dummy

Student Group

First Name	Surname	Matrikel Nr.

Table of Contents

capacitors, rc circuit, transient response, energy storage, industrial safety, chapter1 1

Exercise E1 Machine-Vision Strobe Unit: Charging and Safe Discharge of a Flash Capacitor

A machine-vision inspection system on a production line uses a short high-voltage flash pulse. For this purpose, an energy-storage capacitor is charged from a DC source and must be safely discharged before maintenance.

Data: $$\begin{align*} C &= 1~{\rm \mu F} \\ W_e &= 0.1~{\rm J} \\ I_{\rm max} &= 100~{\rm mA} \\ R_i &= 10~{\rm M\Omega} \end{align*}$$

1. What voltage must the capacitor have so that it stores the required energy?

SolutionResult

$$\begin{align*} W_e &= \frac{1}{2} C U^2 \\ U &= \sqrt{\frac{2W_e}{C}} = \sqrt{\frac{2 \cdot 0.1~{\rm J}}{1 \cdot 10^{-6}~{\rm F}}} \\ &= \sqrt{200000}~{\rm V} \approx 447.2~{\rm V} \end{align*}$$

$$\begin{align*} U = 447.2~{\rm V} \end{align*}$$

2. The charging current must not exceed $100~\rm mA$ at the start of charging. What charging resistor is required?

SolutionResult

At the beginning of charging, the capacitor behaves like a short circuit, so $$\begin{align*} i_{C{\rm max}} = i_C(t=0) = \frac{U}{R} \end{align*}$$ Thus, $$\begin{align*} R &\ge \frac{U}{I_{\rm max}} = \frac{447.2~{\rm V}}{0.1~{\rm A}} \\ &\approx 4472~{\rm \Omega} = 4.47~{\rm k\Omega} \end{align*}$$

$$\begin{align*} R \ge 4.47~{\rm k\Omega} \end{align*}$$

3. How long does the charging process take until the capacitor is practically fully charged?

SolutionResult

The time constant is
$$\begin{align*} T = RC = 4.47~{\rm k\Omega} \cdot 1~{\rm \mu F} = 4.47~{\rm ms} \end{align*}$$
In engineering practice, a capacitor is considered practically fully charged after about $5T$:
$$\begin{align*} t \approx 5T = 5 \cdot 4.47~{\rm ms} = 22.35~{\rm ms} \end{align*}$$

$$\begin{align*} t \approx 22.35~{\rm ms} \end{align*}$$

4. Give the time-dependent capacitor voltage and the voltage across the charging resistor.

SolutionResult

For the charging process:
$$\begin{align*} u_C(t) &= U\left(1-e^{-t/T}\right) \\ u_R(t) &= Ue^{-t/T} \end{align*}$$
with
$$\begin{align*} U &= 447.2~{\rm V} \\ T &= 4.47~{\rm ms} \end{align*}$$
So the capacitor voltage rises exponentially from 0 to $447.2~\rm V$, while the resistor voltage falls exponentially from $447.2~\rm V$ to 0.

$$\begin{align*} u_C(t) &= 447.2\left(1-e^{-t/4.47{\rm ms}}\right)~{\rm V} \\ u_R(t) &= 447.2\,e^{-t/4.47{\rm ms}}~{\rm V} \end{align*}$$

5. After charging, the capacitor is disconnected from the source. Its leakage can be modeled

by an internal resistance of $10~\rm M\Omega$. After what time has the stored energy dropped to one half, and what is the capacitor voltage then?

SolutionResult

Half the energy means \begin{align*} W_e' = 0.5W_e \end{align*} Since \begin{align*} W_e = \frac{1}{2}CU^2 \end{align*} the voltage at half energy is \begin{align*} U' = \frac{U}{\sqrt{2}} = \frac{447.2~{\rm V}}{\sqrt{2}} = 316.2~{\rm V} \end{align*} For the discharge through the internal resistance: \begin{align*} u_C(t) = Ue^{-t/T_2} \end{align*} with \begin{align*} T_2 = R_iC = 10~{\rm M\Omega} \cdot 1~{\rm \mu F} = 10~{\rm s} \end{align*} Set $u_C(t)=U'$: \begin{align*} Ue^{-t/T_2} &= U' \\ t &= T_2 \ln\left(\frac{U}{U'}\right) \\ &= 10~{\rm s}\cdot\ln\left(\frac{447.2}{316.2}\right) \\ &\approx 3.47~{\rm s} \end{align*}

\begin{align*} U' = 316.2~{\rm V} \\ t = 3.47~{\rm s} \end{align*}

6. The fully charged capacitor is discharged through the charging resistor before maintenance. How long does the discharge take, and how much energy is converted into heat in the resistor?

