# ws2022_exam_r

## Student Group

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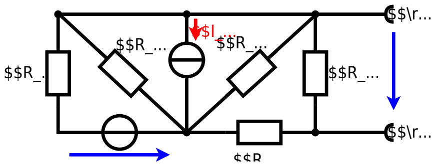
**Exercise E1 Equivalent linear Source**
**(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

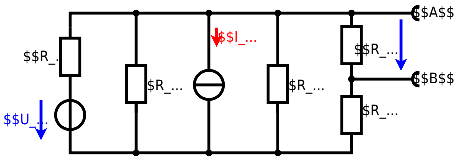
> \begin{align*} U_{\rm s} &= U_{\rm AB} &&= 4.5 ~\rm{V}\\ R_{\rm i} &= R_{\rm AB} &&= 6 ~\Omega \end{align*}



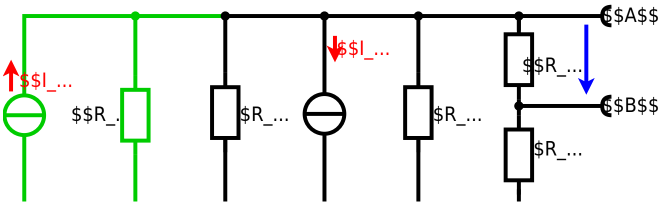

Calculated the internal resistance $R_\rm i$ and the source voltage $U_\rm s$ of an equivalent linear voltage source on the connectors $\rm A$ and $\rm B$. \begin{align*} R_1=5.0 ~\Omega, && U_2=6.0 ~\rm{V}, && R_3= 10 ~\Omega, \\ I_4=4.2 ~\rm{A}, && R_5=10 ~\Omega , && R_6=7.5 ~\Omega, \\ R_7=15 ~\Omega && && \end{align*} Use equivalent sources in order to simplify the circuit!

Solution

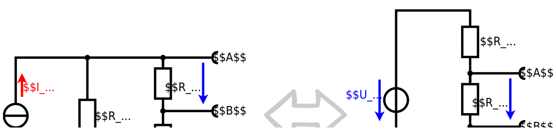
> The best thing is to re-think the wiring like rubber bands and adjust them:




The linear voltage source of $U_2$ and $R_1$ can be transformed into a current source $I_2={{U_2}\over{R_1}}$ and $R_1$:




Now a lot of them can be combined. The resistors $R_1$, $R_3$, $R_5$ are in parallel, like also $I_2$ and $I_4$: \begin{align*} R_{135} &= R_1||R_3||R_5\\ I_{24} &= I_2 - I_4 = {{U_2}\over{R_1}} - I_4 \end{align*} The resulting circuit can again be transformed:



Here, the $U_{24}$ is calculated by $I_{24}$ as the following: \begin{align*} U_{24}

&= R_{135} \cdot I_{24} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot R_1||R_3||R_5 \end{align*}

On the right side of the last circuit, there is a voltage divider given by $R_{135}$, $R_6$, and $R_7$.
Therefore the voltage between $A$ and $B$ is given as: \begin{align*} U_{\rm AB} &= U_{24} \cdot {{R_7}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot {{R_7 \cdot R_1||R_3||R_5}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ \end{align*}

For the internal resistance $R_i$ the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit): \begin{align*} R_{\rm AB} &= R_7 || ( R_6 + R_1||R_3||R_5) \\ \end{align*}

with $R_1||R_3||R_5 = 5 ~\Omega || 10 ~\Omega || 10 ~\Omega = 5 ~\Omega || 5 ~\Omega = 2.5 ~\Omega$:

\begin{align*} U_{\rm AB} &= ({{6.0 ~\rm{V}}\over{5.0 ~\Omega}} - 4.2 ~\Omega) \cdot {{15 ~\Omega \cdot 2.5 ~\Omega}\over{7.5 ~\Omega + 15 ~\Omega + 2.5 ~\Omega}} \\ R_{\rm AB} &= 15 ~\Omega|| ( 7.5 ~\Omega + 2.5 ~\Omega) \\ \end{align*}

**Exercise E1 Temperature-dependent Resistance**
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

A thermistor is used as a temperature sensor in a refrigeration system. The thermistor has a resistance of $10 ~\rm{k}\Omega$ at $+25 ~°\rm{C}$.
Its temperature coefficients are: $\alpha=0.01 ~{{1}\over{\rm{K}}}$ and $\beta=71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}}$
The temperature inside the refrigeration system can reach down to $-40 ~°\rm{C}$.

