)$$

**Exercise E6 Pure Resistor Network Simplification**  
**(written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved at once, the result is given.  $R_{AB}$  and the value between  $A$  and  $B$ .

Solution

$$R_{AB} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2 = R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{AB} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel (R_Y + R_2 + 100 \Omega)$$

The switch shall now be open. Calculate the equivalent resistance  $R_{AB}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel (500 \Omega) \parallel (200 \Omega) \parallel (500 \Omega \cdot 200 \Omega) / (500 \Omega + 200 \Omega)$$

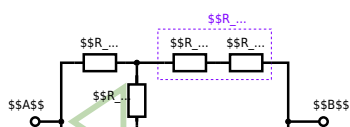
**Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved with  $R_1 = 200 \Omega$ ,  $R_2 = R_3 = 100 \Omega$  and the source  $B = 15 \text{ V}$ .  
 Result given:  $R_{\text{eq}} = B$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.

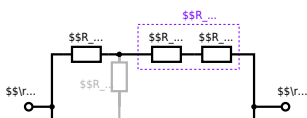


Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as: 
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series: 
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{eq}$  between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E1 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



Calculate the internal resistance  $R_{int}$  and the source voltage  $U_{s}$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3=10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$ . Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{135} + I_1 \cdot R_1$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0\Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5\Omega || 10\Omega || 10\Omega = 5\Omega || 5\Omega = 2.5\Omega$ :

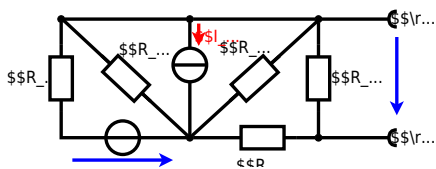
$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega}$$

$$R_{AB} = 15\Omega || (7.5\Omega + 2.5\Omega)$$

### Exercise E3 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.  
Result

$$U_s = U_{AB} = 4.5\text{V} \quad R_i = R_{AB} = 6\Omega$$



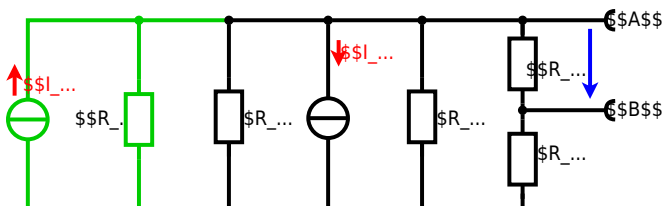
Calculate the internal resistance  $R_{int}$  and the source voltage  $U_s$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  
 $R_1=5.0 \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3=10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$   
 Use equivalent sources in order to simplify the circuit!

**Solution**

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{135} + I_{24} \cdot R_4$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} = \frac{6.0 \text{ V}}{7.5 \Omega + 15 \Omega + 2.5 \Omega} = 0.2 \text{ V}$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} = \frac{6.0 \text{ V}}{7.5 \Omega + 15 \Omega + 2.5 \Omega} = 0.2 \text{ V}$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5) = 5 \Omega || (10 \Omega + 10 \Omega) = 5 \Omega || 20 \Omega = 2.5 \Omega$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} = 0.2 \text{ V}$$

**Exercise E5 Charging Capacitors**  
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a RC circuit. It consists of a DC voltage source  $U$ , a resistor  $R_1$ , a resistor  $R_2$ , and a capacitor  $C$ . The switch  $S_1$  is initially open. The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution:** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

$$U_{eq} = \frac{U \cdot R_2}{R_1 + R_2} = \frac{12 \text{ V} \cdot 2 \Omega}{2 \Omega + 2 \Omega} = 3 \text{ V}$$

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .



The circuit contains a voltage source  $U=12\text{ V}$ , a switch  $S_1$ , a resistor of  $R_1=20\text{ }\Omega$  and a capacitor of  $C=100\text{ }\mu\text{F}$ . The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first task. At the moment  $t_0=0\text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0)=0\text{ V}$ .



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1)=0.5 \cdot U$ .

Solution



So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1)=0.5 \cdot U$ :  

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5$ .  
 An equivalent linear voltage source can be given with  $U_s = U \cdot \frac{R_B}{R_1 + R_B}$  and  $R_i = R_1 \parallel R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ . The internal resistance is given by substituting the ideal voltage source with its resistance ( $R_i = 0\text{ }\Omega$ , short-circuit).  

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1\text{ ms}/(10\text{ }\Omega \cdot 100\text{ }\mu\text{F})})$$

**Exercise E4 Charging Capacitors**  
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (as shown in the figure) consists of a DC voltage source  $U=6\text{ V}$ , a resistor  $R_1=20\text{ }\Omega$ , a capacitor  $C=20\text{ }\mu\text{F}$  and a light bulb  $R_B=20\text{ }\Omega$ . The switch  $S_1$  is open. The voltage across the capacitor is again  $0\text{ V}$  at the moment  $t_0=0\text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2=1\text{ ms}$  after closing the switch.