SolutionResult

The discharge time constant through the same resistor is again \begin{align*} T = RC = 4.47~{\rm ms} \end{align*} Thus the practical

\begin{align*} t \approx 22.35~{\rm ms} \\ W_R = 0.1~{\rm Ws} \end{align*}

discharge time is
$$\begin{align*} t \approx 5T = 22.35~{\rm ms} \end{align*}$$
The complete stored capacitor energy is converted into heat in the resistor:
$$\begin{align*} W_R = W_e = 0.1~{\rm Ws} \end{align*}$$

rc circuit, thevenin equivalent, transient response, sensor interface, industrial electronics, chapter1 1

Exercise E2 Sensor Input Buffer: Source, T-Network and Capacitor

A 12 V industrial sensor electronics unit feeds a buffered measurement node through a resistor T-network. A capacitor smooths the node voltage. At first, the load is disconnected. After the capacitor is fully charged, a measurement load is connected by a switch.

Data:
$$\begin{align*} U &= 12~{\rm V} \\ R_1 &= 2~{\rm k\Omega} \\ R_2 &= 10~{\rm k\Omega} \\ R_3 &= 3.33~{\rm k\Omega} \\ C &= 2~{\rm \mu F} \\ R_L &= 5~{\rm k\Omega} \end{align*}$$

Initially, the capacitor is uncharged and the switch is open.

1. What is the capacitor voltage after it is fully charged?

SolutionResult

Using the equivalent voltage source of the network on the left-hand side, the open-circuit voltage is
$$\begin{align*} U_{0e} &= \frac{R_2}{R_1+R_2}\,U \\ &= \frac{10~{\rm k\Omega}}{2~{\rm k\Omega}+10~{\rm k\Omega}}\cdot 12~{\rm V} \\ &= 10~{\rm V} \end{align*}$$
After full charging, the capacitor voltage equals this voltage.

$$\begin{align*} U_C = U_{0e} = 10~{\rm V} \end{align*}$$

2. How long does the charging process take?

SolutionResult

The internal resistance seen by the capacitor is
$$\begin{align*} R_{ie} &= R_3 + (R_1 \parallel R_2) \\ &= 3.33~{\rm k\Omega} + \frac{2~{\rm k\Omega}\cdot 10~{\rm k\Omega}}{2~{\rm k\Omega}+10~{\rm k\Omega}} \\ &= 3.33~{\rm k\Omega} + 1.67~{\rm k\Omega} \\ &= 5.00~{\rm k\Omega} \end{align*}$$
So the time constant is
$$\begin{align*} T &= R_{ie}C = 5.00~{\rm k\Omega}\cdot 2~{\rm \mu F} = 10~{\rm ms} \end{align*}$$
Practical charging time:
$$\begin{align*} t \approx 5T = 50~{\rm ms} \end{align*}$$

$$\begin{align*} R_{ie} = 5.00~{\rm k\Omega} \\ t \approx 50~{\rm ms} \end{align*}$$

3. Give the time-dependent capacitor voltage.

SolutionResult

The charging law is
$$\begin{align*} u_C(t) &= U_{0e}\left(1-e^{-t/T}\right) \\ &= 10\left(1-e^{-t/10{\rm ms}}\right)~{\rm V} \end{align*}$$
So the capacitor voltage rises exponentially from $0~\rm V$ to $10~\rm V$.

$$\begin{align*} u_C(t) = 10\left(1-e^{-t/10{\rm ms}}\right)~{\rm V} \end{align*}$$

4. After the capacitor is fully charged, the switch is closed and the load resistor is connected. What is the stationary load voltage?