1. Calculate the resistance of the thermistor at $-40 ~°\rm{C}$.

Solution

\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \text{with } \Delta T = T_{\rm end} - T_{\rm start}\\ R &= 10 ~\rm{k}\Omega \cdot \left(1 + 0.01 ~{{1}\over{\rm{K}}} \cdot (-40~°\rm{C} - 25~°\rm{C} ) + 71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}} \cdot (-40~°\rm{C} - 25~°\rm{C})^2 \right) \\ \end{align*}

Result

\begin{align*} R &= 6.5 ~\rm{k}\Omega \\ \end{align*}

2. Additionally, explain ... Give an explanation for your answer.

Resistors transfer electrical energy out of the circuit and generate heat. Therefore, a resistor tends to heat up the refrigeration system.

Result

The power of the resistor $P = U \cdot I = R \cdot I^2 = {{U^2}\over{R}}$ is equivalent to the heat flow.
Therefore, with constant $U$ and increasing $R$ the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

**Exercise E2 Analyzing complex Impedances**
**(written test, approx. 14 % of a 60-minute written test, WS2022)**

2. Now a third ideal component $\underline{X}_2$ shall be connected in series to the two components. ($R$ and $\underline{X}_1$) shall be given.
This component shall be dimensioned in such a way that the current and voltage are in phase. Calculate the component value.

3. Calculate the phase and absolute value of the current through the circuit with $U = 50 ~V$, $f = 300 ~Hz$.

Result

After analysis, the following formula for the impedance was extracted \begin{align*} \underline{Z} = \left( {{2}\over{3+4{\rm j}}}+5{\rm j} \right) \Omega \end{align*}

Solution

1. Calculate the physical values of the two components.

\begin{align*} C = 103 ~\rm{µF} \end{align*}

Solution

\begin{align*} R &= 0.24 ~\Omega \\ L &= 2.26 ~\rm{mH} \end{align*}

\begin{align*} |\underline{I}| &= 10.67 ~\rm{A} \\ \varphi_i &= -87.06° \end{align*}

Solution

\begin{align*} \underline{I} &= {{\underline{U}}\over{\underline{Z}}} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \cdot {{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }\over{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }} \end{align*}

The current and voltage are in phase once there is only a pure ohmic (= pure real) resulting impedance $\underline{Z} + \underline{X}_2$.
Therefore, the component mus be a capacitor with the same absolute value of impedance: $|\underline{X}_C| = |\underline{X}_L|$ \begin{align*} X_C &= {{1}\over{\omega \cdot C}} = X_L \\ C &= {{1}\over{\omega \cdot X_L}} \\ &= {{1}\over{2\pi \cdot 300 ~\rm{Hz} \cdot 4.68 ~\Omega}} \\ \end{align*}

\begin{align*} \underline{Z} &= \left( {{2}\over{3+4{\rm j}}} + 5{\rm j} \right) ~\Omega \\ &= \left( {{2}\over{3+4{\rm j}}} \cdot {{3-4{\rm j}}\over{3-4{\rm j}}} + 5{\rm j} \right) ~\Omega \\ &= \left( {{2}\over{9+16 }} \cdot (3-4{\rm j} ) + 5{\rm j} \right) ~\Omega \\ &= (0.24 - 0.32{\rm j} + 5{\rm j}) ~\Omega \\ &= 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega \\ &= R + {\rm j} X_L \\ \end{align*}

