task_7el8zlzjglaazxtw_with_calculation

Student Group

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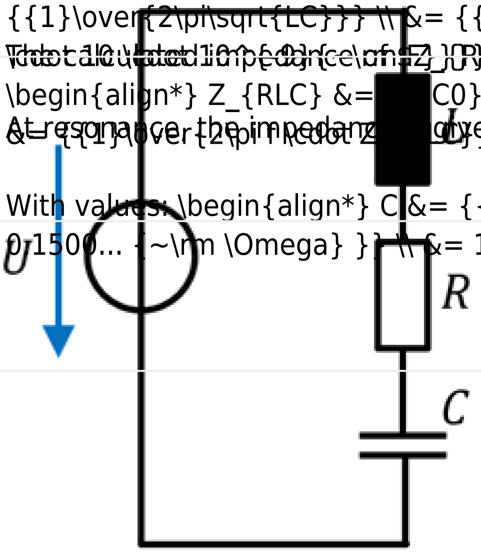
resonant circuit, exam ee2 SS2022

Exercise E1 Series Resonant Circuit
(written test, approx. 10 % of a 120-minute written test, SS2022)

A real capacitor can be modeled as an ideal capacity C in series with a resistance R and an inductance L.
A capacitor shall be given with the following values:

- $C=10 ~\rm nF$
- $R=88 ~\rm m\Omega$
- $L=60 ~\rm pH$



2. What is the resonance frequency of the circuit?

Path

The resonance frequency is given as \begin{align*} f_{\rm r} &= {{1}\over{2\pi\sqrt{LC}}} \\ &= {{1}\over{2\pi\sqrt{60 \cdot 10^{-12}{~\rm H} \cdot 10 \cdot 10^{-9}{~\rm F}}}} \end{align*}

At resonance, the impedance is given purely by the resistor.

Result

- $f_r = 205.5 ~\rm MHz$
- $Z_{RLC} = 88 ~\rm m\Omega$

3. The impedance magnitude of the circuit shall be replaced by an ideal capacity C_0. In this case, the magnitude of the impedance of C_0 would be $X_{C0}=Z_{RLC}$.
Which value would C_0 have for the given f_0?

Path

The calculated impedance Z_{RLC} has to be set equal to X_{C0}:
\begin{align*} Z_{RLC} &= X_{C0} \\ &= {{1}\over{2\pi f \cdot C}} \\ \rightarrow C &= {{1}\over{2\pi f \cdot Z_{RLC}}} \end{align*}

With values: \begin{align*} C &= {{1}\over{2\pi 100 \cdot 10^{6} {~\rm MHz}\cdot 0.1500... {~\rm \Omega} }} \\ &= 10.6... ~\rm nF \end{align*}

Result

- $C_0 = 10.6 ~\rm nF$

1. What is the impedance $\underline{Z}_{RLC}$ of this real capacitor for $f_0=100 ~\rm MHz$? (Phase and magnitude)

Path

The impedance $\underline{Z}_{RLC}$ is given by: \begin{align*} \underline{Z}_{RLC} &= R + \underline{X}_{L} + \underline{X}_{C} \\ &= R + {\rm j}\omega L - {{\rm j}\over{\omega C}} \\ &= R + {\rm j}\cdot \left(\omega L - {{1}\over{\omega C}} \right)\\ &= R + {\rm j}\cdot {X}_{LC} \\ \end{align*}

Putting in the numbers, only for the reactive part ${X}_{LC}$: \begin{align*} {X}_{LC} &= 2\pi\cdot \quad \quad f_0 \quad \quad \cdot \quad L \quad \quad \quad \; - {{1}\over{2\pi \cdot \quad \quad f_0 \quad \quad \cdot \quad \quad C \quad\quad}} \\ &= 2\pi \cdot 100 \cdot 10^{6}{~\rm Hz} \cdot 60 \cdot 10^{-12}{~\rm H} - {{1}\over{2\pi \cdot 100 \cdot 10^{6}{~\rm Hz} \cdot 10 \cdot 10^{-9}{~\rm F}}} \\ &= -121.45... ~\rm m\Omega \\ \end{align*}

With the real and imaginary parts, we can derive the magnitude and phase:
\begin{align*} Z_{RLC} &=\sqrt{R^2 + {X}_{LC}^2} \\ &=\sqrt{(88 ~\rm m\Omega)^2 + (-121.45 ~\rm m\Omega)^2} \\ &= 150.0... ~\rm m\Omega \\ \end{align*}

\begin{align*} \varphi &=\arctan \left({ {{{X}_{LC}}\over{R}}} \right)\\ &=\arctan \left({ {{-121.45 ~\rm m\Omega}\over{88 ~\rm m\Omega}}} \right)\\ &= -0.9437... = -54.07...° \\ \end{align*}