**Solution** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_B$ .  

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 3\text{ V}$$

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

Solution

The ideal voltage source is  $U = 12 \text{ V}$ . The internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .



The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ }\Omega$  and a capacitor of  $C = 100 \text{ }\mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first asks. At the moment  $t_0 = 0$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0$ .

First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .



Solution

An equivalent linear voltage source can be given with  $U$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ . The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \text{ }\Omega$ , short-circuit).  $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$

$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2/(10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$   
So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_C(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ :  $u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$   
It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5)$



The absolute value of the impedance is  $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$  and the phase angle is  $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$ .  
 With the complex part comes the physical value:  $X_L = \omega L$  and  $X_C = \frac{1}{\omega C}$ .  
 The phase  $\phi$  is given by  $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$ .

**Exercise E3 Impedances at different Frequencies**  
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with a resistor  $R_1 = 1.00 \text{ k}\Omega$ , a capacitor  $C_1 = 40 \text{ nF}$  and an inductor  $L_1 = 4.7 \text{ }\mu\text{H}$  connected in series, the current  $I = 10 \text{ mA}$  flows through the circuit. The resistor  $R_1$  shall have the same absolute value of the impedance as a capacitor  $C_2 = 40 \text{ nF}$  at  $f_2 = 4 \text{ MHz}$ .

**Solution**

$R_1 = 1.00 \text{ k}\Omega$   
 $R_2 = 10.0 \text{ k}\Omega$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $L$  combined is given by  $Z_{RL} = R + j\omega L$ .  
 Parallel circuit means that the voltage is the same on  $R_2$  and  $C_2$ .  
 $Z_{RC} = \frac{R_2 \cdot (-j/\omega C_2)}{R_2 - j/\omega C_2}$ . Since  $Z_{RC}$  is perpendicular to  $Z_{RL}$ , this can be simplified to  $Z_{RC} = \frac{R_2}{1 - j\omega C_2 R_2}$ .  
 $Z_{RC}$  is perpendicular to  $Z_{RL}$  (It has to, since  $R_2$  is perpendicular to  $j\omega L$  and  $-j/\omega C_2$  is perpendicular to  $R_2$ ).  
 Therefore, the resulting current of the parallel circuit is given as:  
 $I_{RC} = \frac{I_{R2} + I_{C2}}{\sqrt{1 + (\omega C_2 R_2)^2}}$   
 This can be rearranged to get  $R_2 = \frac{R_1 \sqrt{1 + (\omega C_2 R_2)^2}}{1 - (\omega C_2 R_2)^2}$ .  
 Back to the first formula:  $R_2 \cdot I_{RC} = X_{C2} \cdot I_{RC}$   
 $R_2 = X_{C2} \cdot \frac{1}{\sqrt{1 + (\omega C_2 R_2)^2}}$

**Exercise E6 Impedances at different Frequencies**  
 (written test, approx. 18 % of a 60-minute written test, WS2022)

**Resistor**  $R_1$  shall have the same absolute value of the impedance as a capacitor  $C_1 = 40 \text{ nF}$  at  $f_1 = 4 \text{ MHz}$ .

**Solution**

$$R_1 = 1.00 \text{ } \Omega$$

$$R_2 = 10.0 \text{ } \Omega$$

A series circuit means that the current is constant on every component.

The equivalent impedance for  $R$  and  $L$  combined is given by

$$Z_{RL} = R + j\omega L$$

Parallel circuit means that the voltage is the same on  $R_1$  and  $C_1$

$$\frac{1}{Z_{RC}} = \frac{1}{R_1} + \frac{1}{j\omega C_1}$$

Since  $R_1$  and  $C_1$  are perpendicular to each other, the resulting current of the parallel circuit is given as:

$$I_{RC} = I_{R1} + I_{C1}$$

This circuit is a parallel circuit

$$I_{RC} = \frac{U}{Z_{RC}} = \frac{U}{\sqrt{R_1^2 + \left(\frac{1}{\omega C_1}\right)^2}}$$

Back to the first formula:

$$R_3 \cdot I_{RC} = X_{C3} \cdot I_{RC}$$

$$R_3 = \frac{X_{C3} \cdot I_{RC}}{I_{RC}}$$

$$R_3 = \frac{1}{2\pi \cdot f \cdot C_3} \cdot \frac{1}{\sqrt{R_1^2 + \left(\frac{1}{\omega C_1}\right)^2}}$$

**Exercise E1 Complex Impedance Circuit**  
 (written test, approx. 15 % of a 60-minute written test, WS2022)

**1. Calculate the current  $i(t)$  through the resistor  $R_1$  in the circuit shown in the figure. The voltage source is  $u(t) = 3.0 \text{ V} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$ .**