The absolute value $|\underline{I}|$ can be calculated as \begin{align*} |\underline{I}| &= {{50 ~\rm{V}}\over{| 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega |}} \\ &= {{50 ~\rm{V}}\over{\sqrt{ (0.24 ~\Omega)^2 + (4.68 ~\Omega)^2 }}} \end{align*}

With the complex part comes the physical value: \begin{align*} X_L &= \omega L \\ L &= {{X_L}\over{2\pi \cdot f}} \\ &= {{4.68 ~\Omega}\over{2\pi \cdot 300 ~\rm{Hz}}} \\ \end{align*}

The phase $\varphi_i$ can be calculated as \begin{align*} \varphi_i &= \arctan \left( {{\Im()}\over{\Re()}} \right) \\ &= \arctan \left( {{-4.68 ~\Omega}\over{0.24 ~\Omega}} \right) \\ \end{align*}

**Exercise E1 Complex Impedance Circuit**
**(written test, approx. 15 % of a 60-minute written test, WS2022)**

2. Calculate the impedances $|\underline{Z}_L|$, $|\underline{Z}_C|$, and $|\underline{Z}|$, and the current $I$ of the circuit.

3. Draw the complete phasor diagram of $\underline{Z}_L$, $\underline{Z}_C$, $\underline{Z}_R$ and $\underline{Z}$ such that $\underline{Z}$ is given by the diagram.
Choose an appropriate scaling factor and write it down.

Result

A linear voltage source is given by the voltage $u(t) = 3.0 ~{\rm V} \cdot \sin(2\pi \cdot 15 ~{\rm kHz} \cdot t)$ with an internal resistance of $10 ~\Omega$.
This linear source is connected with an inductor of $330 ~ {\rm µH}$ and a capacitor of $0.22 ~{\rm µF}$, all in series.

Solution

Solution

Result

\begin{align*} I = 107.3 {\rm mA} \end{align*}

\begin{align*} Z_L &= 31.1 ~\Omega \\ Z_C &= 48.2 ~\Omega \\ Z &= 19.8 ~\Omega \end{align*}

1. Draw the circuit diagram of the given circuit.
Label all components, voltages, and currents.

\begin{align*} Z &= {{\hat{U}}\over{\hat{I}}} \\ \hat{I} &= {{\hat{U}}\over{Z}} \\ \end{align*}

\begin{align*} Z_C &= {{1}\over{2\pi \cdot f \cdot C}}\\ &= {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}}}\\ \end{align*}

Result

With $I = {{1}\over{\sqrt{2}}}\cdot \hat{I}$: \begin{align*} I &= {{1}\over{\sqrt{2}}}\cdot {{\hat{U}}\over{Z}} \\ &= {{1}\over{\sqrt{2}}}\cdot {{3.0 ~{\rm V}}\over{19.28 ~\Omega}} \\ \end{align*}

\begin{align*} Z_L &= 2\pi \cdot f \cdot L \\ &= 2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}\\ \end{align*}

\begin{align*} Z_C &= {{1}\over{2\pi \cdot f \cdot C}}\\ &= {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 330 ~{\rm µH}}}\\ \end{align*}

$$\begin{align*} \underline{Z} &= R + \underline{Z}_L + \underline{Z}_C \\ &= R + j \cdot {Z}_L - j \cdot {Z}_C \\ &= R + j \cdot ({Z}_L - {Z}_C) \\ |\underline{Z}| &= \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2 }\\ \end{align*}$$