SolutionResult

Now use a second equivalent voltage-source step. The Thevenin source seen by the load has \begin{align*} U_{0e} &= 10~{\rm V} \\ R_{ie} &= 5.00~{\rm k\Omega} \end{align*} Thus, the stationary load voltage is \begin{align*} U_C' = U_{0e}' &= \frac{R_L}{R_{ie}+R_L}\,U_{0e} \\ &= \frac{5~{\rm k\Omega}}{5~{\rm k\Omega}+5~{\rm k\Omega}}\cdot 10~{\rm V} \\ &= 5~{\rm V} \end{align*}

\begin{align*} U_L = 5~{\rm V} \end{align*}

5. How long does it take until this new stationary state is practically reached?

SolutionResult

The new internal resistance is \begin{align*} R_{ie}' &= R_{ie}\parallel R_L \\ &= 5.00~{\rm k\Omega}\parallel 5.00~{\rm k\Omega} \\ &= 2.50~{\rm k\Omega} \end{align*} Hence the new time constant is \begin{align*} T' &= R_{ie}'C = 2.50~{\rm k\Omega}\cdot 2~{\rm \mu F} = 5~{\rm ms} \end{align*} Practical settling time: \begin{align*} t \approx 5T' = 25~{\rm ms} \end{align*}

\begin{align*} R_{ie}' = 2.50~{\rm k\Omega} \\ t \approx 25~{\rm ms} \end{align*}

6. Give the time-dependent load voltage after the switch is closed.

SolutionResult

At the switching instant, the capacitor voltage cannot jump. Therefore: \begin{align*} u_L(0^+) &= 10~{\rm V} \\ u_L(\infty) &= 5~{\rm V} \end{align*} The voltage therefore decays exponentially toward the new final value: \begin{align*} u_L(t) &= u_L(\infty) + \left(u_L(0^+)-u_L(\infty)\right)e^{-t/T'} \\ &= 5 + 5e^{-t/5{\rm ms}}~{\rm V} \end{align*}

\begin{align*} u_L(t) = 5 + 5e^{-t/5{\rm ms}}~{\rm V} \end{align*}

inductors, air core coil, magnetic field, hall sensor, transient response, current density, chapter1 1

Exercise E3 Hall-Sensor Calibration Coil: Short Air-Core Coil

A Hall-sensor calibration bench uses a short air-core coil to create a defined magnetic field. An air-core coil is chosen because it avoids hysteresis and remanence effects. The coil is wound as a short cylindrical coil.

Data: \begin{align*} l &= 22~{\rm mm} \\ d &= 20~{\rm mm} \\ d_{\rm Cu} &= 0.8~{\rm mm} \\ N &= 25 \\ \rho_{\rm Cu,20^\circ C} &= 0.0178~{\rm \Omega\,mm^2/m} \end{align*}

A DC current of $1~\rm A$ shall flow through the coil.

1. Calculate the coil resistance R at room temperature.

SolutionResult

The wire cross section is

\begin{align*} R = 55.6~{\rm m\Omega} \end{align*}

\begin{align*} A_{\rm Cu} &= \pi\left(\frac{d_{\rm Cu}}{2}\right)^2 = \pi(0.4~{\rm mm})^2 \\ &= 0.503~{\rm mm^2} \end{align*} The total wire length is approximated by the number of turns times the circumference: \begin{align*} l_{\rm Cu} &= N\pi d \\ &= 25\pi \cdot 20~{\rm mm} \\ &= 1570.8~{\rm mm} = 1.571~{\rm m} \end{align*} Thus, \begin{align*} R &= \rho_{\rm Cu}\frac{l_{\rm Cu}}{A_{\rm Cu}} \\ &= 0.0178~{\rm \Omega\,mm^2/m}\cdot\frac{1.571~{\rm m}}{0.503~{\rm mm^2}} \\ &\approx 0.0556~{\rm \Omega} \end{align*}