**Solution**

Result

$$Z = 48.2 \text{ } \Omega \quad Z_C = 19.8 \text{ } \Omega$$

Draw the circuit diagram of the given circuit

$$Z = \frac{U}{I} \quad I = \frac{U}{Z}$$

$$Z_C = \frac{1}{2\pi \cdot f \cdot C}$$

Result

$$Z = \sqrt{R^2 + (Z_L - Z_C)^2}$$

$$Z = \sqrt{10^2 + (29.8 - 19.8)^2} = 19.8 \text{ } \Omega$$

$$Z = R + j(Z_L - Z_C) = 10 + j(29.8 - 19.8) = 10 + j10 \text{ } \Omega$$

$$Z = \sqrt{10^2 + 10^2} = 14.14 \text{ } \Omega$$





**Exercise E7 Complex Impedance Circuit**  
**(written test, approx. 15 % of a 60-minute written test, WS2022)**

1. Calculate the current  $i(t)$  in the circuit shown in Fig. 1. The voltage source is  $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V. The circuit consists of a resistor of  $10 \Omega$ , an inductor of  $330 \mu\text{H}$ , and a capacitor of  $0.22 \mu\text{F}$ , all in series.

Result:  $Z = 19.8 \Omega$

Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.

$$Z = \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} \quad Z_C = \frac{1}{2\pi \cdot f \cdot C} = \frac{1}{2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} \approx 382 \Omega$$

$$\hat{I} = \frac{\hat{U}}{Z} = \frac{3.0 \text{ V}}{19.8 \Omega} \approx 0.1515 \text{ A} \approx 151.5 \text{ mA}$$

$$i(t) = 151.5 \text{ mA} \cdot \sin(2\pi \cdot 15 \cdot t)$$

$$\underline{Z} = R + \underline{Z}_L + \underline{Z}_C = R + j\omega L - j\omega C$$

$$\underline{Z} = 10 \Omega + j(2\pi \cdot 15 \cdot 330 \cdot 10^{-6}) - j(2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6})$$

$$\underline{Z} = 10 \Omega + j0.0314 \Omega - j0.0207 \Omega = 10 \Omega + j0.0107 \Omega$$

$$|\underline{Z}| = \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2} = \sqrt{10^2 + 0.0107^2} \approx 10 \Omega$$

$$\hat{I} = \frac{\hat{U}}{|\underline{Z}|} = \frac{3.0 \text{ V}}{10 \Omega} = 0.3 \text{ A}$$

$$i(t) = 0.3 \text{ A} \cdot \sin(2\pi \cdot 15 \cdot t)$$

$$Z = \frac{U}{I} = \frac{3.0 \text{ V}}{0.3 \text{ A}} = 10 \Omega$$

$$Z_C = \frac{1}{2\pi \cdot f \cdot C} = \frac{1}{2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} \approx 382 \Omega$$

$$Z_L = \omega L = 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} \approx 0.0314 \Omega$$

$$\underline{Z} = R + jZ_L - jZ_C = 10 \Omega + j0.0314 \Omega - j382 \Omega$$

$$|\underline{Z}| = \sqrt{10^2 + (0.0314 - 382)^2} \approx 382 \Omega$$

$$\hat{I} = \frac{\hat{U}}{|\underline{Z}|} = \frac{3.0 \text{ V}}{382 \Omega} \approx 7.85 \text{ mA}$$

$$i(t) = 7.85 \text{ mA} \cdot \sin(2\pi \cdot 15 \cdot t)$$

$$Z = \frac{U}{I} = \frac{3.0 \text{ V}}{7.85 \text{ mA}} \approx 382 \Omega$$

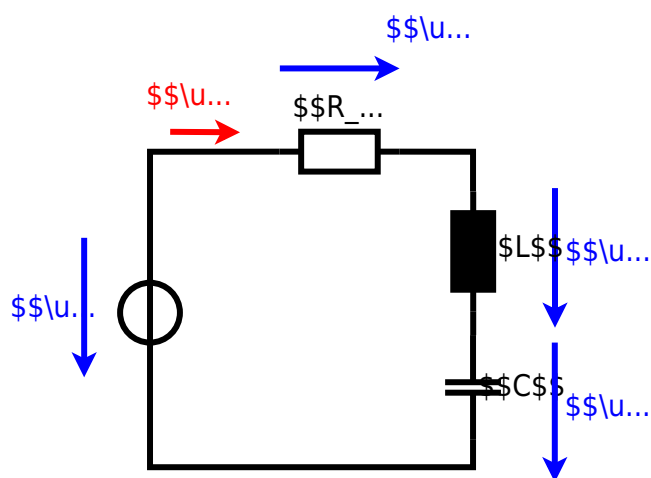
$$Z_C = \frac{1}{2\pi \cdot f \cdot C} = \frac{1}{2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} \approx 382 \Omega$$

$$Z_L = \omega L = 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} \approx 0.0314 \Omega$$

$$\underline{Z} = R + jZ_L - jZ_C = 10 \Omega + j0.0314 \Omega - j382 \Omega$$

$$|\underline{Z}| = \sqrt{10^2 + (0.0314 - 382)^2} \approx 382 \Omega$$





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Last update: **2023/07/24 17:53**