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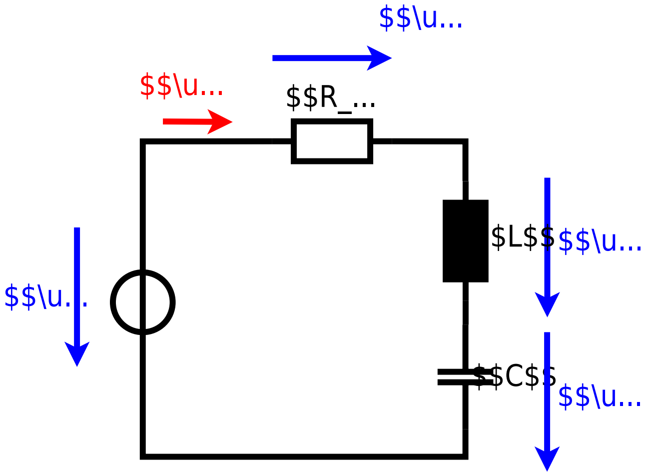
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$$\u...
$$R_...
$L$$
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## Exercise E3 Impedances at different Frequencies
## (written test, approx. 18 % of a 60-minute written test, WS2022)

Calculate the resistor values which have to be used in the following circuits.

1. A resistor $R_1$ shall have the same absolute value of the impedance as a capacitor $C_1=40 ~{\rm nF}$ at $f_1=4 ~{\rm MHz}$.

2. A $RL$ series circuit with $L_2=4.7 ~\rm µH$ is used in an AC voltage source with $U_2=1.0 ~\rm V$, $f_2=450 ~\rm kHz$, which generates a current of $I_2=60 ~\rm mA$.

3. A $RC$ parallel circuit with $C_3$ [illegible] with $I_{3}=1.1 ~\rm A$, $f_3=200$ [illegible] generates a current $I_{3R}=1.0 ~\rm A$ through $R_3$.

Solution

\begin{align*} R_1 &= 1.00 ~\Omega \\ \end{align*}

Result

\begin{align*} R_2 &= 10.0 ~\Omega \\ \end{align*}

Solution

\begin{align*} R_1 &= {\underline{X}_{C1}} \\ &= {{1}\over{2\pi \cdot f_1\cdot C_1}} \\ &= {{1}\over{2\pi \cdot 4 ~{\rm MHz} \cdot 40 ~\rm nF}} \\ \end{align*}

Solution

A series circuit means that the current is constant on every component.

The equivalent impedance for $R$ and $L$ combined is given by \begin{align*} {{\underline{U}}\over{\underline{I}}} &= R_2 + \underline{X}_{L2} \\ &= R_2 + {\rm j} \cdot \omega L \end{align*} Since ${\rm j} \cdot \omega L$ is perpendicular to $R_2$ this can be simplified to: \begin{align*} \left| {{\underline{U}}\over{\underline{I}}} \right|^2 &= |R_2|^2 + |\underline{X}_{L2}|^2 \\ \left( {{U}\over{I}} \right)^2 &= {R_2}^2 + {X_{L2}}^2 \\ \end{align*}

This can be rearranged to get $R_2$: \begin{align*} R_2 &= \sqrt{ \left( {{U}\over{I}} \right)^2 - {X_{L2}}^2 } \\ &= \sqrt{ \left( {{1~\rm V}\over{60~\rm mA}} \right)^2 - (2\pi \cdot 450~{\rm kHz} \cdot 4.7 ~ {\rm µH})^2 } \\ \end{align*}

Parallel circuit means that the voltage is the same on $R_3$ and $C_3$.

$\underline{I}_{3R}$ is perpendicular to $\underline{I}_{3C}$ (It has to, since $R_3$ is perpendicular to ${\rm j}\cdot{X}_{3C}$, too)

Therefore, the resulting current of the parallel circuit is given as: \begin{align*} \underline{I}_{3} &= \underline{I}_{3R} + \underline{I}_{3C} \\ |\underline{I}_{3}|^2 &= |\underline{I}_{3R}|^2 + |\underline{I}_{3C}|^2 \\ \end{align*}

Back to the first formula: \begin{align*} R_3 \cdot {I}_{3R} &= {X}_{3C} \cdot {I}_{3C} \\ R_3 &= {X}_{3C} \cdot {{{I}_{3C}}\over{{I}_{3R}}} \\ &= {{1}\over{2\pi \cdot f \cdot C_3}} \cdot {{\sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2}}\over{{I}_{3R}}} \\ \end{align*}

## Exercise E4 Resistance of a Wire by Resistivity
## (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of a Nichrome wire with a round cross-section is used in an electric oven.