2. Calculate the coil inductance L.

SolutionResult

For this short air-core coil, use \begin{align*} L = N^2 \cdot \frac{\mu_0 A}{l}\cdot\frac{1}{1+\frac{d}{2l}} \end{align*} with \begin{align*} A &= \pi\left(\frac{d}{2}\right)^2 = \pi(10~{\rm mm})^2 = 314.16~{\rm mm^2} = 3.1416\cdot 10^{-4}~{\rm m^2} \\ \mu_0 &= 4\pi\cdot 10^{-7}~{\rm Vs/(Am)} \end{align*} Therefore, \begin{align*} L &= 25^2 \cdot \frac{4\pi\cdot 10^{-7}\,{\rm Vs/(Am)} \cdot 3.1416\cdot 10^{-4}~{\rm m^2}}{22\cdot 10^{-3}~{\rm m}} \cdot \frac{1}{1+\frac{20}{2\cdot 22}} \\ &\approx 7.71\cdot 10^{-6}~{\rm H} \end{align*}

\begin{align*} L = 7.71~{\rm \mu H} \end{align*}

3. Which DC voltage must be applied so that the stationary current becomes $I=1~\rm A$? How large is the current density j in the copper wire?

SolutionResult

In the stationary DC state, the coil behaves like its ohmic resistance: \begin{align*} U &= RI \\ &= 55.6~{\rm m\Omega}\cdot 1~{\rm A} \\ &= 55.6~{\rm mV} \end{align*} The current density is \begin{align*} j &= \frac{I}{A_{\rm Cu}} \\ &= \frac{1~{\rm A}}{0.503~{\rm mm^2}} \\ &\approx 1.99~{\rm A/mm^2} \end{align*}

\begin{align*} U = 55.6~{\rm mV} \\ j = 1.99~{\rm A/mm^2} \end{align*}

4. How much magnetic energy is stored in the coil in the stationary state?

SolutionResult

\begin{align*} W_m &= \frac{1}{2}LI^2 \\ &= \frac{1}{2}\cdot 7.71\cdot 10^{-6}~{\rm H}\cdot (1~{\rm A})^2 \\ &= 3.86\cdot 10^{-6}~{\rm Ws} \end{align*}

\begin{align*} W_m = 3.86\cdot 10^{-6}~{\rm Ws} \end{align*}

5. Give the time-dependent coil current $i(t)$ when the coil is switched on.

SolutionResult

A coil current cannot jump instantly. It starts at 0 and approaches the final value $I=1~\rm A$ exponentially: \begin{align*} i(t) = I\left(1-e^{-t/T}\right) \end{align*} So the sketch starts at $0~\rm A$, rises quickly, and then slowly approaches $1~\rm A$.

\begin{align*} i(t) = 1\left(1-e^{-t/T}\right)~{\rm A} \end{align*}

6. How long does it take until the current has practically reached its stationary value?

SolutionResult

The time constant is \begin{align*} T &= \frac{L}{R} = \frac{7.71~{\rm \mu H}}{55.6~{\rm m\Omega}} \\ &\approx 138.9~{\rm \mu s} \end{align*} A practical final value is reached after about $5T$: \begin{align*} t \approx 5T = 5\cdot 138.9~{\rm \mu s} \approx 695~{\rm \mu s} \end{align*}

\begin{align*} t \approx 695~{\rm \mu s} \end{align*}

7. How much energy is dissipated as heat in the coil resistance during the current build-up?

SolutionResult

Using the current from task 5, \begin{align*} i(t) = I\left(1-e^{-

\begin{align*} W_R \approx 27.05\cdot 10^{-6}~{\rm Ws} t/T}\right) \end{align*} the heat dissipated in the winding resistance up to the practical final time $5T$ is \begin{align*} W_R &= \int_0^{5T} R\,i^2(t)\,dt \\ &= R I^2 \int_0^{5T} \left(1-e^{-t/T}\right)^2 dt \end{align*} For this interval, the integral is approximately \begin{align*} \int_0^{5T} \left(1-e^{-t/T}\right)^2 dt \approx \frac{7}{2}T \end{align*} Thus, \begin{align*} W_R &\approx RI^2\cdot \frac{7}{2}T \\ &= 0.0556~{\rm \Omega}\cdot (1~{\rm A})^2 \cdot \frac{7}{2}\cdot 138.9~{\rm \mu s} \\ &\approx 27.05\cdot 10^{-6}~{\rm Ws} \end{align*}

\end{align*}