The heating element needs to heat the oven to a temperature of $180~°\rm{C}$. For this a power dissipation (= heat flow) of $P=40 ~\rm{W}$ is necessary.

Nichrome is a common Nickel-Chrome alloy for heating elements.

The Nichrome wire has a resistivity of $1.10\cdot 10^{-6} ~\Omega \rm{m}$.

The heating element is $3 ~\rm{m}$ long and has a diameter of $3.57 ~\rm{mm}$.

1. Calculate the resistance $R$ of the heating element.

2. Calculate the current needed to operate it.

Result

Solution

\begin{align*} R=&110.33m ~\Omega \end{align*}

Solution

\begin{align*} P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I=

\sqrt{\frac{P}{R}} = \sqrt{\frac{40 ~\rm{W}}{0.33 ~\Omega}} \end{align*}
\begin{align*} R &= \rho \cdot \frac{l}{A} && | \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} && \\ R &= 1.10\cdot 10^{-6} ~\Omega \rm{m} \cdot \frac{4 \cdot 3~\rm{m}}{(3.57\cdot 10^{-3}~\rm{m})^2 \cdot \pi} && \\ \end{align*}

**Exercise E5 Charging Capacitors**
**(written test, approx. 16 % of a 60-minute written test, WS2022)**

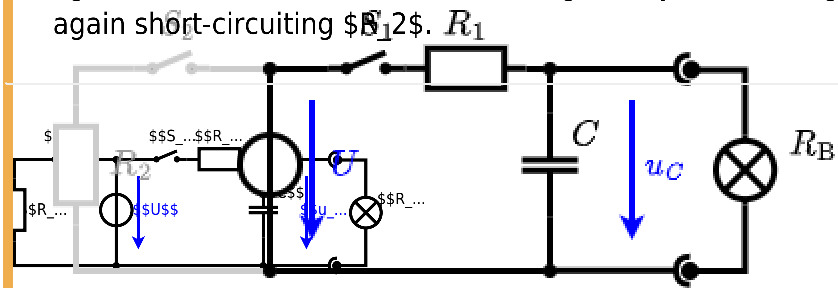
... The voltage across the capacitor is again $0 ~\rm{V}$ at the moment $t_0=0 ~\rm{s}$ when the switch $S_1$ is closed. Calculate the voltage $u_c (t_2)$ across the capacitor at $t_2=1 ~\rm{ms}$ after closing the switch.

Hint: To solve this, first create an equivalent linear voltage source from $U$, $R_1$, and $R_\rm B$.

Solution

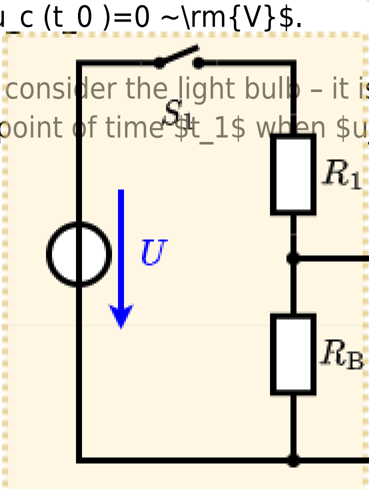
... $U=12~\rm{V}$ independent of this resistor.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting $R_2$.



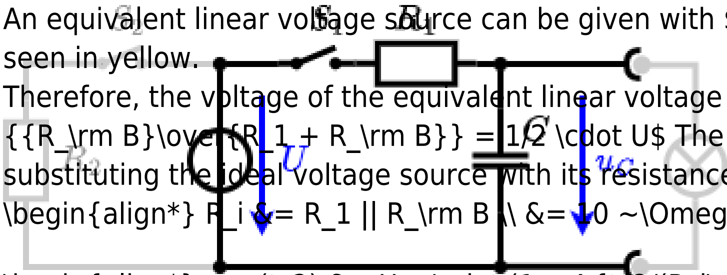

The circuit contains a voltage source $U=12 ~\rm{V}$, a switch $S_1$, a resistor of $R_1=20 ~\Omega$ and a capacitor of $C=100 ~\rm{µF}$.
The switch $S_2$ to an additional consumer $R_2$ will be considered to be open for the first tasks. At the moment $t_0=0 ~\rm{s}$ the switch $S_1$ is closed, the voltage across the capacitor is $u_c (t_0 )=0 ~\rm{V}$.

1. First do not consider the light bulb – it is not connected to the RC circuit.
Calculate the point of time $t_1$ when $u_c (t_1)=0.5\cdot U$.




Solution

An equivalent linear voltage source can be given with $U$, $R_1$, and $R_\rm B$ as seen in yellow.
Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot {{R_\rm B}\over{R_1 + R_\rm B}} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 ~\Omega$, short-circuit). \begin{align*} R_i &= R_1 || R_\rm B \\ &= 10 ~\Omega \end{align*}




So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

\begin{align*} u_c (t_2) &= U_s \cdot (1- e^{t_2/(R_i\cdot C)}) \\ &= {{1}\over{2}} \cdot U \cdot (1- e^{1~\rm{ms}/(10 ~\Omega \cdot 100 ~\rm{µF})}) \end{align*}

The following formula describes the time course of $u_C(t)$ which has to be $u_c (t_1)=0.5\cdot U$: \begin{align*} u_c (t) = U \cdot (1- e^{t/\tau}) = 0.5\cdot U \end{align*} It has to be rearranged to $t$ \begin{align*} (1- e^{t/\tau}) &= 0.5 \\ e^{t/\tau} &= 0.5 \\ t/\tau &= ln(0.5) \\ t &= \tau \cdot ln(0.5) \\ t &= R_1 \cdot C \cdot ln(0.5) \end{align*}

**Exercise E6 Pure Resistor Network Simplification**
**(written test, approx. 13 % of a 60-minute written test, WS2022)**

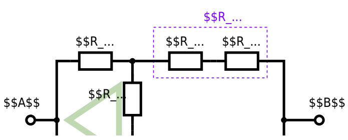
The following circuit with $R_1=200 ~\Omega$, $R_2=R_3=100 ~\Omega$ [illegible] is given.
2. The switch shall now be closed. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Result

Solution

\begin{align*} R_{\rm eq} &= 142.8 ~\Omega \\ \end{align*}

Now a wye-delta transformation is necessary.

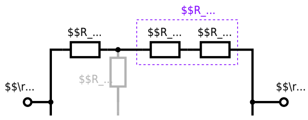


Since $R_2=R_3$ and based on the equations for the transformation, the transformed $R_Y$ is given as: \begin{align*} R_{Y} &= {{R_2 \cdot R_2}\over{R_2 + R_2 + R_2}} \\ &= {{(100 ~\Omega)^2}\over{3 \cdot 100 ~\Omega}} \\ &= {{1}\over{3}} \cdot 100 ~\Omega = 33.33 ~\Omega \end{align*}

The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= R_Y + (R_Y + R_1 + R_1)||(R_Y + R_2)\\ R_{\rm eq} &= 33.33 ~\Omega + (33.33 ~\Omega + 400 ~\Omega)||(33.33 ~\Omega + 100 ~\Omega)\\ \end{align*}

1. The switch shall now be open. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Solution




The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= (R_2 + R_1 + R_1)||(R_2 + R_2)\\ R_{\rm eq} &= (100 ~\Omega + 200 ~\Omega + 200 ~\Omega )&&||(100 ~\Omega + 100 ~\Omega ) &&\\ R_{\rm eq} &= (500 ~\Omega )&&||(200 ~\Omega )&&\\ R_{\rm eq} &= {{500 ~\Omega \cdot 200 ~\Omega }\over{500 ~\Omega + 200 ~\Omega}}&&\\ \end{align*}